Obtaining the value of the dark energy from hyperconical universes

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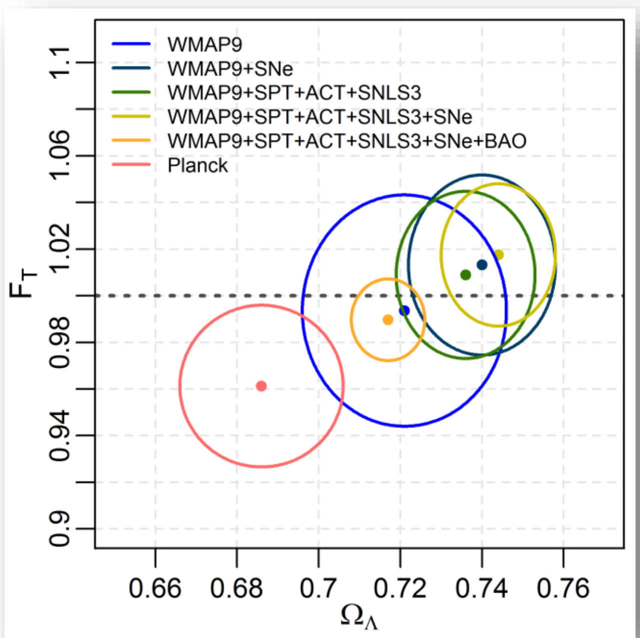
RESULTS & DISCUSSION

I. Introduction

Motivation

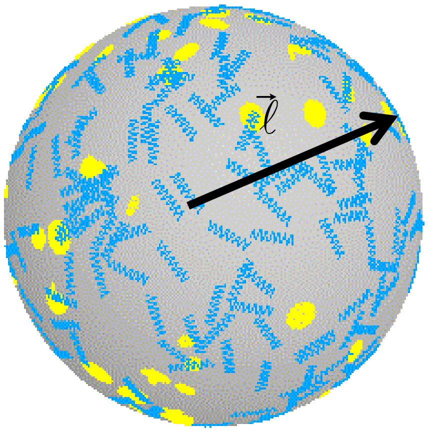


The most simple manifold with linear expansion:

$$S_t^3 = \{\vec{\ell} \in \mathrm{R}^4 : |\vec{\ell}| = R(t)\}$$

$$|\vec{\ell}'| = R(t_o) \qquad a(t) := \frac{R(t)}{R(t_o)} = \frac{t}{t_o}$$

$$\boxed{\vec{\ell}(t) = a(t)\vec{\ell}' \in S_t^3}$$



Similar idea

1/*H* = *t* Dirac-Milne model (2012) [k < 0]

$$d\vec{\ell}_{FRW} = a(t)d\vec{\ell}' \quad \Rightarrow \quad ds^2_{FRW} = dt^2 - a(t)\ \ d\vec{\ell}'$$

Proper-time preservation: homogeneous case

$$\boxed{d\vec{\ell} = a(t)d\vec{\ell}' + \vec{\ell}'\,da(t)} \quad \Rightarrow \quad ds^2 = dt^2 - d\vec{\ell}^2 \neq ds^2_{FRW}$$

Local-time preservation: inhomogeneous case

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II. General considerations

$$S_t^3 = \{(\vec{r}, u) \in \mathrm{R}^4 : \; \vec{r}^{\,2} + u^2 = v^2 t^2\} \subset \mathrm{H}^4$$

Equivalence in time

The local time of an observer in H^4 is the same that in $\mathrm{R}^{1,3}$

Let be x_0, $x \in \mathrm{R}^{1,3}$ two static points, $x_0 = (t_0, \mathbf{0})$, $x = (t, \mathbf{0})$, with $t > t_0 > 0$

Let be x_0', $x' \in \mathrm{H}^4$ their extension, $x_0 = (t_0, \mathbf{0}, \; vt_0)$, $x = (t, \mathbf{0}, \; vt)$

$$\exists g : \; \left| x' - x_0' \right|_g = \left| x - x_0 \right|_\eta = t - t_0 > 0$$

