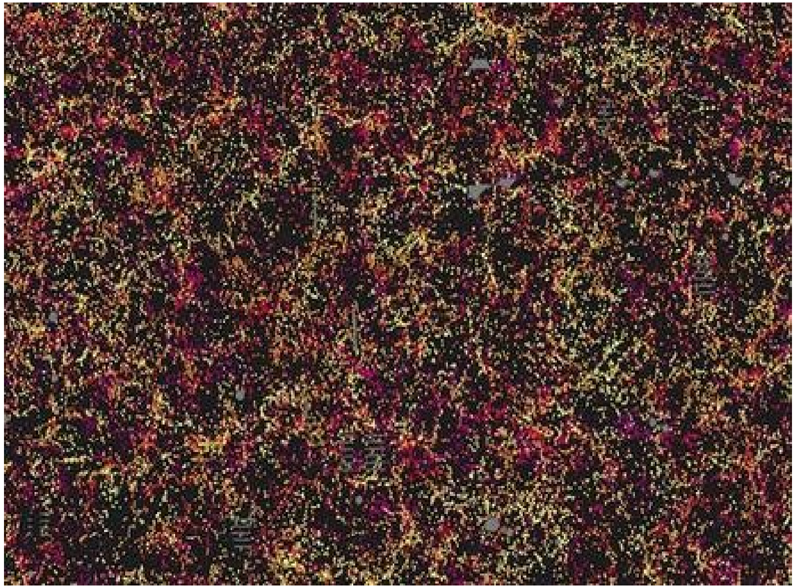
Uncertain theory predictions and Gaussian emulation of likelihoods

Marcos Pellejero Ibáñez, Giovanni Arico, Raul Angulo, Matteo Zennaro, Sergio Contreras…

Donostia International Physics Center (San Sebastián)

Big Picture

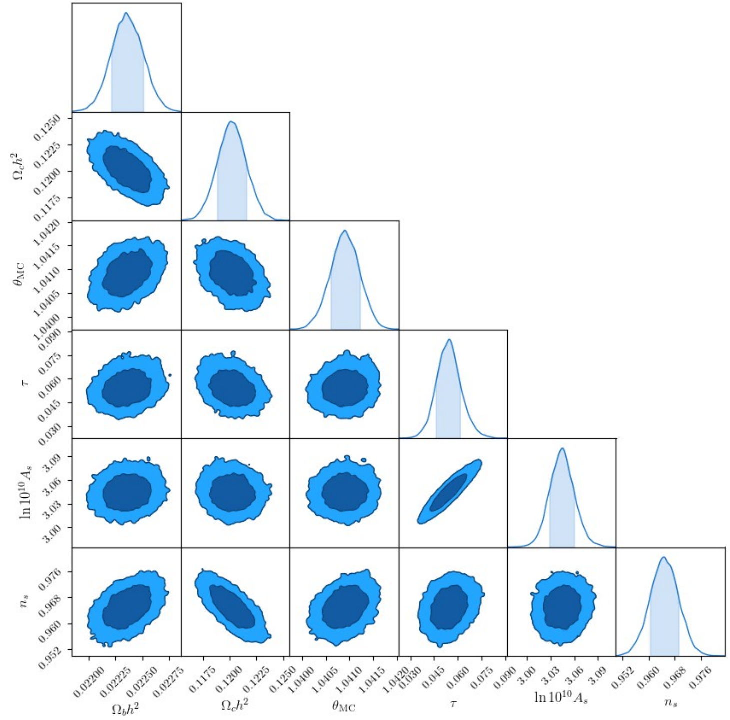
- Go from:



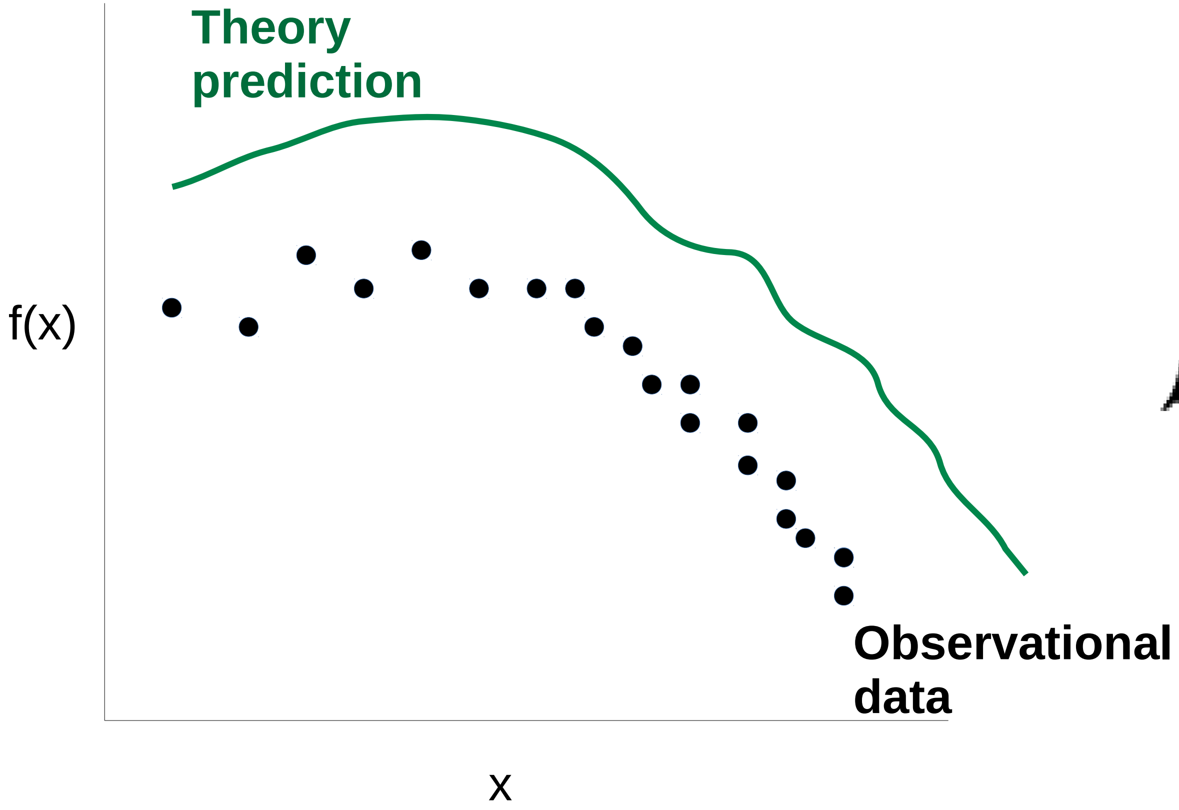
Credit Sloan Digital Sky Survey and its
Baryon Oscillation Spectroscopic Survey

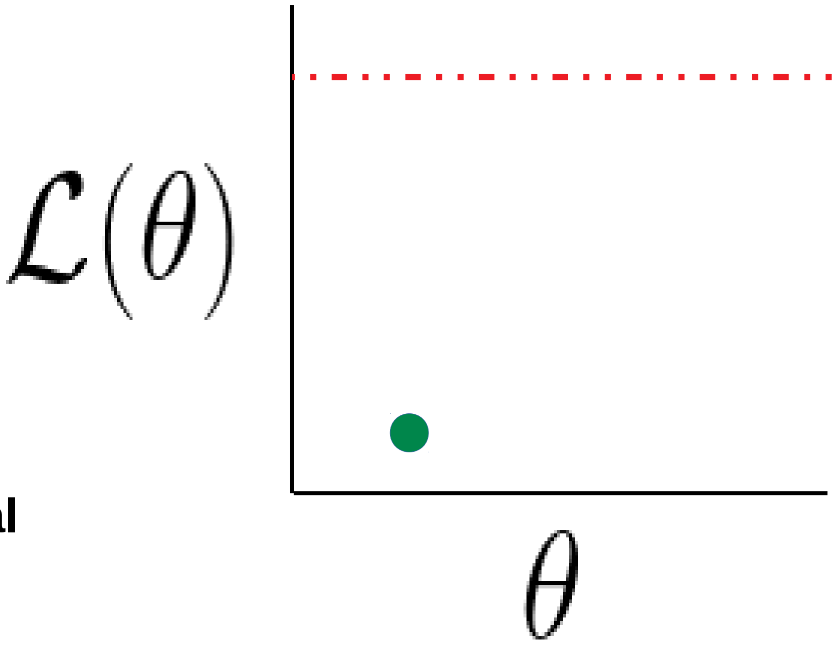
- To:

Likelihood function: Theoretical model



Simple example





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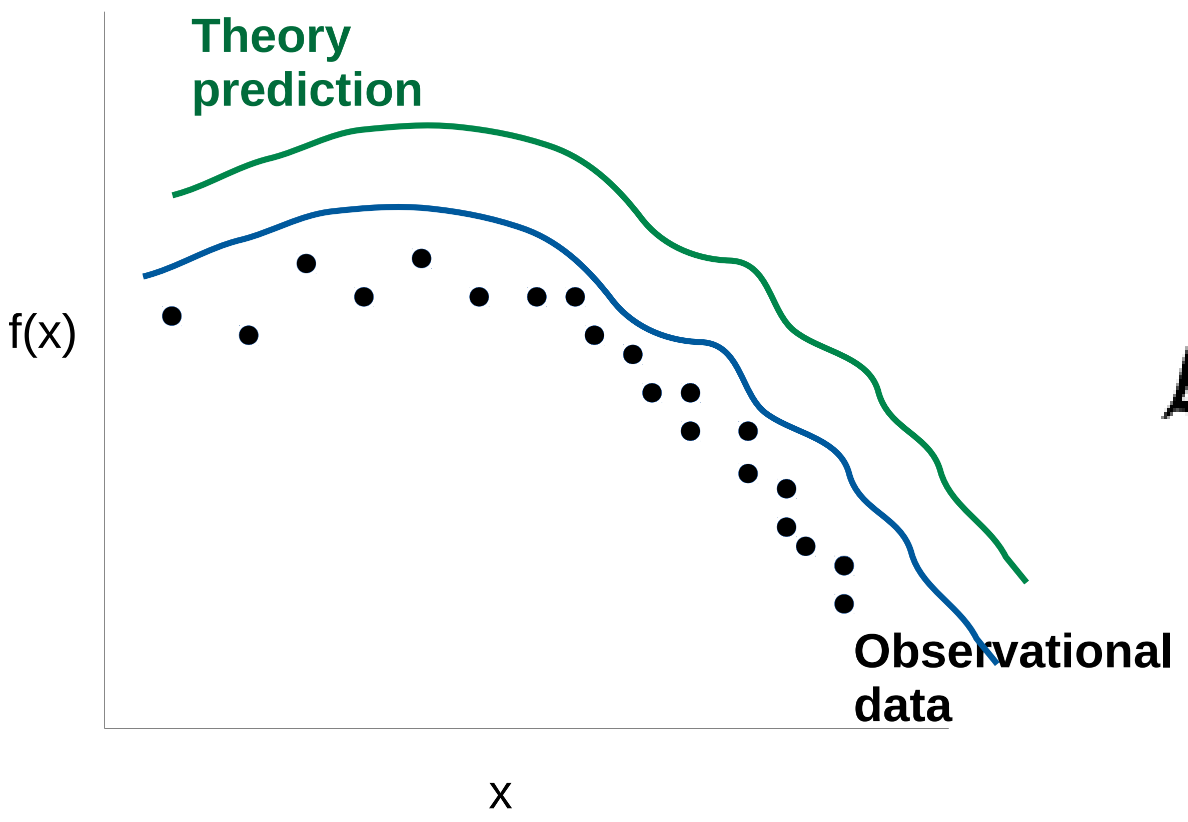

