# Non-comoving cosmology

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Universidad Complutense de Madrid and IPARCOS

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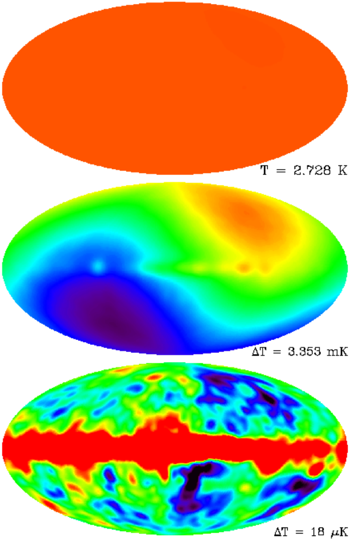
# Observations




[Credit: COBE collaboration]

*Homogeneity and isotropy*

Multiple probes:

- Cosmic microwave background
- Galaxy number counts
- Expansion history

Some anomalies (?):

- Large-scale bulk flows
- CMB dipolar anomaly
- Radio dipole

[Schwarz et al. CQG 2016]

[Buchert et al. IJMPD 2016]

# Mathematical embodiment

| | | |
|---|---|---|
| Homogeneity<br>Isotropy | $\rightarrow$ | RW metric<br>$\mathrm{d}s^2 = a^2(\tau)\left(-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2\right)$ |

From Einstein equations $(G^{\mu}{}_{\nu} = 8\pi G T^{\mu}{}_{\nu})$

$$G^0{}_i = 0 \qquad \rightarrow \qquad T^0{}_i = 0$$

$$G^i{}_j \propto \delta^i{}_j \qquad \rightarrow \qquad T^i{}_j = 0\ , \quad i \neq j$$

# Perfect fluids

$$T^{\mu}{}_{\nu} = (\rho + P) u^{\mu} u_{\nu} + \delta^{\mu}{}_{\nu} P$$

**Single fluid** ($\gamma \equiv (1 - \beta^2)^{-1/2}$)

$$T^{0}{}_{i} = (\rho + P)\gamma^2 \beta_i = 0$$

$$T^{i}{}_{j} = P\delta^{i}{}_{j} + (\rho + P)\gamma^2 \beta^i \beta_j$$

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**Multicomponent** $(\beta \ll 1)$

$$T^{0}{}_{i} \simeq \sum_s (\rho + P)\beta_{s\,i} = 0$$

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- $\Lambda$CDM assumes $\boldsymbol{\beta}_s = 0 \quad \forall s$ (i.e. for every component)
- Background isotropy requires $\sum_s (\rho_s + P_s)\boldsymbol{\beta}_s = 0$ , $\quad (\beta_s \ll 1)$

# Standard lore

$$T_{\mu\nu} = \underbrace{T^{(\gamma)}_{\mu\nu} + T^{(b)}_{\mu\nu} + T^{(\nu)}_{\mu\nu}}_{\textbf{Visible sector}} + \underbrace{T^{(\Lambda)}_{\mu\nu} + T^{(\mathsf{CDM})}_{\mu\nu} + \ldots}_{\textbf{Dark sector}}$$

**Visible sector**

- Non-gravitational interactions
- Essentially decoupled at late times
- Tightly coupled in the early Universe

**Dark sector**

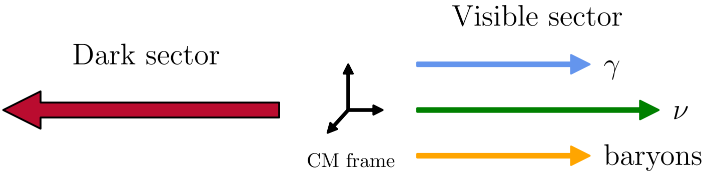
- Only observed gravitationally
- $\Lambda$ and CDM behaviour at late times
- Early Universe behaviour?

# Non-comoving cosmology

## Center of mass condition

$$\sum_s (\rho_s + P_s)\boldsymbol{\beta}_s = 0 \ , \qquad s = \gamma, b, \nu, DS$$

[Maroto JCAP 2006] [Maroto, Beltran Jimenez PRD 2007]




The background is homogeneous and isotropic (FLRW) in the CM frame.

