VII Meeting on Fundamental Cosmology

Semiclassical avoidance of singularities in cosmology and black holes

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Introduction

- Semiclassical gravity: GR + QFT in curved spacetimes,

$$G_{\mu\nu} = 8\pi \left[T_{\mu\nu}^{CL} + l_P^2 \left\langle T_{\mu\nu}^{QM} \right\rangle_0 \right]. \tag{1}$$

- $\left\langle T_{\mu\nu}^{QM} \right\rangle_0$ is the renormalised vacuum expectation value of the stress-energy tensor operator, constructed from a quantum field operator and its covariant derivatives.
  - For a given field, it is a function of the metric and its derivatives.
  - It is zero in flat spacetime.
  - It can be calculated analytically for very few spacetimes.

Semiclassical cosmology

For a homogeneous and isotropic universe,

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2)\right], \qquad (2)$$

and a conformally coupled field, it has been shown that[1]

$$\begin{aligned}\left\langle T^{QM}_{\mu\nu}\right\rangle_0 = \frac{\alpha}{3}\left(g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R + RR_{\mu\nu} - \frac{1}{4}R^2 g_{\mu\nu}\right) + \\ \beta\left(\frac{2}{3}RR_{\mu\nu} - R_\mu{}^\rho R_{\nu\rho} + \frac{1}{2}R_{\rho\sigma}R^{\rho\sigma}g_{\mu\nu} - \frac{1}{4}R^2 g_{\mu\nu}\right)\end{aligned} \qquad (3)$$

[1] P. C. W. Davies, S. A. Fulling, S. M. Christensen, and T. S. Bunch, Ann. Phys. (N.Y.) 109, 108 (1977).

Energy content near the initial singularity

Consider a classical universe in which for $a \to 0$ we have

$$\rho^{CL} \sim \frac{1}{a^s}. \tag{4}$$

The leading order terms close to the singularity for the quantum contributions to the energy density and pressure are

$$\begin{aligned} \rho^{QM} &\sim \frac{1}{a^{2s}}\left[\frac{1}{(s/2-1)^2}(-3\alpha+\beta)+3\alpha\right], \\ p^{QM} &\sim (2s-3)\frac{1}{3}\rho^{QM}. \end{aligned} \tag{5}$$

- We get a larger divergence for the quantum term, $\rho^{QM} \sim (\rho^{CL})^2$.
- The sign of the quantum energy depends of the coefficients α, β.

Some bouncing solutions

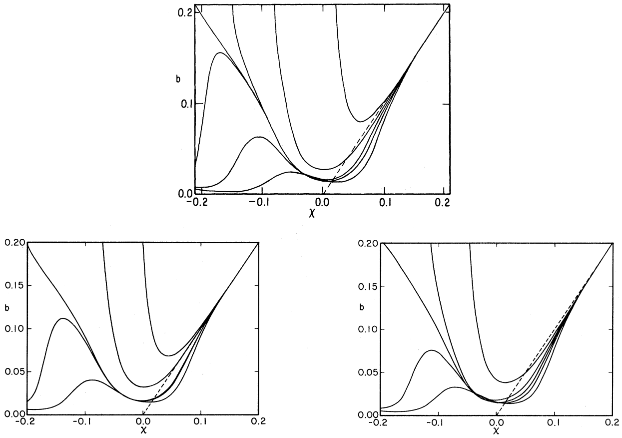

Figure: Asymptotically classical solutions for $k = 0, 1$ and -1. Constants $\beta = 6\alpha = 1/480\pi^2$. Source: Paul Anderson, Phys. Rev. D 28, 2695 (1983).

Black holes problems

What about black hole singularities?

- There are no known solutions for $\langle T_{\mu\nu}^{QM} \rangle$ in spacetimes approaching the formation of black holes.
- Approximations for spherically symmetric models fail near the origin.
- But there are some indications that something similar may occur...

Oppenheimer-Snyder model

The Oppenheimer-Snyder model for black-hole formation consists of a collapsing homogeneous ball of dust, the interior of which behaves as a patch of a $k = 1$ contracting universe,

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2)\right]. \tag{6}$$

- The classical energy density behaves as $\rho^{CL} \sim 1/a^3$.
- An approximation (through dimensional reduction) suggests the quantum contribution to be $\rho^{QM} \sim -1/a^5$.

  $\rightarrow$ A full semiclassical treatment of the problem will give a substantially different result from its classical counterpart.

Conclusion

- The semiclassical theory provides a first approach toward quantum corrections of classical gravity.
- Cases in which curvature is large (nearly Planckian) are among those which would have substantial corrections.

Thank you for your attention!