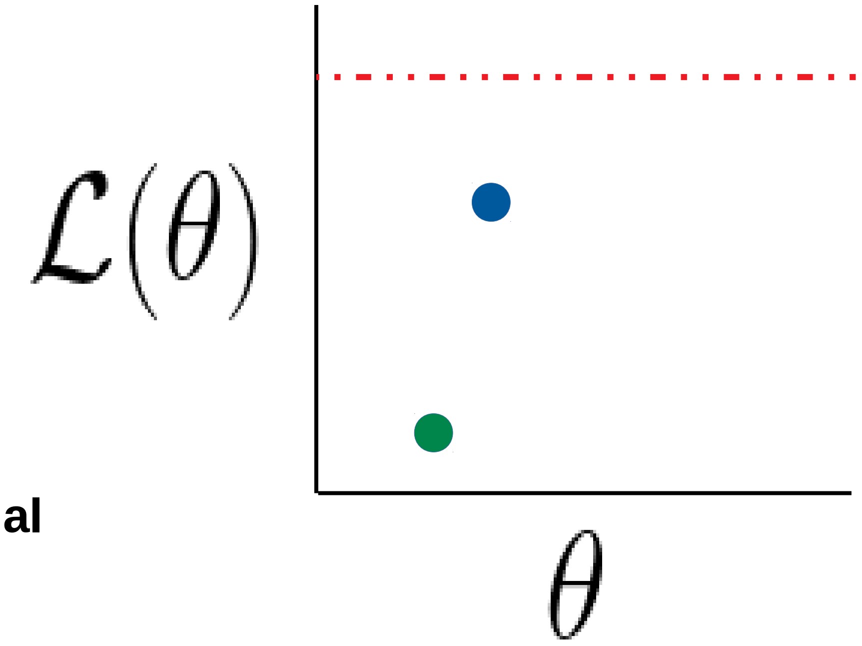

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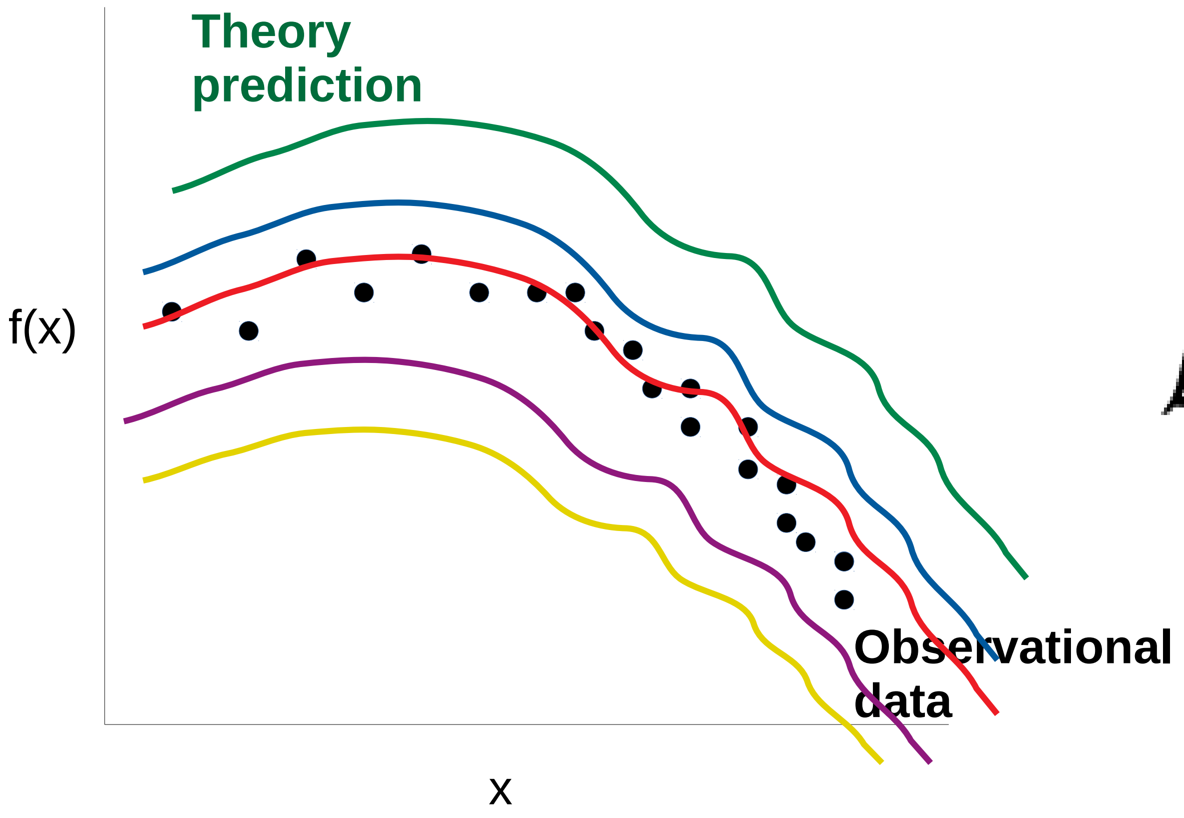
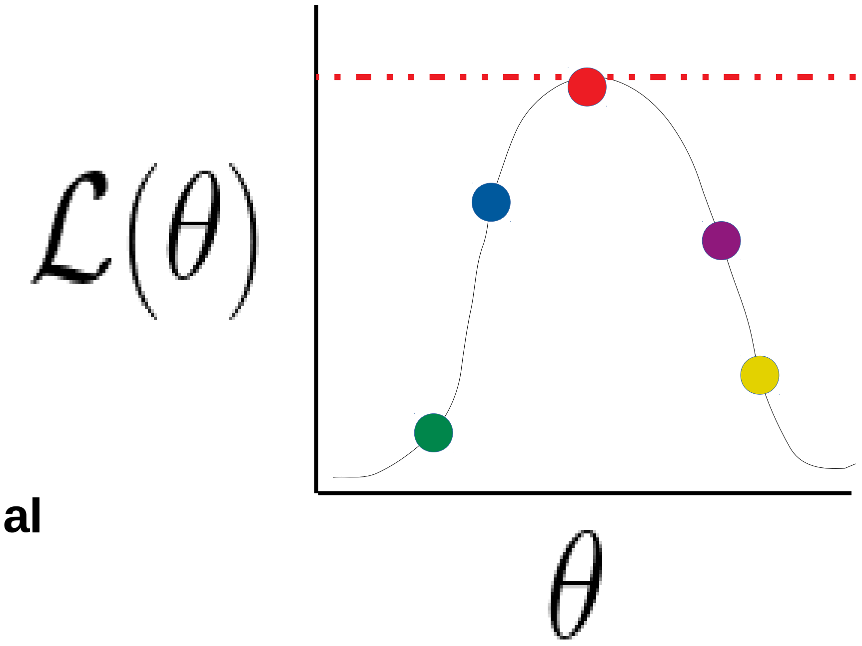


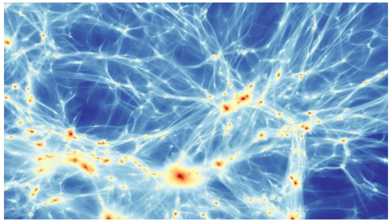


Problems

1) Expensive evaluations of the likelihood (advance modeling).

Complicated loop evaluations, high-n integrals in PT, modeling observational effects

Direct N-body simulations



2) Expensive posterior inferences. Very long chains to achieve convergence, high dimensionality.

3) Noisy evaluations. Cosmic variance from simulations, uncertainties in the group finder or in the gravity solver.

Traditional MCMC not adequate

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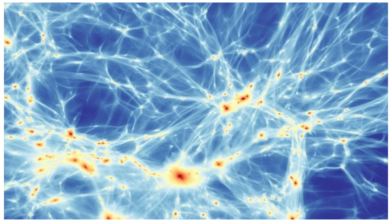

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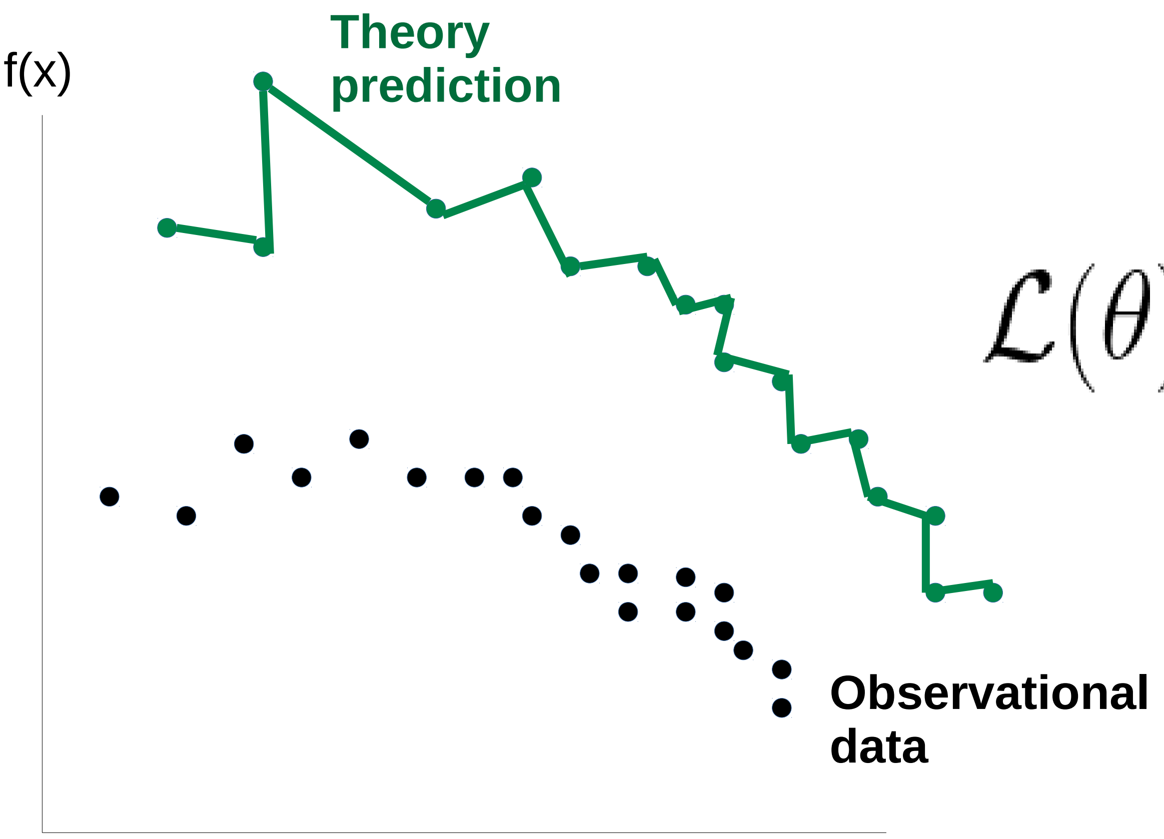