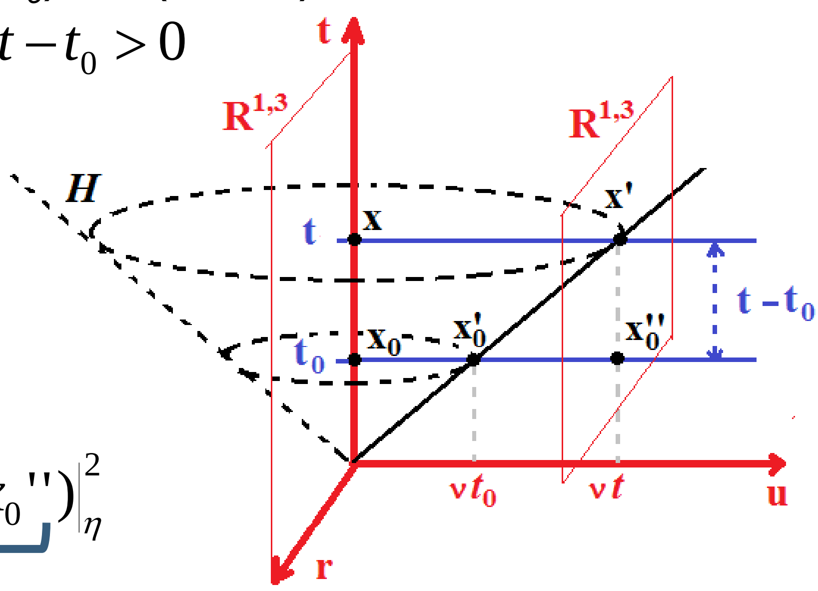
$$\exists x_0'' \in M \; : \; \left| x' - x_0'' \right|_\eta = \left| x - x_0 \right|_\eta$$

$x_0'' = (t_0, \mathbf{0}, t/t_0 \cdot vt_0)$

$$\exists g : \; \left| d(x' - x_0') \right|_g^2 = \left| d(x' - x_0'') \right|_\eta^2$$

$$X := x'(\vec{r}) \in \mathrm{H}^4$$

$$X_D := x'(\vec{r}) - x_0'' = \left(t - t_0, \; \frac{t}{t_0} \vec{r}\,', \; -vt \left(1 - \sqrt{1 - \frac{r'^2}{v^2 t_0^2}} \right) \right)$$



$k := v^{-2}$ *Curvature*

II. General considerations

Hypothesis of local equivalences and projection

Spatial projection

The local space of an observer in H^4 is the same that in $R^{1,3}$

That is, the spatial distance is given by a **locally conformal projection**
Let be *r*' the comoving distance in H^4, the spatial distance measured by an observer is:

$$\hat{r}' := f_{\hat{r}}^{\alpha}(r')$$

where

$$f_{\hat{r}}^{\alpha}(r') \in \{ {}_a f_{\hat{r}}^{\alpha}(r'),\ {}_b f_{\hat{r}}^{\alpha}(r'),\ \ldots \}$$

$${}_a f_{\hat{r}}^{\alpha}(r') = t_0 \gamma' \Delta^{\alpha}(\gamma'/\gamma_{\max}')$$

Distorted stereographic projection

$$\gamma' = \gamma'(r') := \sin^{-1}(r'/t_0)$$

$$\Delta^{\alpha}(x) := \frac{1}{(1-x)^{\alpha}}$$

$${}_b f_{\hat{r}}^{\alpha}(r') = 2t_0 \tan^{-1} \frac{\Delta^{\alpha}(\gamma'/\gamma_{\max}')}{2}$$

Inverse of distorted stereographic projection

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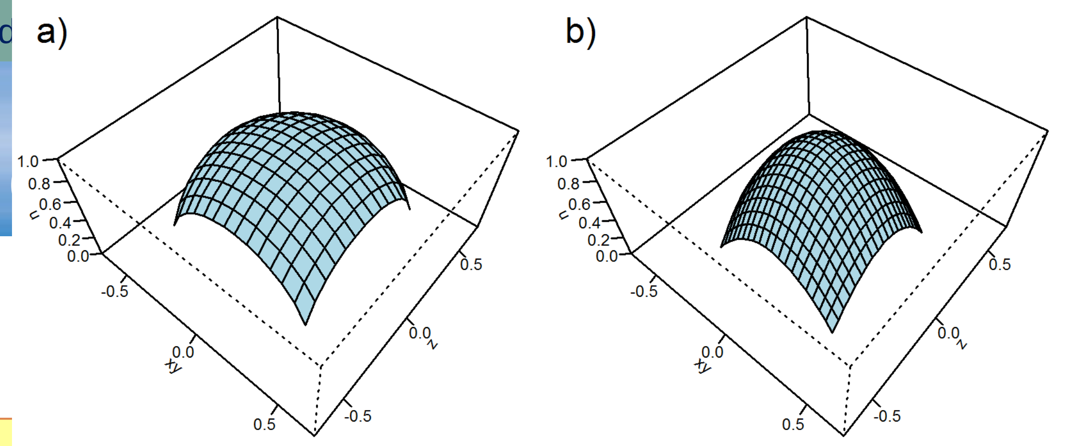
III. Theoreti