# Kinetic theory

Two frames

$$\begin{cases} \mathcal{O}(\boldsymbol{p}) & \rightarrow \quad \text{CM frame} \qquad\qquad \text{(RW background)} \\ \tilde{\mathcal{O}}(\tilde{\boldsymbol{p}}) & \rightarrow \quad \text{Fluid rest frame} \qquad (\tilde{p}^\mu = \Lambda^\mu{}_\nu(\beta) p^\nu) \end{cases}$$

The background distribution function satisfies

$$f_0(\boldsymbol{p}) = \tilde{f}_0(\tilde{p})$$

e.g. for photons $\rightarrow$ boosted blackbody spectrum in the CM frame

$$\tilde{f}_0(\tilde{p}) = \underbrace{\frac{1}{\mathrm{e}^{\,\tilde{p}/\tilde{T}} - 1}}_{\text{Isotropic in the fluid rest frame}} = \frac{1}{\mathrm{e}^{\,p/T(\boldsymbol{p})} - 1} = f_0(\boldsymbol{p}) \ , \qquad T(\boldsymbol{p}) = \underbrace{\gamma \left(1 - \frac{\boldsymbol{p} \cdot \boldsymbol{\beta}}{p}\right) \tilde{T}}_{\text{Dipolar modulation in the CM frame}}$$

# Bulk velocities

Collisionless species: Neutrinos and CDM

$$\dot{\boldsymbol{\beta}}_\nu = 0 \qquad \rightarrow \qquad \boldsymbol{\beta}_\nu \propto \text{const.}\ , \qquad \text{(Radiation)}$$

$$\dot{\boldsymbol{\beta}}_c = -\mathcal{H}\boldsymbol{\beta}_c \qquad \rightarrow \qquad \boldsymbol{\beta}_c \propto a^{-1}\ , \qquad \text{(Matter)}$$

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Coupled species: Photon-baryon plasma

$$\dot{\boldsymbol{\beta}}_\gamma = \quad 0 \quad -\frac{1}{\tau_c}(\boldsymbol{\beta}_\gamma - \boldsymbol{\beta}_b)\ , \qquad \tau_c \equiv a n_e \sigma_T$$
$$\dot{\boldsymbol{\beta}}_b = -\mathcal{H}\boldsymbol{\beta}_b + \frac{1}{R\tau_c}(\boldsymbol{\beta}_\gamma - \boldsymbol{\beta}_b)\ , \qquad R \equiv \frac{3\rho_b}{4\rho_\gamma}$$

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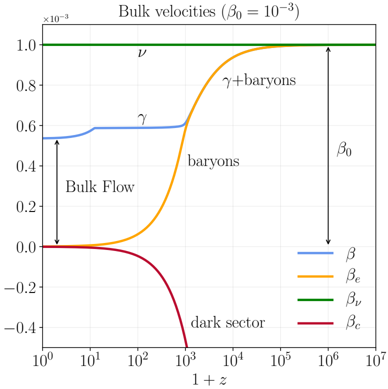
Tight-coupling ($\tau_c \ll 1$)

$$\boldsymbol{\beta}_\gamma = \boldsymbol{\beta}_b + \mathcal{O}(\tau_c) = \frac{\boldsymbol{\beta}_0}{1+R} + \mathcal{O}(\tau_c)$$

# Bulk velocities (II)




1. Visible sector is tightly coupled with velocity $\beta_0$
2. Photon-baryon plasma evolves as $\beta_{\gamma b} = \beta_0 \left(1 + \frac{3\Omega_b a}{4\Omega_\gamma}\right)^{-1}$
3. After decoupling: $\beta_\gamma =$ const., $\beta_b \propto a^{-1}$

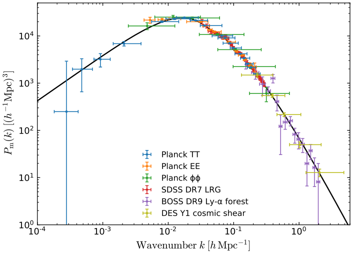
Bulk velocity constraints (kSZ from Planck '13)

$$\beta_0 < 1.6 \times 10^{-3} \text{ (95\% CL)}$$

# Perturbations

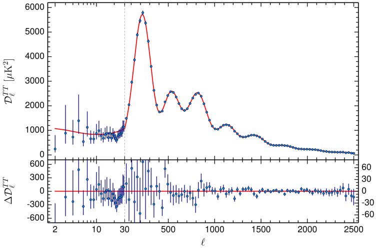
FLRW background is preserved, what about perturbations?