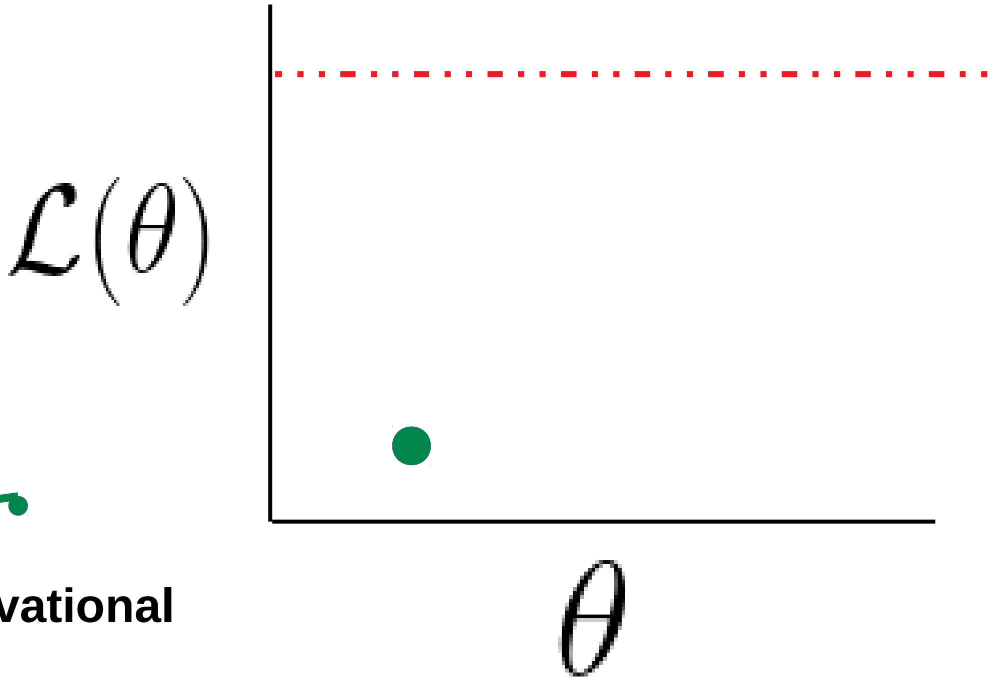

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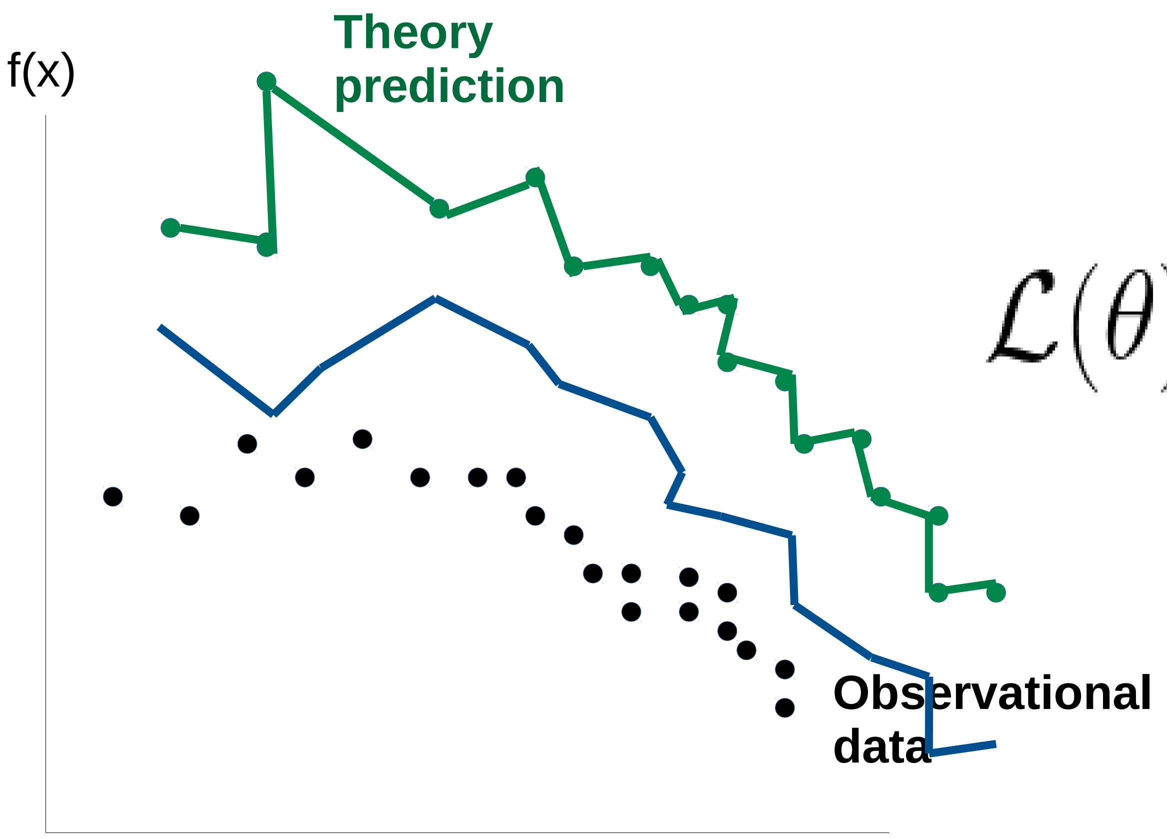

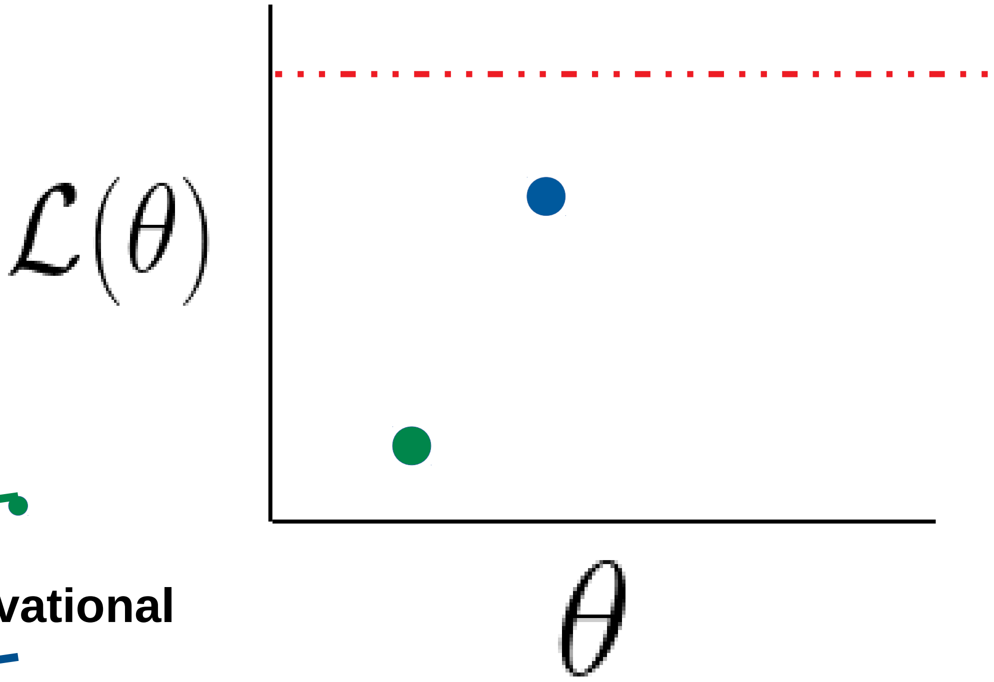

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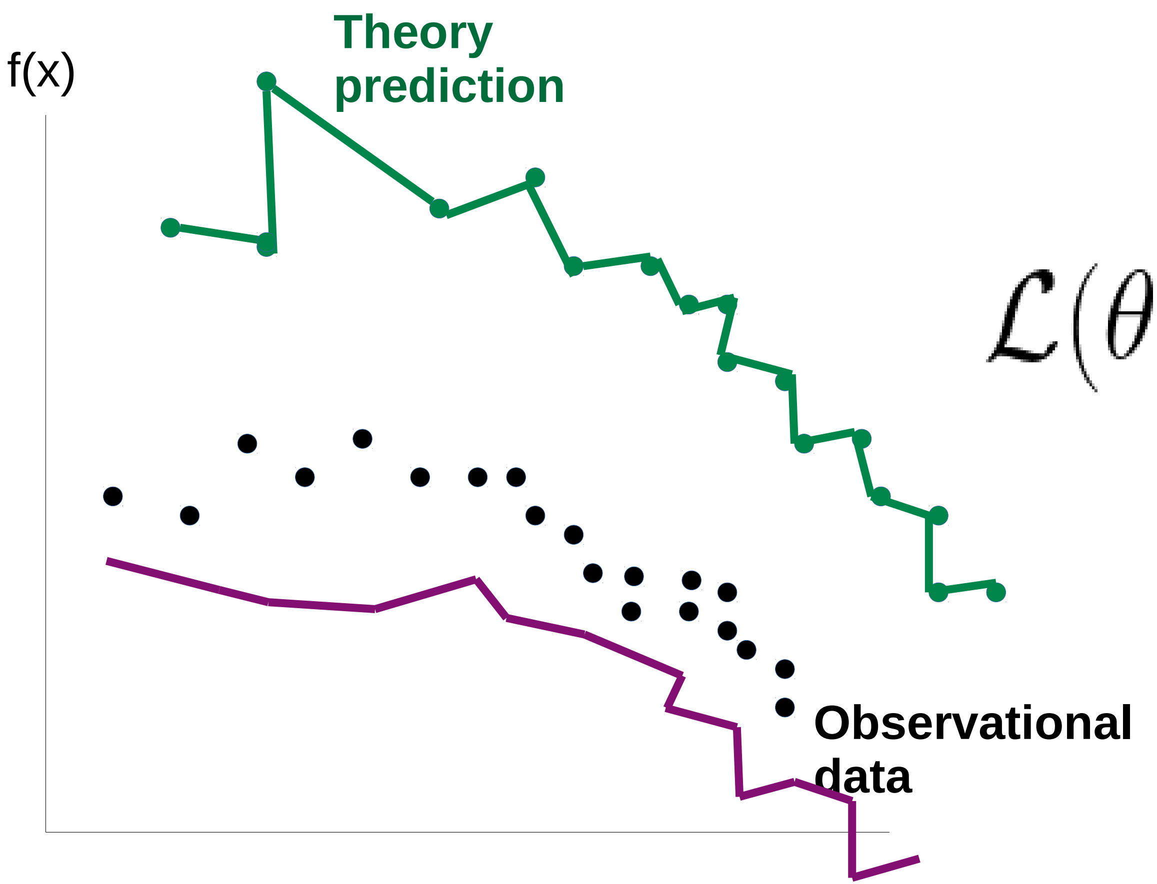