A. Metric tensor

Differential line element

$$dX_D := x'(\vec{r}) - x_0'' = \left(t - t_0,\quad \frac{t}{t_0}\vec{r}',\quad -vt\left(1-\sqrt{1-\frac{r'^2}{v^2 t_0^2}}\right)\right) dt - vt\left(1-\frac{r'^2}{v^2 t_{O'}^2}\right)^{-\frac{1}{2}} \frac{r'}{v^2 t_{O'}^2} dr'$$

$$dX^2\big|_{D,\eta} = dt^2\left(1+2v^2\sqrt{1-\frac{r'^2}{v^2 t_{O'}^2}}-2v^2\right) - \left(\frac{t}{t_{O'}}\right)^2 \left(\frac{dr'^2}{1-\frac{r'^2}{t_{O'}^2}} + r'^2 d\Sigma^2\right) - 2\frac{t}{t_{O'}}\frac{r'}{t_{O'}}\frac{dr'\,dt}{\sqrt{1-\frac{r'^2}{v^2 t_{O'}^2}}}$$

Metric tensor

Change of coordinates

$$b(r') := \sqrt{1-\frac{kr'^2}{t_{O'}^2}}$$

$$g_{00} = 1 + 2k^{-1}(b-1)$$

$$g_{r'r'} = -\frac{a^2}{b^2} \qquad g_{\theta\theta} = -a^2 r'^2$$

$$g_{0r'} = -a\frac{\dot{a}r'}{b} \qquad g_{\varphi\varphi} = -a^2 r'^2 \sin^2\theta$$

$$t' := t\sqrt{g_{00}}$$

$$a(t', r') = \frac{t'}{t_0\sqrt{g_{00}}}$$

$$g_{00}' = 1$$

$$g_{r'r'}' = g_{r'r'} - \frac{g_{0r'}^2}{g_{00}} = -a(t',r')^2 \frac{1-k^{-1}(1-b)^2}{b^2 g_{00}}$$

$$g_{0r'}' = 0$$

III. Theoretical features

B. Redshift-distance relation

Redshift and comoving distance

$$z := \frac{\lambda}{\lambda_0} - 1 = \frac{a_0}{a} - 1$$

$$dt' = -\frac{a}{a_0}\frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}dr'$$

$$\frac{dz}{dr'} = \frac{dz}{dt'}\frac{dt'}{dr'} = \frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}H_{hyp}(z)$$

$$H_{hyp}(z) = \frac{1}{t'} = \frac{1+z}{t_0}$$

$$\xi_k\left(\frac{r'}{t_0}\right) := \int_0^z \frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}\frac{dr'}{t_0} = \int_0^z \frac{dz'}{1+z'}$$

$$r_{hyp}' = t_0\ \xi_k^{-1}\left(\ln(1+z)\right)$$

$$\hat{r}_{hyp}' = f_r^{\alpha}\left(r_{hyp}'\right)$$

III. Theoretical features

C. Ricci curvature

$$R_{\alpha\beta} := R^{\mu}_{\alpha\mu\beta} = -\partial_{\beta}\Gamma^{\mu}_{\mu\alpha} + \partial_{\mu}\Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\rho\beta}\Gamma^{\rho}_{\alpha\mu} + \Gamma^{\mu}_{\rho\mu}\Gamma^{\rho}_{\alpha\beta} \qquad \Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_{\alpha} g_{\nu\beta} + \partial_{\beta} g_{\alpha\nu} - \partial_{\nu} g_{\alpha\beta}\right)$$

locally

$$R_{r'r'} = -\left(\frac{\dot a}{a}\right)^2 \frac{2k^2}{2b - b^2 + k - 1} g_{r'r'}$$