## Large scale structure (LSS)




[Credit: Planck collaboration]

## Temperature anisotropies

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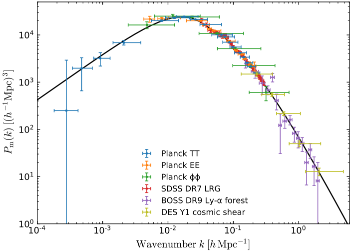
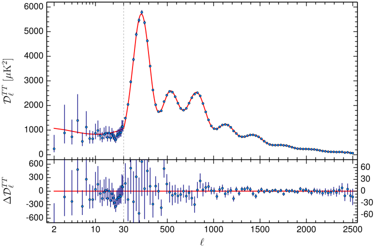



[Credit: Planck collaboration]

$$P_m(k) \quad \to \quad \text{No effect } \mathcal{O}(\beta)$$

**Temperature anisotropies**

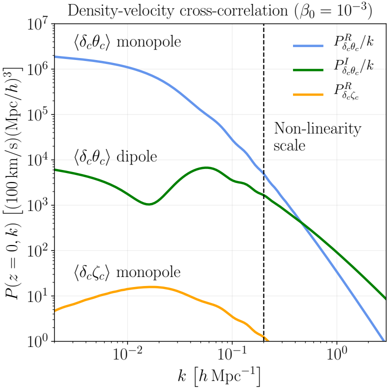



[Credit: Planck collaboration]

$$C_\ell \quad \to \quad \text{No effect } \mathcal{O}(\beta)$$

But there are new signatures. . .

# Effects on LSS




Any perturbation can be splitted

$$\delta(\tau,\boldsymbol{k}) = \underbrace{\delta^R(\tau,k)}_{\Lambda\text{CDM}} + \underbrace{\mathrm{i}(\hat{\beta}\cdot\hat{k})\delta^I(\tau,k)}_{\text{New } \mathcal{O}(\beta)}$$

We recover $\Lambda$CDM results for $\langle\delta\delta\rangle$

$$|\delta(\tau,\boldsymbol{k})|^2 = |\delta^R(\tau,k)|^2 + \mathcal{O}\left(\beta^2\right)$$

But we have new effects for cross-correlations $\langle\delta\theta\rangle$

$$\delta\theta^* = \delta^R\theta^{R\,*} + \mathrm{i}(\hat{\beta}\cdot\hat{k})\left(\delta^I\theta^{R\,*} - \delta^R\theta^{I\,*}\right) + \mathcal{O}\left(\beta^2\right)$$

# Effects on CMB

CMB temperature anisotropy observed from the solar frame

$$\Theta_\odot(\hat{n}_\odot) = \overbrace{\hat{n}_\odot \cdot \boldsymbol{d}_{\mathsf{kin}}}^{\text{Kinematic dipole}} + \frac{1}{4} \overbrace{(1 + \hat{n}_\odot \cdot \boldsymbol{d}_{\mathsf{mod}})}^{\text{Dipolar modulation}} \mathcal{F}_\gamma^{\Lambda\mathsf{CDM}} \overbrace{(\hat{n}_\odot - \boldsymbol{\nabla}(\hat{n}_\odot \cdot \boldsymbol{d}_{\mathsf{a}}))}^{\text{Aberration}}$$

$\Lambda$CDM result

$$\boldsymbol{d}_{\mathsf{kin}} = \boldsymbol{\beta}^{\odot}_{\mathsf{CMB}}$$
$$\boldsymbol{d}_{\mathsf{mod}} = \boldsymbol{\beta}^{\odot}_{\mathsf{CMB}}$$
$$\boldsymbol{d}_{\mathsf{a}} = \boldsymbol{\beta}^{\odot}_{\mathsf{CMB}}$$

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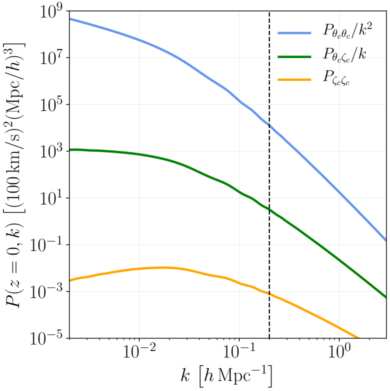
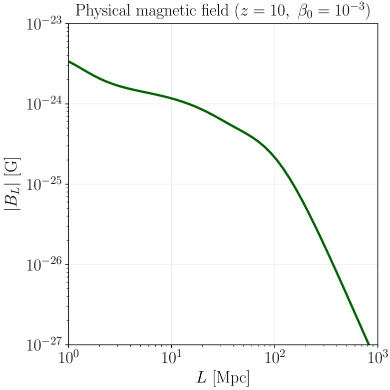
$$+ \frac{1}{4} \overbrace{(\hat{n}_\odot \cdot \boldsymbol{\beta}) \mathcal{F}_\gamma^{\beta}(\hat{n}_\odot)}^{\text{Intrinsic anisotropy}} + \mathcal{O}\left(\beta^2\right)$$