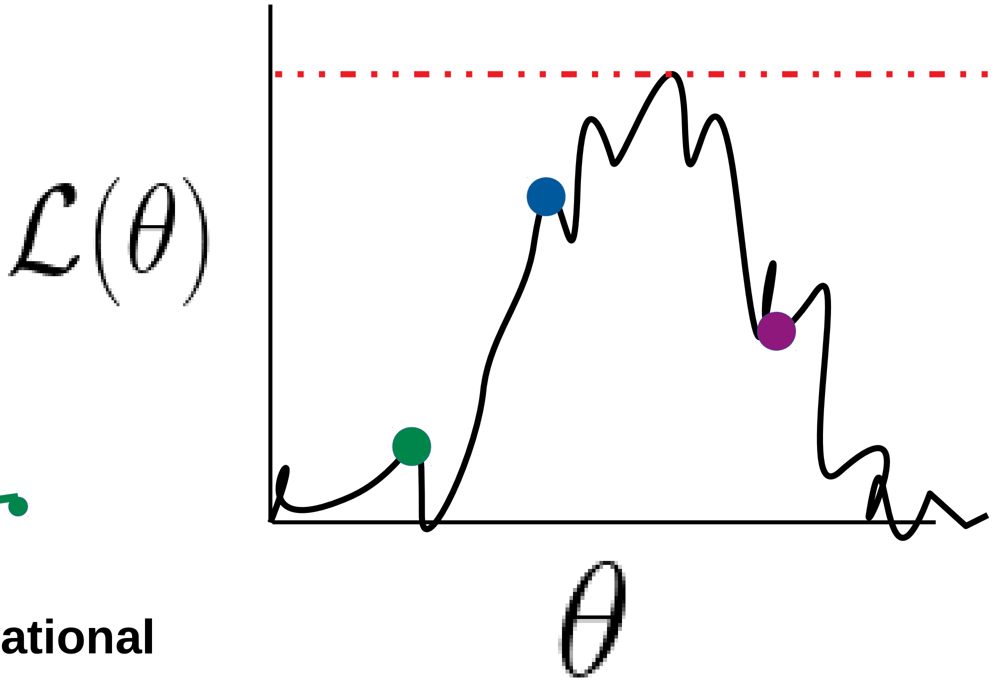

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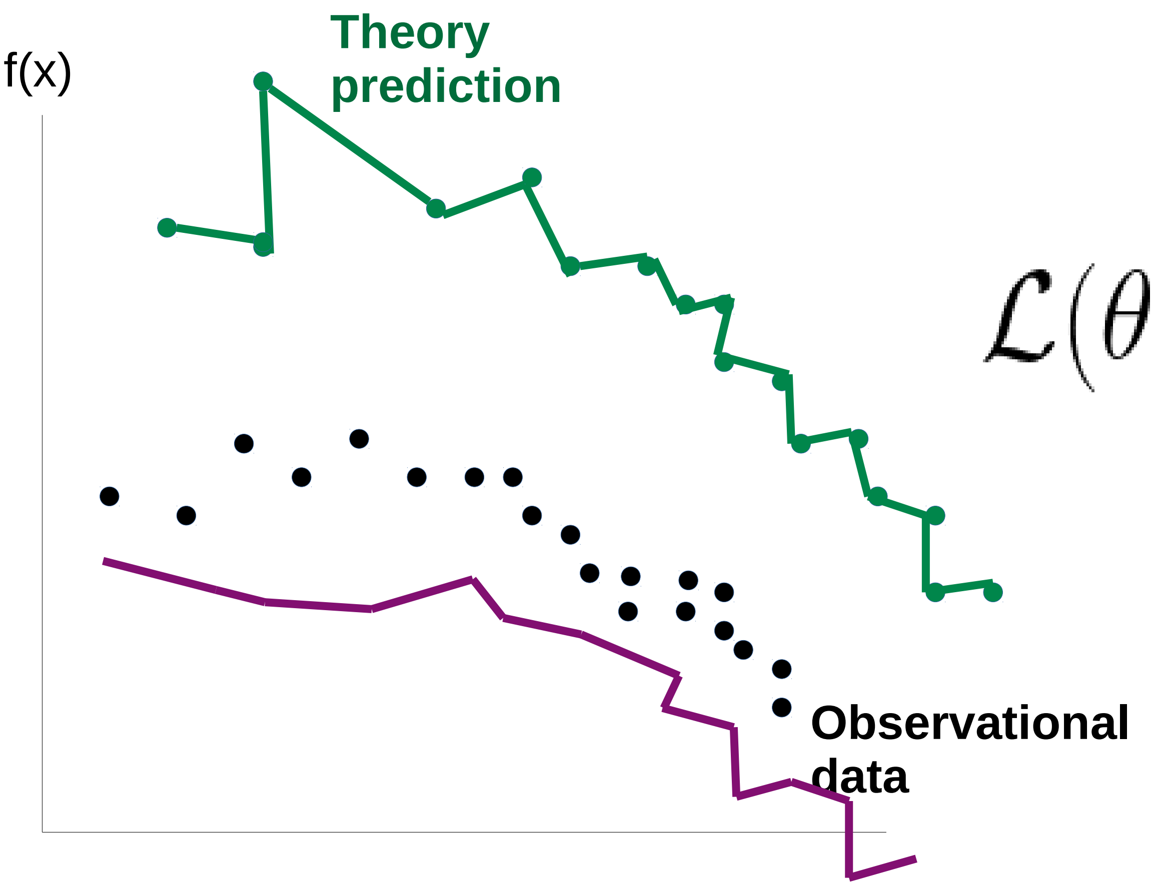
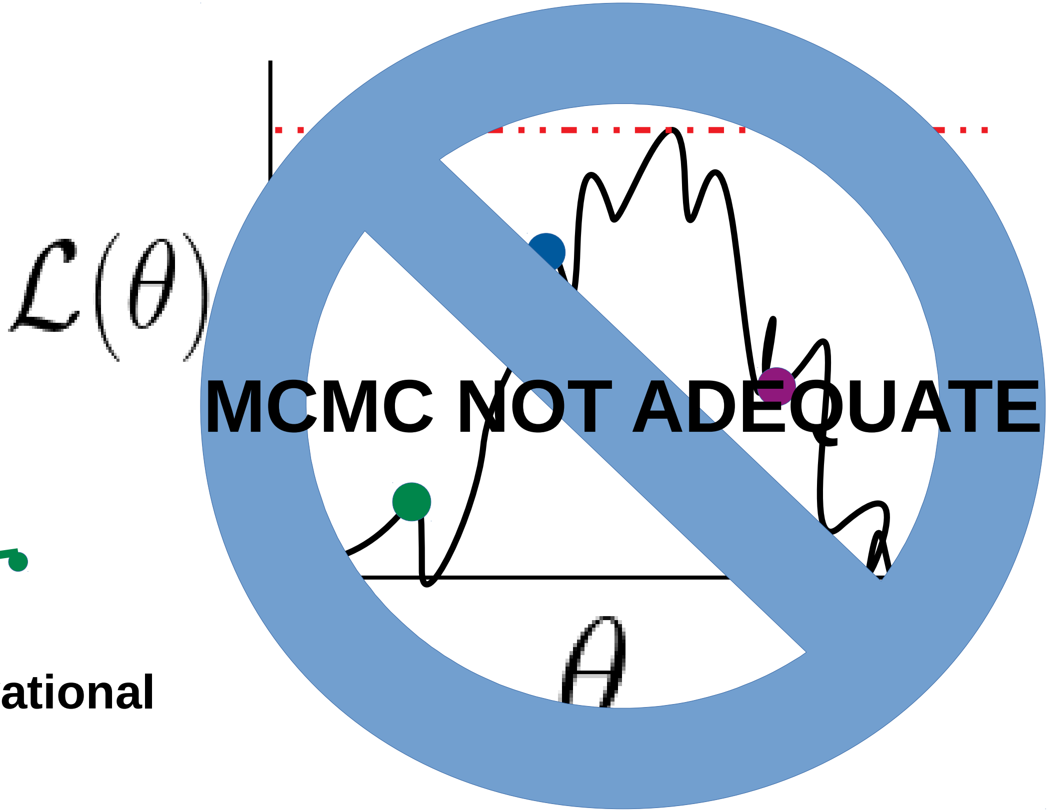
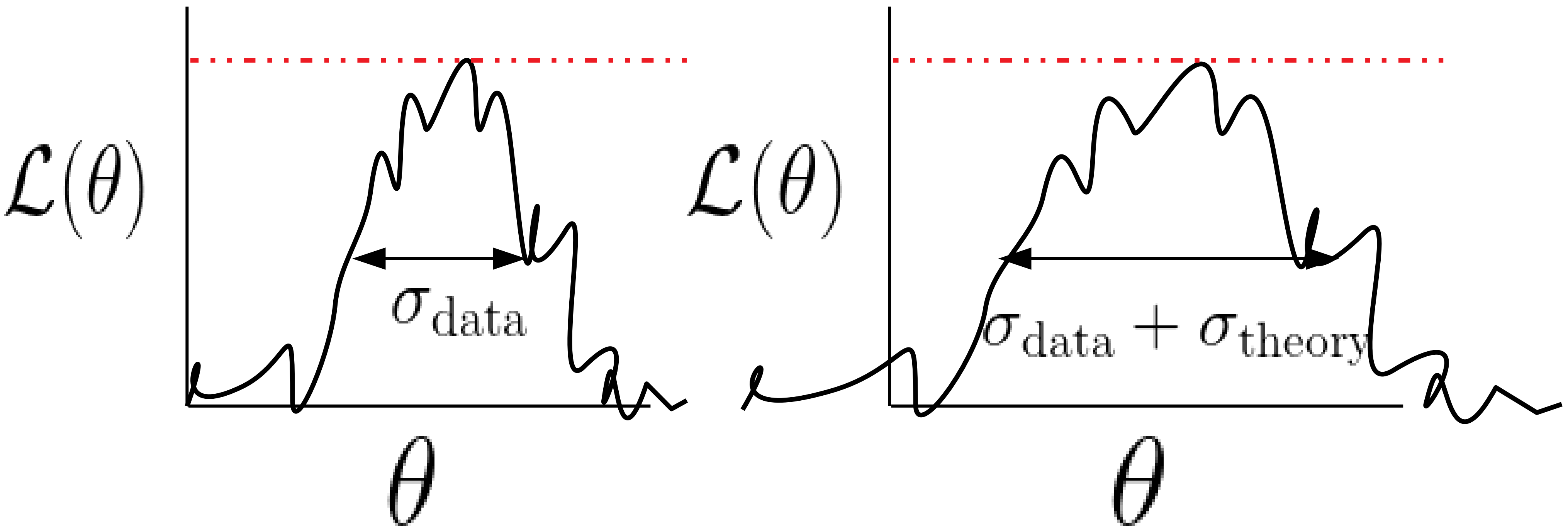
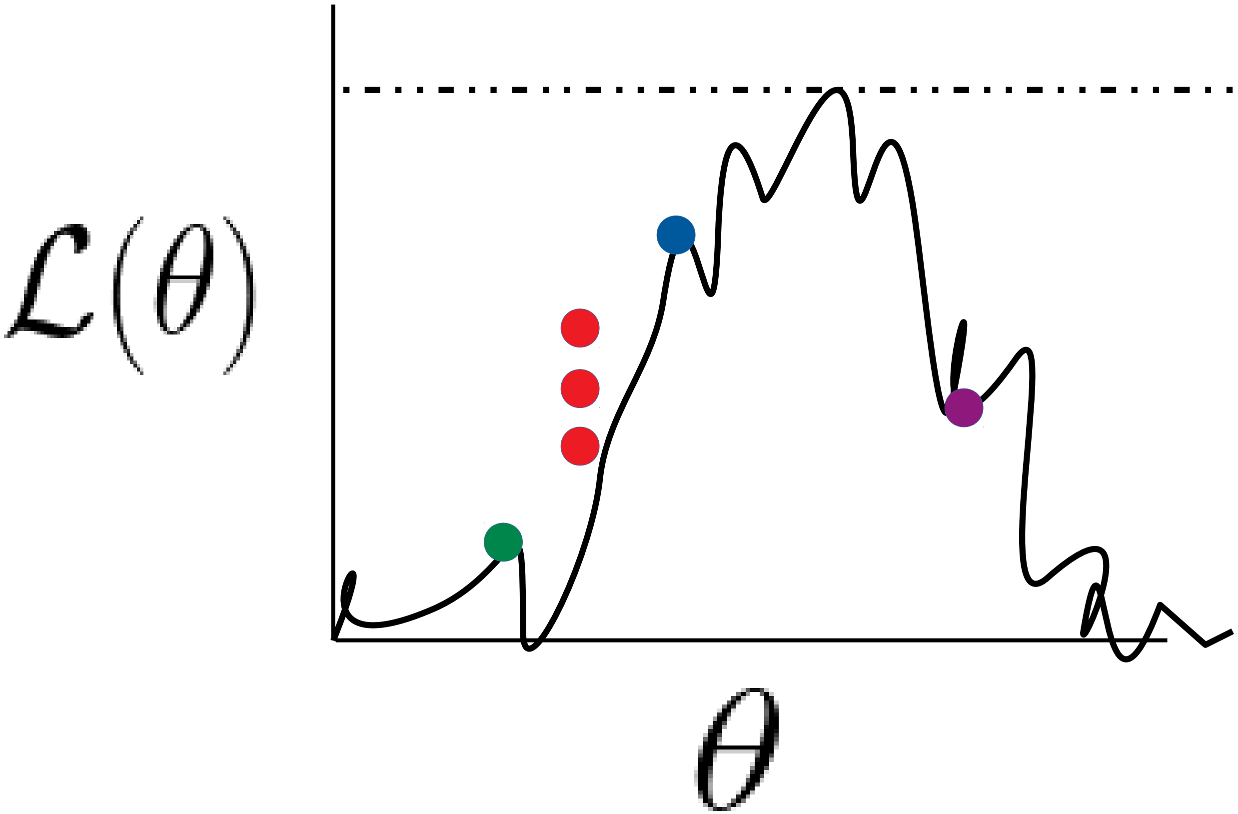




1) Include theory error



2) Reconstruction (regression) of the likelihood: Gaussian Process



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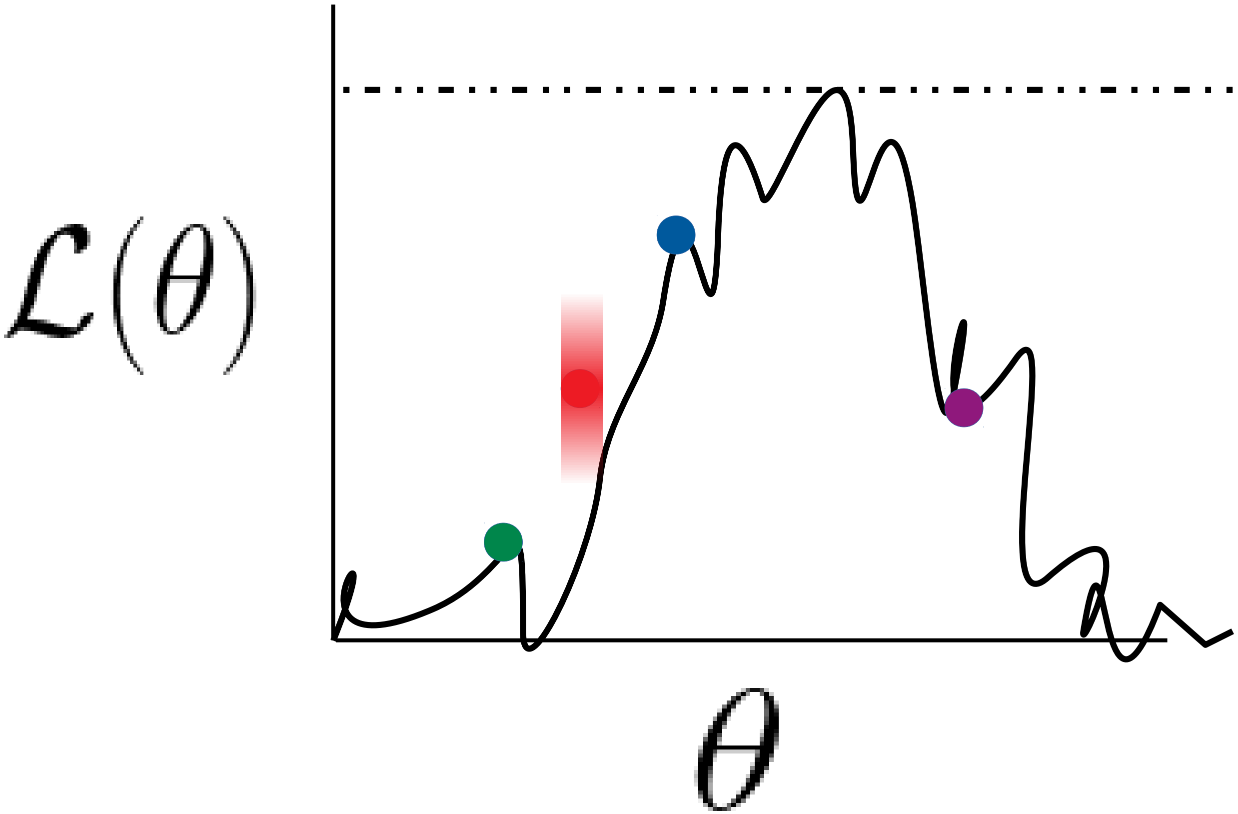