$$R_{\theta\theta} = -\left(\frac{\dot a}{a}\right)^2 \frac{(4b - b^2 + 2k - 3)k^2}{2b - b^2 + k - 1} g_{\theta\theta}$$

$$R_{\varphi\varphi} = -\left(\frac{\dot a}{a}\right)^2 \frac{(4b - b^2 + 2k - 3)k^2}{2b - b^2 + k - 1} g_{\varphi\varphi}$$

$$R_{ij} \approx -2k\left(\frac{\dot a}{a}\right)^2 g_{ij} \quad \Rightarrow \quad R \approx -6k\left(\frac{\dot a}{a}\right)^2$$

k ≡ 1

$$R^{FRW}_{ij} = -\left[\cancel{\frac{\ddot a}{a}} + 2\left(\frac{\dot a}{a}\right)^2 + \cancel{\frac{2K}{a^2}}\right] g^{FRW}_{ij} \quad \Rightarrow \quad R^{FRW} \approx -6\left(\frac{\dot a}{a}\right)^2$$

K ≡ 0

III. Theoretical features

D. Arnowitt-Deser-Misner (ADM) equations

$$\mathcal{S} = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g}\,(R - R_u + \mathcal{L}_M) \qquad R_u \approx -\frac{6k_o}{t^2} = -\frac{k_o}{2}\left(g^{ij}\partial_t g_{ij}\right)^2$$

$$\mathcal{L} = -g_{ij}\partial_t \pi^{ij} + lh + 2l_i \pi^{ij}{}_{;j} - 2\partial_i s^i + \sqrt{-g}R_u$$

$$\pi^{ij} = \sqrt{-g}\left[\left(\Gamma^0_{pq} - g_{pq}\Gamma^0_{rs}g^{rs}\right)g^{ip}g^{jq} - \frac{\partial R_u}{\partial(\partial_t g_{ij})}\right]$$

$l := (g^{00})^{-1/2}$ lapse

$l_i := g_{0i}$ shift

$g := \det g_{\mu\nu}$ total det

$g_s := \det g_{ij}$ spatial

$$\pi^{ii} \approx \sqrt{-g}\left[\left(-g_{ii}\frac{k_o r'^2}{2t_{\hat{o}}^2 t} + g_{ii}\delta^m_m \frac{k_o r'^2}{2t_{\hat{o}}^2 t}\right)(g^{ii})^2 + k_o (g^{ii})^2 \partial_t g_{ii}\right] \approx$$

$$\approx \sqrt{-g}\frac{k_o g^{ii}}{t}\left(\frac{k_o r'^2}{t_{\hat{o}}^2} + 2\right) \approx 2\sqrt{-g}\frac{k_o g^{ii}}{t} = \sqrt{-g}k_o (g^{ii})^2 \partial_t g_{ii}$$

$$\partial_t g_{ij} = 2\frac{1}{\sqrt{-g_s g^{00}}}\left(\pi_{ij} - \frac{1}{2}\pi g_{ij}\right) + \nabla_i g_{0j} + \nabla_j g_{0i}$$

$$\partial_t g_{ii} = \frac{1}{\sqrt{-g_s g^{00}}}\pi^{ii} {g_{ii}}^2$$

⇒ $k_o = k = 1$

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IV. Obtaining parameters

Equivalent proper distance: Obtaining cosmological parameters

$$\hat{r}_{\Lambda CDM}' = \hat{r}_{hyp}'$$

$$t_0 \equiv 1$$

$$\hat{r}_{\Lambda CDM}' = r_{\Lambda CDM}' = \int_0^z \frac{1}{H_{\Lambda CDM}(z')} dz'$$

$$\hat{r}_{hyp}' = f_r^{\alpha}(r_{hyp}') = \\ = f_r^{\alpha} \circ \xi_k^{-1} \circ \ln(1+z)$$

$$H_{hyp} = 1 + z$$

$$\int_0^z \frac{1}{H_{\Lambda CDM}(z')} dz' = \hat{r}_{hyp}' =: \int_0^z \frac{1}{\hat{H}_{hyp}(z')} dz'$$

$$H_{\Lambda CDM} = H_0 \sqrt{\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_\Lambda}$$

$$\hat{H}_{hyp} := \left(\frac{d}{dz} \circ f_r^{\alpha} \circ \xi_k^{-1} \circ \ln(1+z) \right)^{-1}$$