$\Lambda$CDM result + New contributions

$$\boldsymbol{d}_{\mathsf{kin}} = \boldsymbol{\beta}^\odot_{\mathsf{CMB}}$$
$$\boldsymbol{d}_{\mathsf{mod}} = \boldsymbol{\beta}^\odot_{\mathsf{CMB}} - 4\boldsymbol{\beta}$$
$$\boldsymbol{d}_{\mathsf{a}} = \boldsymbol{\beta}^\odot_{\mathsf{CMB}} - \boldsymbol{\beta}$$

# Bonus: Magnetic fields and vorticity



# Summary

- We have extended $\Lambda$CDM with **one** addtional parameter $\beta_0$ to accomodate a non-comoving dark sector.
- The background evolution is not modified (FLRW).
- The initial velocity of the visible sector in the CM frame is constrained

$$\beta_0 < 1.6 \times 10^{-3} \text{ (95\% CL)}$$

- The matter and temperature power spectra are not modified.
- New signatures appear as deviations from statistical isotropy.
- The new coupling between scalar and vector modes leads to production of vorticity and magnetic fields.

## Reference


*Non-comoving Cosmology*
JCAP 06 (2019) 041, arXiv:1903.11009
J.A.R. Cembranos, A.L. Maroto, HVR.

# Backup: Perfect fluids with bulk velocities

Assuming $w \neq -1$

$$\dot{\rho} = \frac{(v^2 - 3)(1 + w)}{1 - wv^2}\mathcal{H}\rho + \frac{\dot{w}}{1 - wv^2}v^2\rho$$

$$\dot{v} = \frac{(1 - v^2)(3w - 1)}{1 - wv^2}\mathcal{H}v + \frac{\dot{w}}{1 + w}\frac{1 - v^2}{1 - wv^2}v$$

*Radiation*, $w = 1/3$

$$\rho = \rho_0 a^{-4}$$

$$v = v_0 = \text{const.}$$

*Matter*, $w = 0$

$$\rho = \frac{\rho_0}{a^2\sqrt{v_0^2 + a^2(1 - v_0^2)}}$$

$$v = \frac{v_0}{\sqrt{v_0^2 + a^2(1 - v_0^2)}}$$

$$\gamma^2\rho = \frac{\rho_0}{a^4(1 - v_0^2)}\sqrt{v_0^2 + a^2(1 - v_0^2)}$$

# Backup: Perfect fluids with bulk velocities

Analytic expressions for a generic equation of state $w(a)$ can be obtained in the regime of small velocities

$$\rho = \rho_0 \exp\left(-3\int \frac{\mathrm{d}a}{a}\,(1+w)\right) + \mathcal{O}\big(v^2\big)$$

$$v = \frac{v_0(1+w_0)}{a^4(1+w)} \exp\left(3\int \frac{\mathrm{d}a}{a}\,(1+w)\right) + \mathcal{O}\big(v^2\big)$$

For the particular case $w = \text{const.}$,

$$\rho = \rho_0\, a^{-3(1+w)} + \mathcal{O}\big(v^2\big)$$

$$v = v_0\, a^{-(1-3w)} + \mathcal{O}\big(v^2\big)$$

## Backup: Dark sector behaviour

Center of mass condition

$$\sum_s T_s{}^i{}_0 = 0 \quad \rightarrow \quad T_{DS}{}^0{}_i = -\sum_{s=\gamma,b,\nu} T_s{}^0{}_i$$

The evolution of the total energy-momentum tensor is described by

$$\dot{\delta} + 3\mathcal{H}(c_{\mathsf{s}}^2 - w)\delta + (1+w)\,\theta - (1+w)\left(3\dot{\phi} - k^2(B - \dot{E})\right) = 0$$

$$\dot{\theta} + (1-3w)\,\mathcal{H}\theta + \frac{\dot{w}}{1+w}\theta - \frac{k^2}{1+w}c_{\mathsf{s}}^2\delta + \frac{4k^2}{3\,(1+w)}\sigma - k^2\psi = 0$$

we assume

- The dark sector is subdominant with respect to neutrinos and photons at early times, i.e. before the matter-domination era.
- There is a transition to a CDM behaviour at late times.