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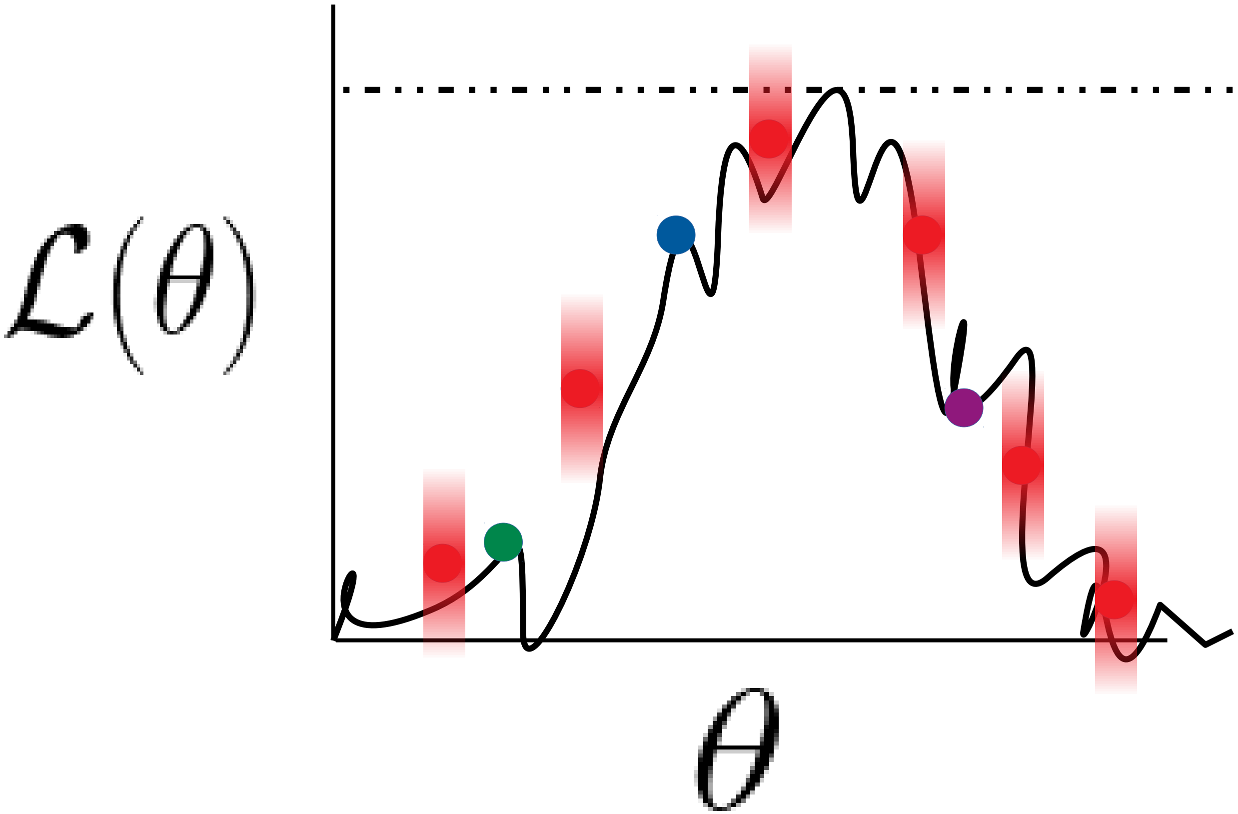

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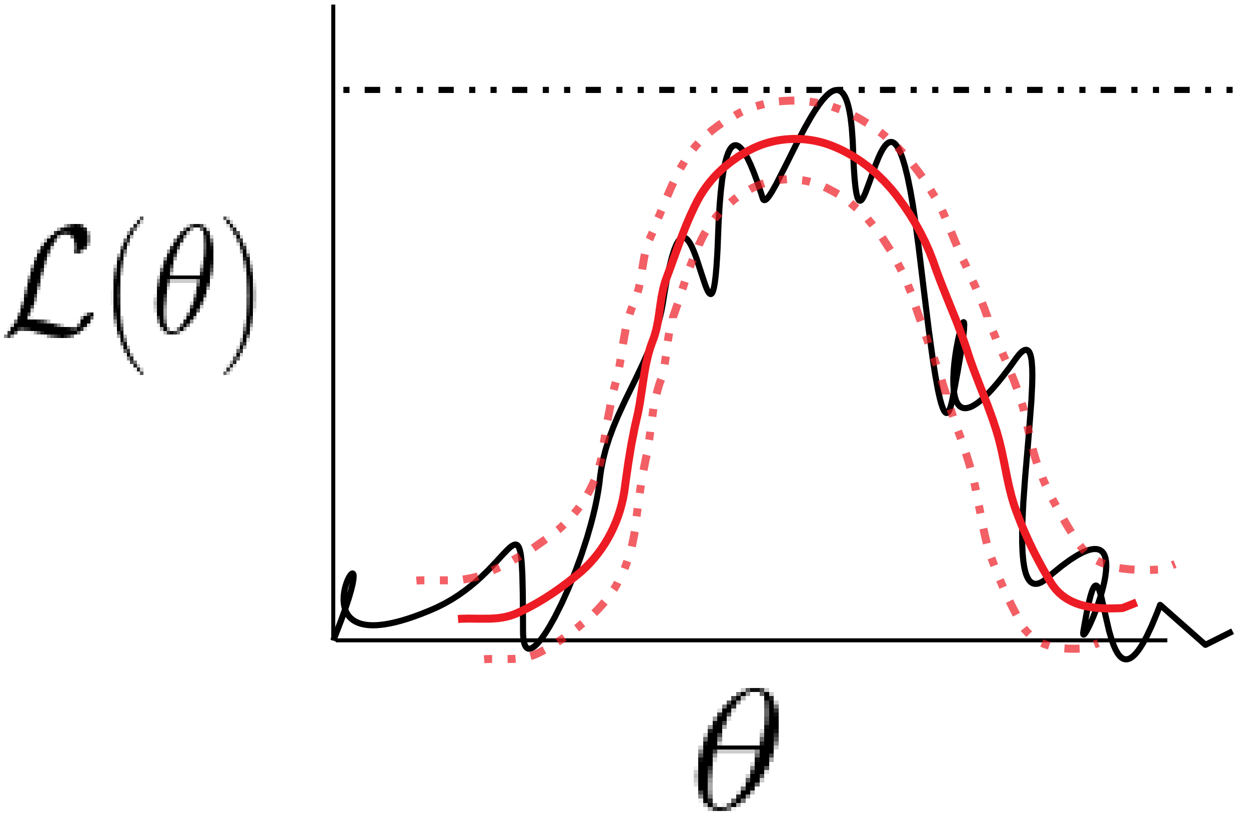
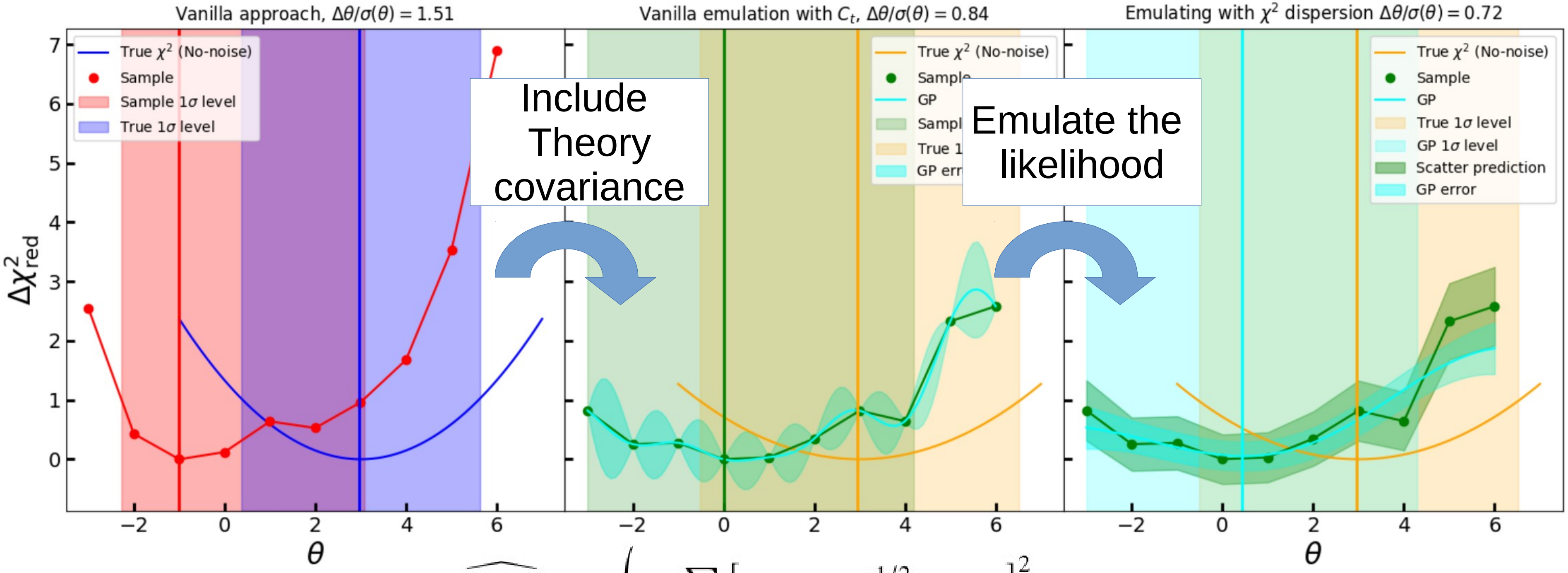


Noise on the likelihood function: toy model and Gaussian Process



$$\widehat{X^{\rm T}C^{-1}X} \sim \mathcal{N}\left(n + \sum_i \left[(C_d + C_t)^{-1/2}_{ij}(d-t)_j\right]^2,\right.$$

$$\left. 2n + 4\sum_i \left[(C_d + C_t)^{-1/2}_{ij}(d-t)_j\right]^2\right).$$

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Problems

Minimal amount of likelihood function evaluations to recover the true one?