IV. Obtaining parameters

Equivalent proper distance: Obtaining cosmological parameters

$$H_{\Lambda}^{(2)} = \sqrt{\Omega_r + \Omega_m + \Omega_\Lambda} + \frac{4\Omega_r + 3\Omega_m}{\sqrt{\Omega_r + \Omega_m + \Omega_\Lambda}} \frac{z}{2} + $$

$$+ \frac{8\Omega_r^2 + (24\Omega_\Lambda + 12\Omega_m)\Omega_r + 12\Omega_m\Omega_\Lambda + 3\Omega_m^2}{(\Omega_r + \Omega_m + \Omega_\Lambda)^{3/2}} \frac{z^2}{8}$$

$$\hat{H}^{(2)} = 1 + \frac{\gamma_k - 2\alpha\sqrt{k}}{\gamma_k} z + \frac{5\alpha^2 k - 2\alpha\sqrt{k}\gamma_k - 3\alpha k + m\gamma_k^2}{2\gamma_k^2} z^2$$

$$m = \begin{cases} 2 & if \quad f^{\alpha} =_a f^{\alpha} \\ \frac{5}{2} & if \quad f^{\alpha} =_b f^{\alpha} \end{cases}$$

$$\alpha,\ \Omega_\Lambda,\ \Omega_m,\ \Omega_r \approx 0,\ k \equiv 1$$

IV. Obtaining parameters

Equivalent proper distance: Obtaining cosmological parameters

$$\Omega_\Lambda = \frac{3\alpha^2 k + 2\alpha\sqrt{k}\gamma_k - \alpha k}{2\gamma_k^2} + \omega_\Lambda$$

$$\Omega_m = \frac{2\alpha k(1-3\alpha)}{\gamma_k^2} + \omega_m$$

$$\Omega_r = \frac{9\alpha^2 k - 2\alpha\sqrt{k}\gamma_k - 3\alpha k}{2\gamma_k^2} + \omega_r$$

$$(\omega_\Lambda, \omega_m, \omega_r) = \begin{cases} \left(\frac{1}{2},\ 0,\ \frac{1}{2}\right), & f^\alpha = {}_a f^\alpha \\ \left(\frac{6+k}{12}, \frac{-k}{3}, \frac{2+k}{4}\right), & f^\alpha = {}_b f^\alpha \end{cases}$$

$$\Omega_r \approx 9.0 \pm 0.5 \cdot 10^{-5}$$
$$k \equiv 1 \equiv t_0$$

$$\alpha = 0.2830219501(1) \pm c_\alpha i$$
$$\Omega_\Lambda = 0.6937181(2) \pm c_\omega i$$
$$\Omega_m = 0.306192(6) \mp c_\omega i$$

$$(c_\alpha, c_\omega) = \begin{cases} (0.204263(4),\ 0.260076(4)), & f^\alpha = {}_a f^\alpha \\ (0.320386(2),\ 0.407928(3)), & f^\alpha = {}_b f^\alpha \end{cases}$$

$$\exists f^\alpha \rightarrow c_\alpha \quad such \quad as \quad c_\omega = 0$$

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V. Conclusions

- We have considered an **inhomogeneous model** with linear expansion that **does not depends on the matter** content.
- The Hyperconical model is **consistent with $k \equiv 1$**, and thus **freedom** for fitting is given by (locally conformal) **spatial projections f^{α}**.
- However, there exists a unique local **projection $f^{\alpha}(r \ll t)$ compatible with the ΛCDM model** (that is, where model provides real values).
- Thanks to this compatibility, the Hyperconical model **predicts** that **Ω_{Λ} = 0.6937181(2) and Ω_m = 0.306192(6)** for $t_o \equiv 1 \equiv k$ and Ω_r = (9.0 ± 0.5) $\cdot 10^{-5}$, **which is compatible** with the Planck Mission results (Ω_{Λ} = 0.6911 ± 0.0062 and Ω_m = 0.3089 ± 0.0062).

Questions

Thanks to Antonio López-Maroto and Rutwig Campoamor
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Thanks for your attention

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Description of the inhomogeneous model used:
R. Monjo, Phys. Rev. D, 96, 103505 (2017). doi:10.1103/PhysRevD.96.103505. arXiv:1710.09697

Geometric interpretation of the dark energy:
R. Monjo, Phys. Rev. D, 98, 043508 (2018). doi:10.1103/PhysRevD.98.043508. arXiv:1808.09793