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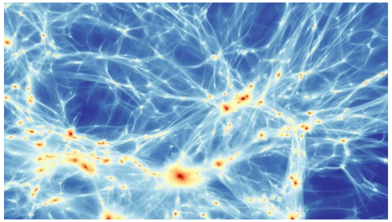

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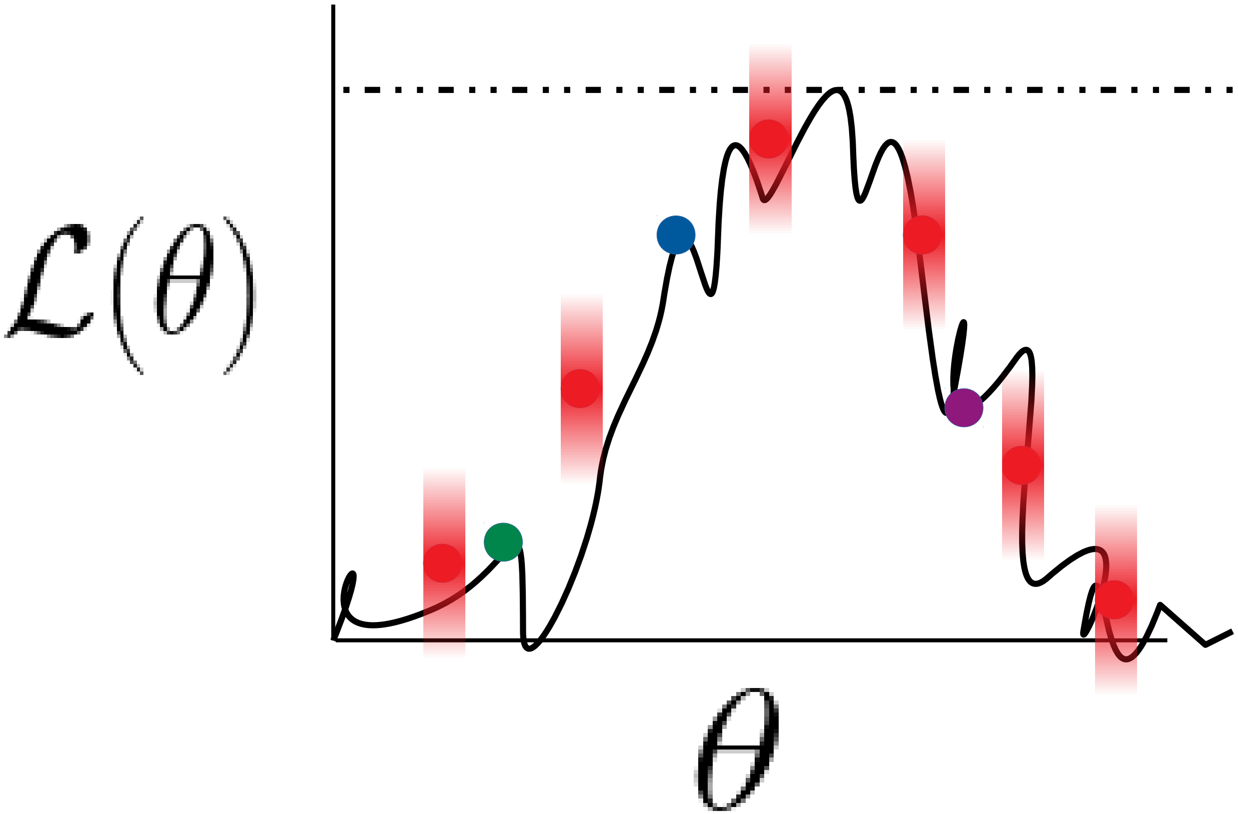
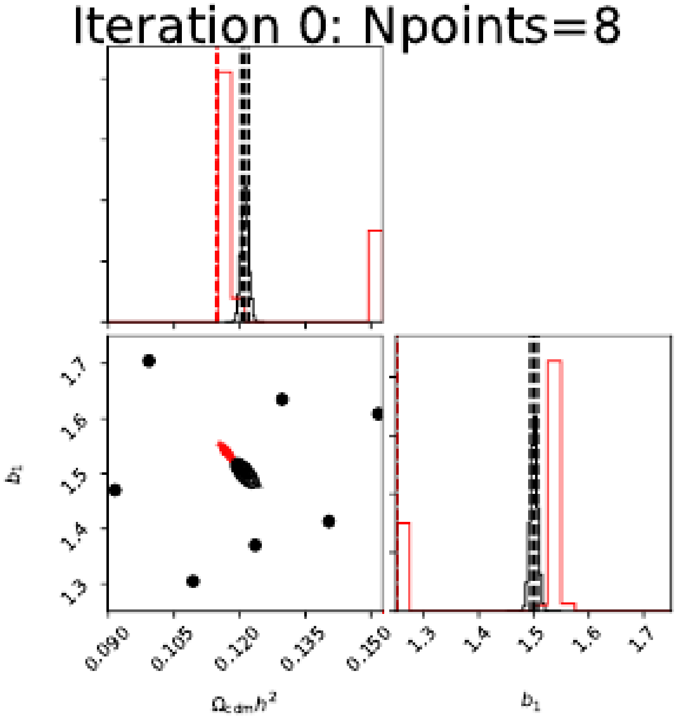
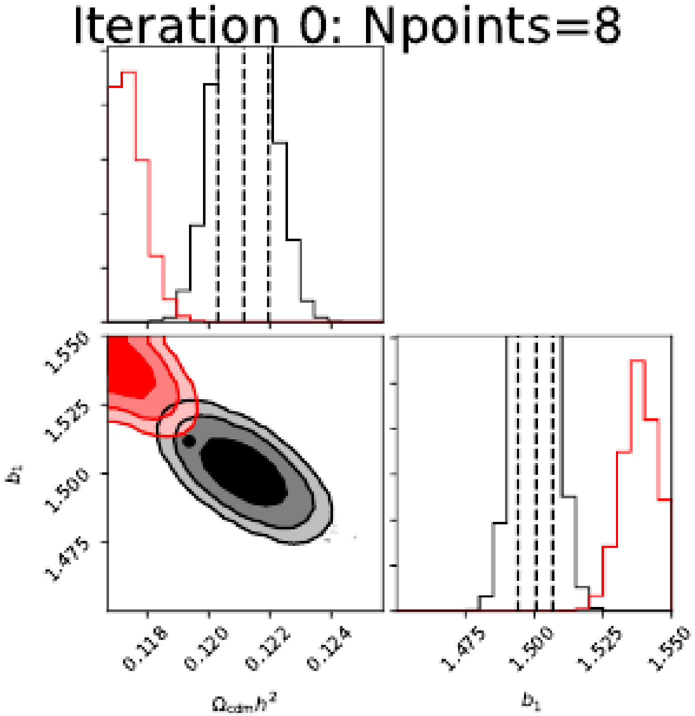


Iterative process with Bayesian Optimization of Gaussian Process





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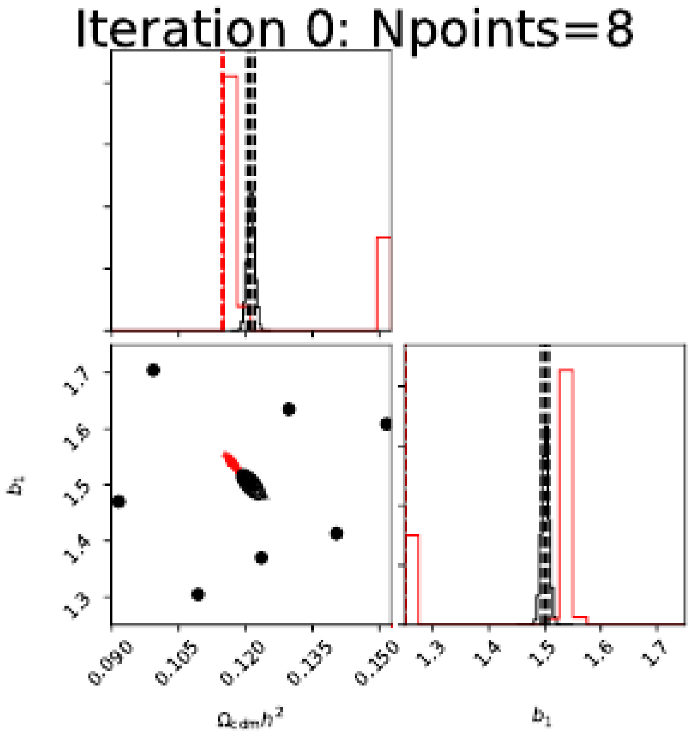

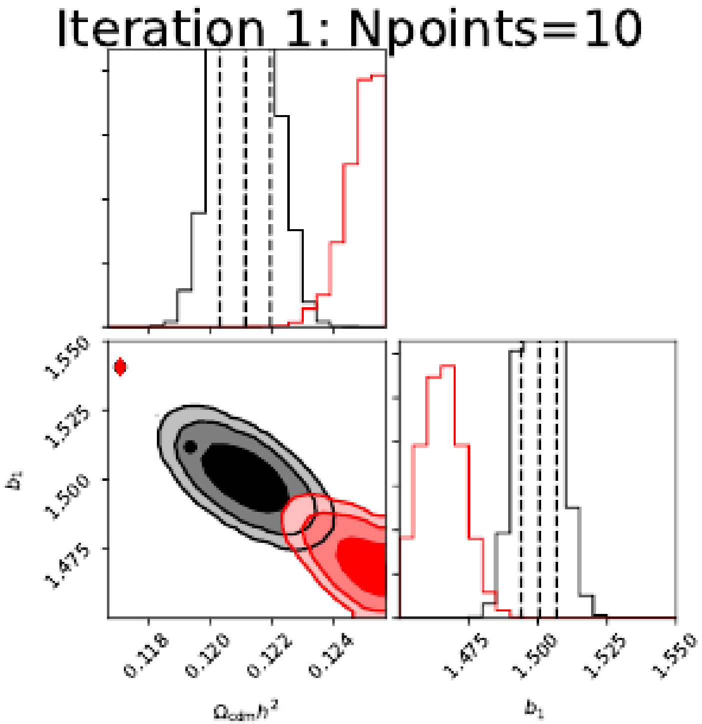

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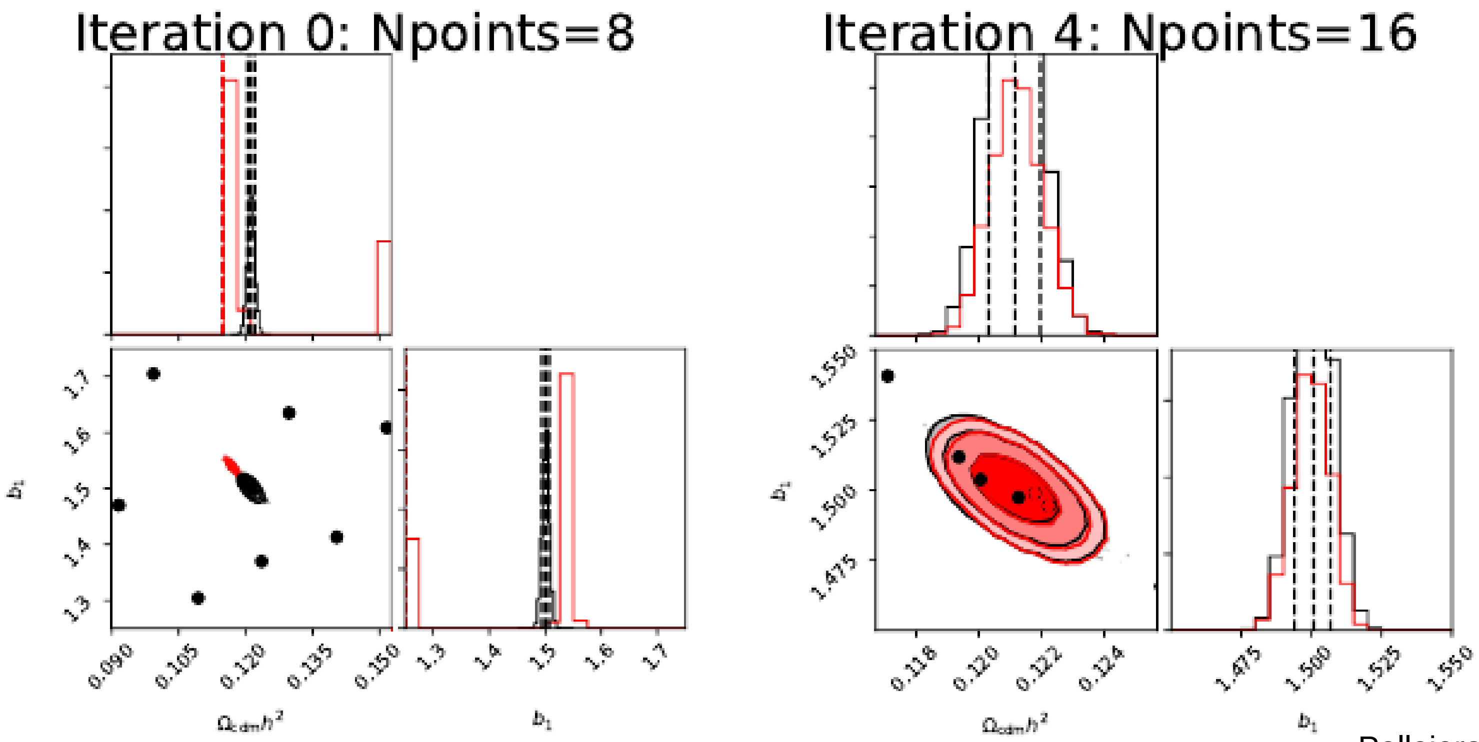

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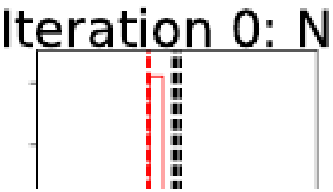

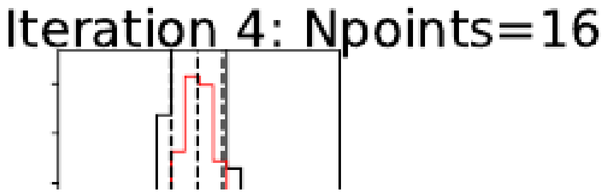

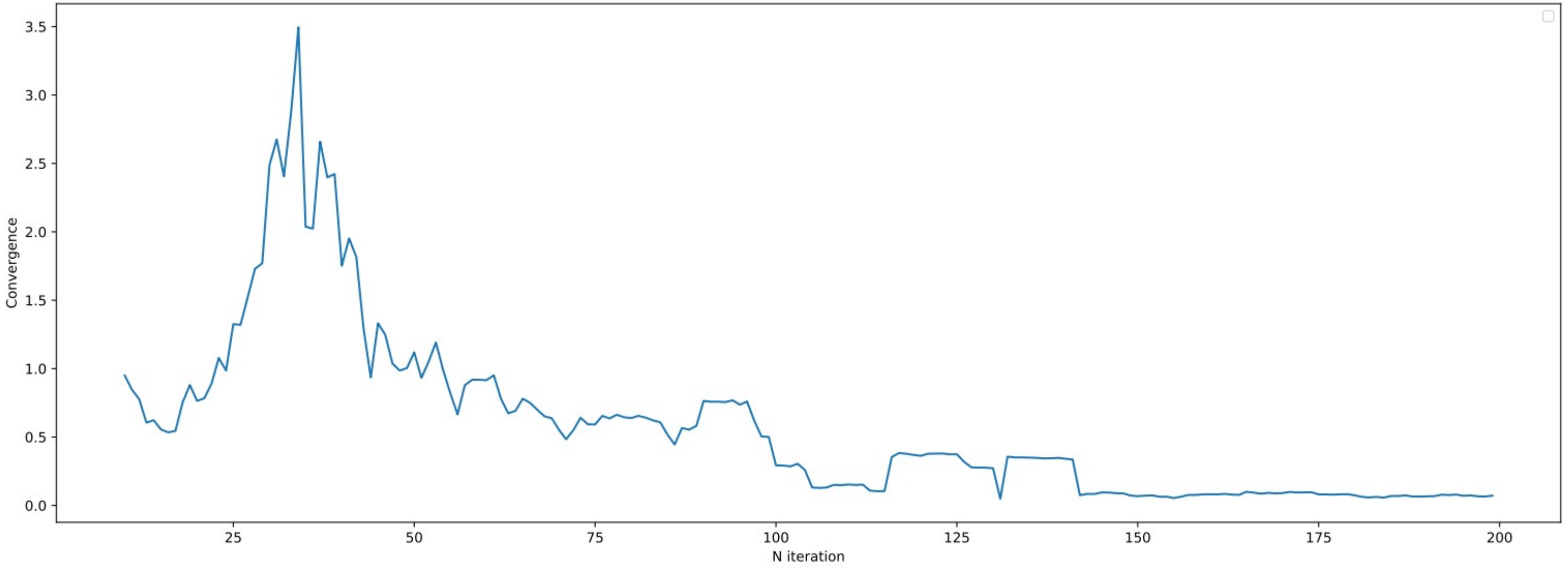

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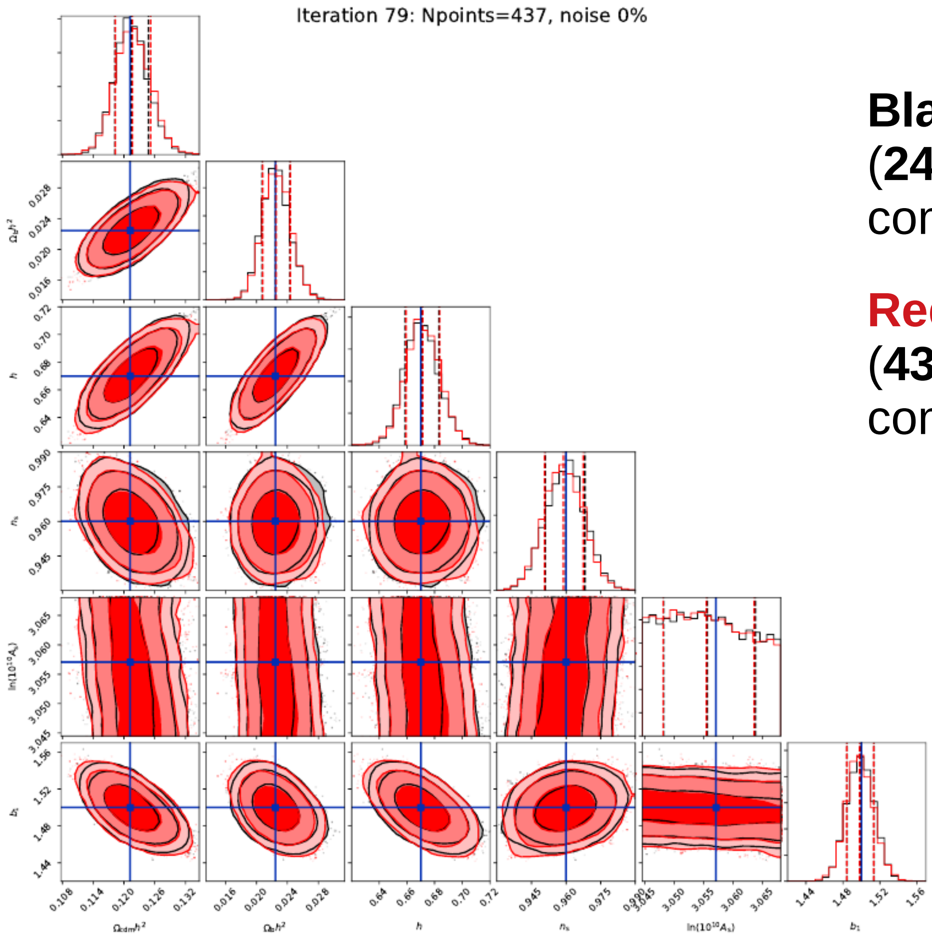
In higher dimensions: clustering



Black: MCMC constraints (**24000** likelihood compuations)

Red: Iterative constraints (**437** likelihood computations)

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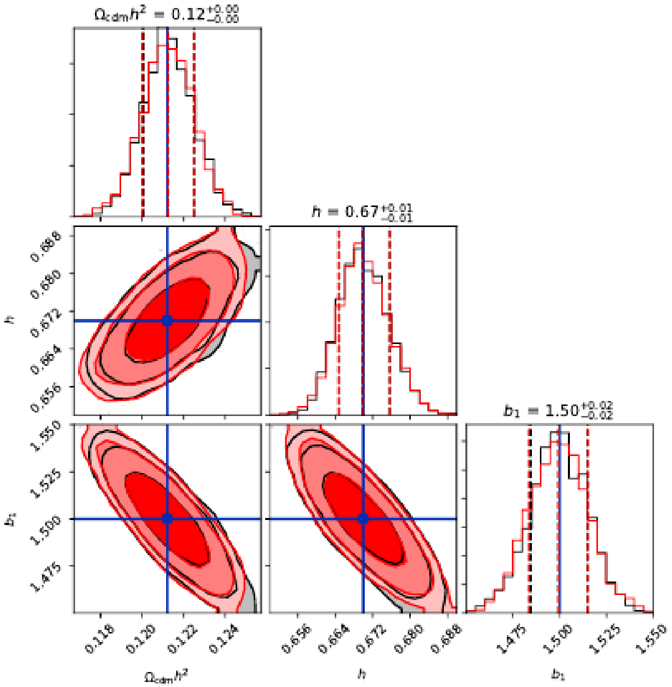
Summary

- **Noisy likelihood**: Include **theory errors** and use **Gaussian Process** to recover “true” results.
- Use iterative **Bayesian Optimization process** to get minimal computations of the likelihood function.

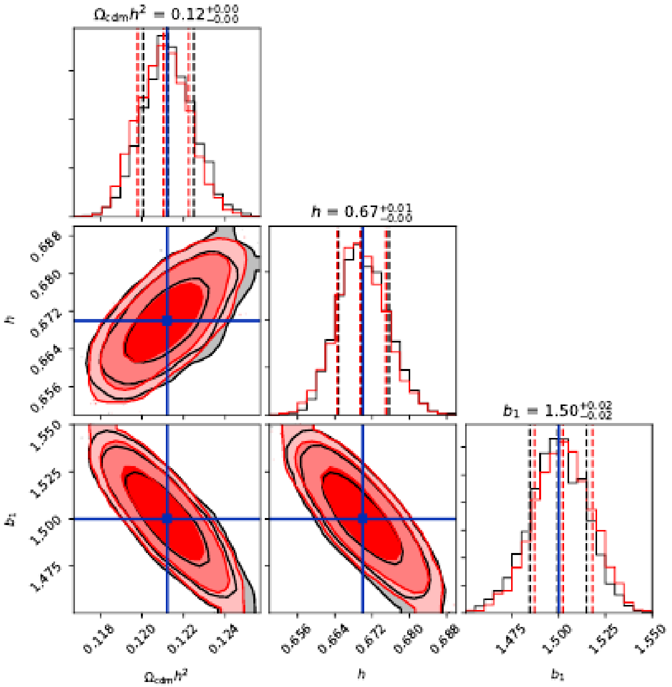
¡Muchas gracias!

Appendix: Combining ideas

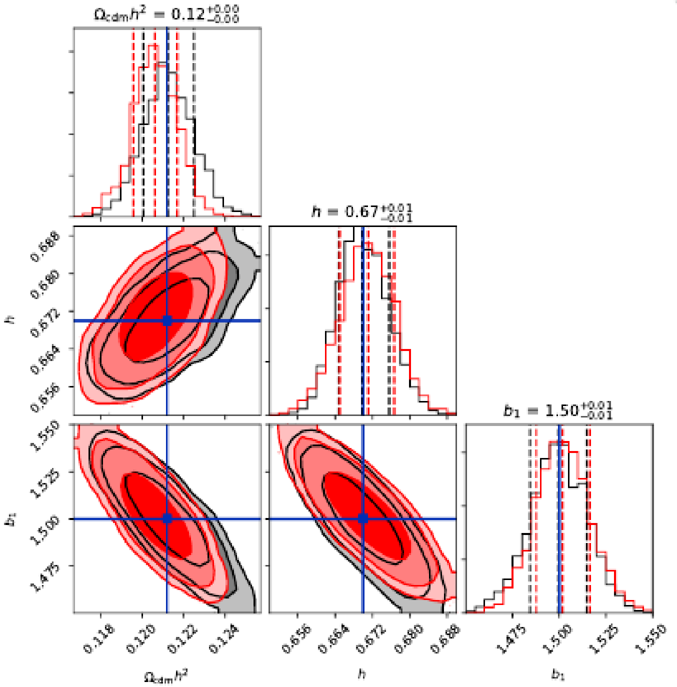
No noise



10% noise



50% noise



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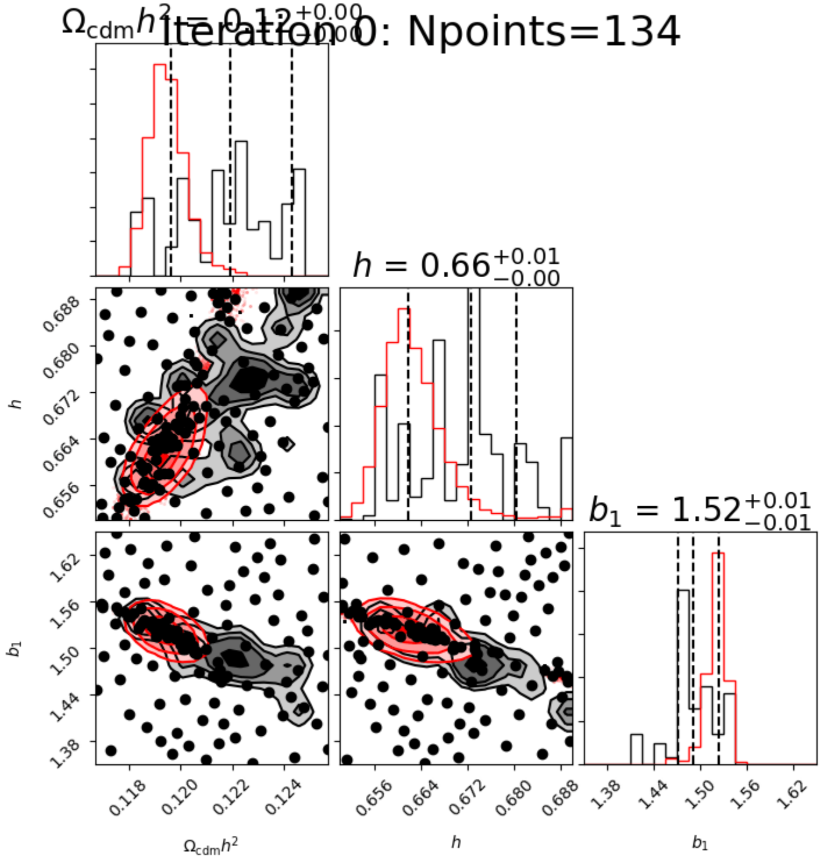
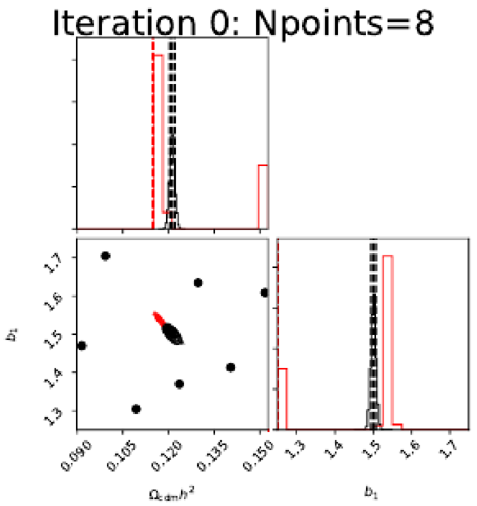
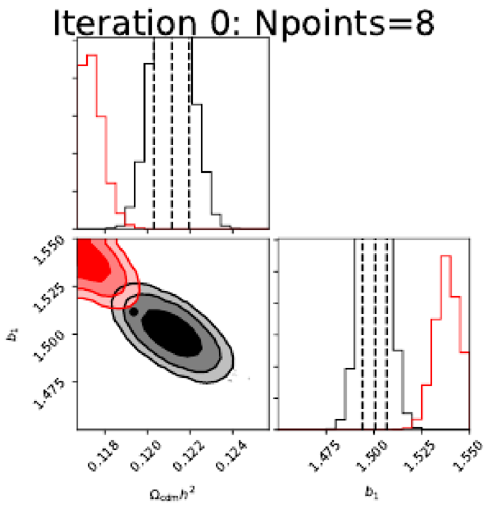
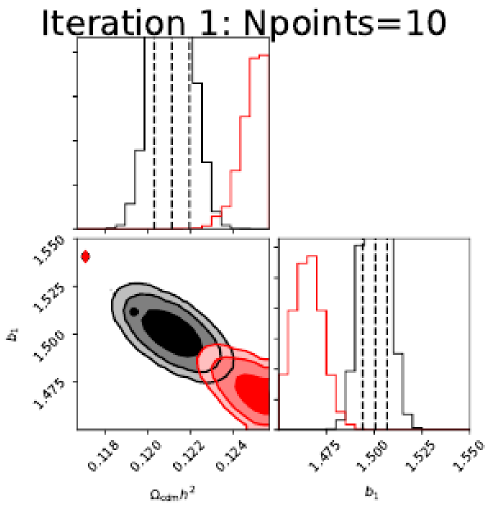
Appendix: MCMC performance

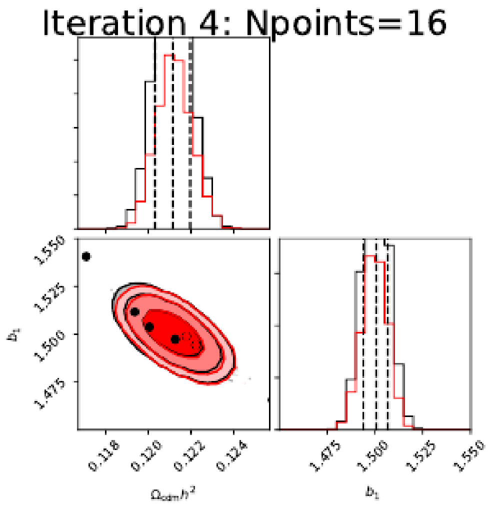


Appendix: Iterative process with Bayesian Optimization of Gaussian Process

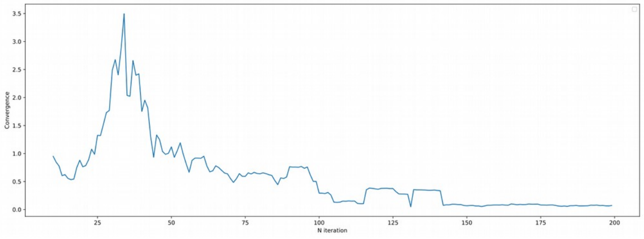








Convergence criteria



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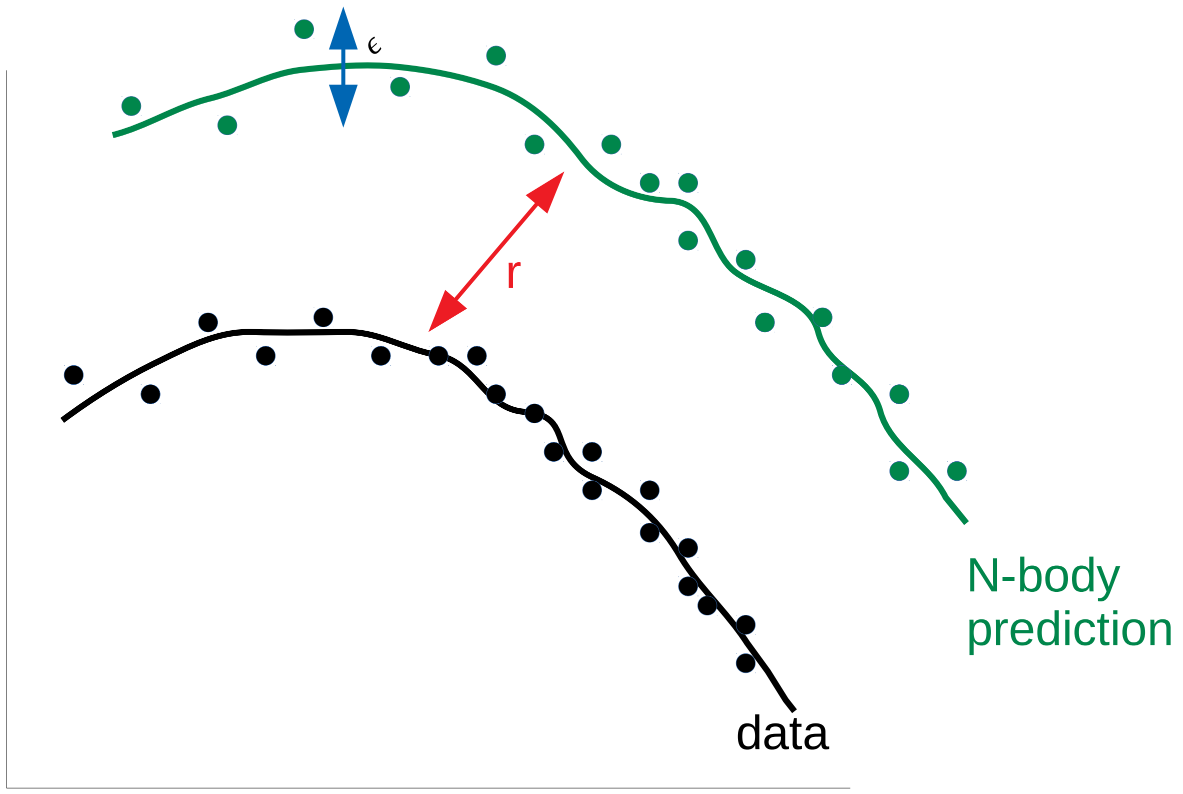
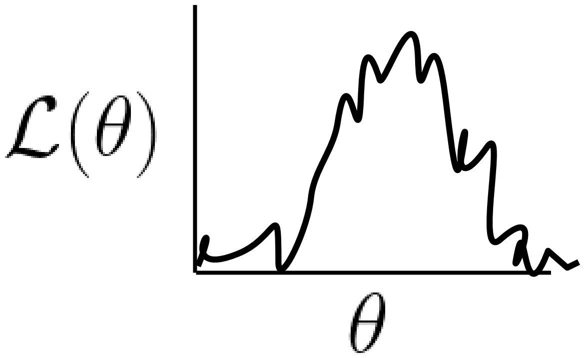
Appendix

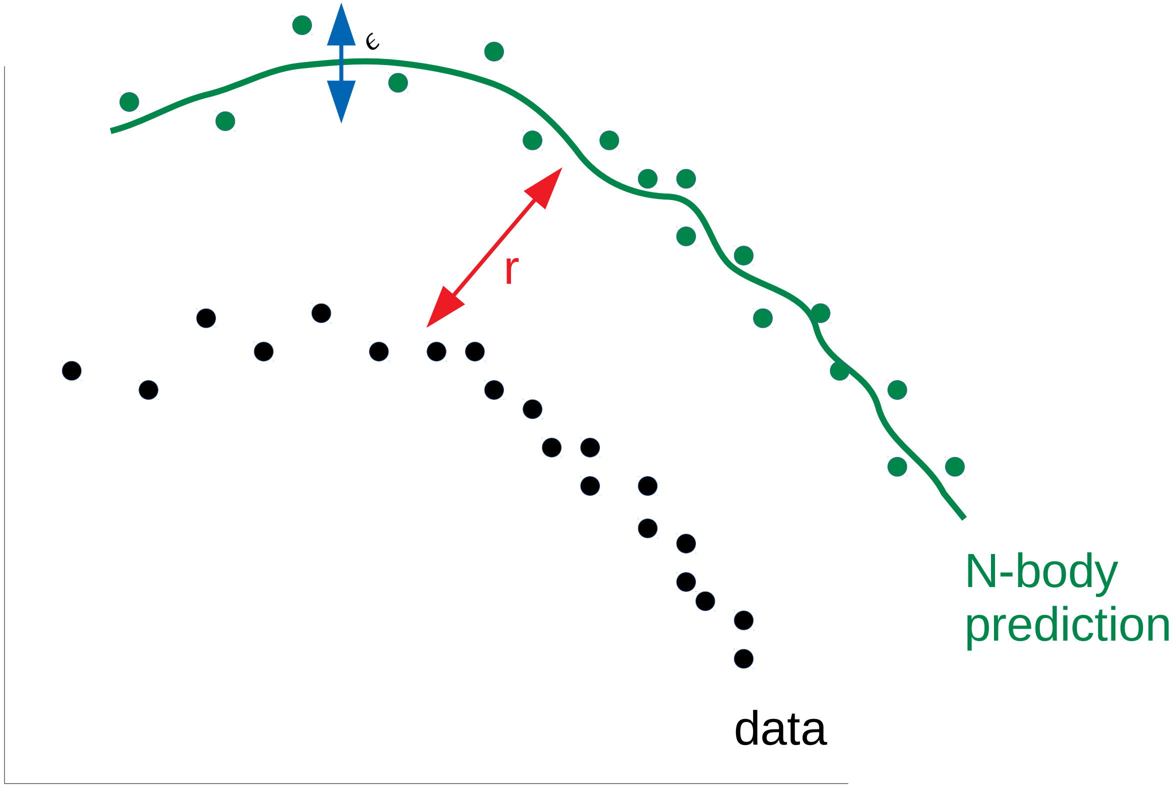


Simple example:

$$\epsilon = \epsilon(\theta)!!$$

Translates into noise
in the likelihood.





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