Stability in quadratic torsion theories

VII Meeting on fundamental Cosmology

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General Relativity

A particular modification

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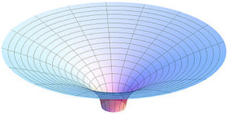
Stress-energy ⇔  Spin distribution ⇔ 

CURVATURE

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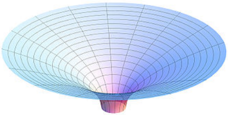
Stress-energy ⇔  Spin distribution ⇔ TORSION $T^{\mu}_{\cdot\nu\sigma} \equiv \tilde{\Gamma}^{\mu}_{\cdot[\nu\sigma]}$

CURVATURE

Quadratic torsion theory

Goals

Goals

Theory of gravity with torsion close to **GR**

Quadratic torsion theory

The action

The most general quadratic Lagrangian density:

$$\mathcal{L} = -\lambda\widehat{R} + \frac{1}{12}(4a + b + 3\lambda)T_{\mu\nu\rho}T^{\mu\nu\rho} + \frac{1}{6}(-2a + b - 3\lambda)T_{\mu\nu\rho}T^{\nu\rho\mu}$$
$$+\frac{1}{3}(-a + 2c - 3\lambda)T^{\mu}T_{\mu} + \frac{1}{6}(2p + q)\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\mu\nu\rho\sigma}$$
$$+\frac{1}{6}(2p + q - 6r)\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\rho\sigma\mu\nu} + \frac{2}{3}(p - q)\widehat{R}_{\mu\nu\rho\sigma}\widehat{R}^{\mu\rho\nu\sigma}$$
$$+(s + t)\widehat{R}_{\nu\sigma}\widehat{R}^{\nu\sigma} + (s - t)\widehat{R}_{\nu\sigma}\widehat{R}^{\sigma\nu} + \mathcal{L}_M$$

parameters: λ, a, b, c, p, q, r, s and t

nine-parameter

[1] Phys. Rev. D **21** 3269 (1980)
[2] Gen. Relat. Gravit. **21** 1107 (1989)

Quadratic torsion theory

Conditions

Constraints on the theory:

1. To recover GR when $T^{\mu}_{\cdot\nu\rho} = 0$
2. Stability in weak-gravity: Ghost & tachyon free

Reduction to GR

$$\mathcal{L}_g\big|_{T=0} = -\lambda R + (p-r) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 2s R_{\mu\nu} R^{\mu\nu}$$

$$\downarrow$$

Conditions: $\boxed{p = r \ , \quad s = 0}$

Stability analysis

Strategy

Strategy to tackle the problem

- Take the decoupling limit: $g_{\mu\nu} = \eta_{\mu\nu}$ $\eta = (+,-,-,-)$
- Only vectorial (T_μ) and pseudo-vectorial (S_μ) d.o.f. :

$$T^{\alpha}_{\cdot\mu\nu} = \frac{1}{3}(T_\mu \delta_\nu{}^\alpha - T_\nu \delta_\mu{}^\alpha) + \frac{1}{6} g^{\alpha\beta} \epsilon_{\beta\mu\nu\sigma} S^\sigma$$

- Analyse the kinetic terms in $\mathcal{L}$
- Analyse the potential terms in $\mathcal{L}$

Stability analysis

Ghost-free sector

$$\mathcal{L}_g = \frac{8}{9}(p+s+t)F_{\mu\nu}(T)F^{\mu\nu}(T) + \frac{16}{3}(p-r+2s)\partial_\mu T^\mu \partial_\nu T^\nu$$

$$+\frac{1}{18}(2p+t)F_{\mu\nu}(S)F^{\mu\nu}(S) + \frac{1}{6}q\partial_\mu S^\mu \partial_\nu S^\nu - \mathcal{V}(T,S)$$

Stability analysis

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$$\downarrow$$

	T^μ	S^μ
Ghost-free	$p-r+2s=0$ $p+s+t<0$	$q=0$ $2r+t<0$

[3] Phys. Rev. D **81** 063519 (2010)

Stability analysis

Tachyon-free sector

Weak torsion regime:

$$\mathcal{V}^{(2)}(T,S) = -\frac{2}{3}(c+3\lambda)T_\mu T^\mu - \frac{1}{24}(b+3\lambda)S_\mu S^\mu$$

Stability analysis

Tachyon-free sector

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$$c+3\lambda > 0 \qquad\qquad b+3\lambda > 0$$

Strong torsion regime:

$$\mathcal{V}^{(4)}(T,S) = -\frac{64}{27}(p-r+2s)T_\alpha T^\alpha T_\beta T^\beta$$

$$-\frac{1}{108}(p-r+2s)S_\alpha S^\alpha S_\beta S^\beta - \frac{8}{81}(2p+3q-4r+2s)T_\alpha S^\alpha T_\beta S^\beta$$

$$-\frac{8}{81}(p+r+4s)T_\alpha T^\alpha S_\beta S^\beta$$

Stability analysis

Tachyon-free sector

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$$-\tfrac{8}{81}(p+r+4s)T_\alpha T^\alpha S_\beta S^\beta$$

$\longrightarrow$ annihilated with $p+3s=0$
\+ ghost-free

Quadratic torsion theory

Conditions analysis

	T^μ	S^μ
Ghost-free	$p - r + 2s = 0$ $p + s + t < 0$	$q = 0$ $2r + t < 0$
Tachyon-free (Weak torsion)	$c + 3\lambda > 0$	$b + 3\lambda > 0$
Tachyon-free (General torsion)	$p + 3s = 0$ $c + 3\lambda > 0$	$p + 3s = 0$ $b + 3\lambda > 0$
Reduction to GR when $T^{\alpha}_{\cdot\mu\nu} = 0$	$p - r = 0$ $s = 0$	

Quadratic torsion theory

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E. Sezgin & P. van Nieuwenhuizen:

[1] Phys. Rev. D **21** 3269 (1980)

[4] Phys. Rev. D **24** 1677 (1981)

Quadratic torsion theory

Conditions analysis

	T^μ	S^μ
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$p + s + t < 0$ $\quad$ $2r + t > 0$

Conclusions

Goals

Theory of gravity with torsion close to **GR**

Results

$$\mathcal{L} = -\lambda\widehat{R} + \frac{1}{12}(4a + b + 3\lambda)T_{\mu\nu\rho}T^{\mu\nu\rho} + \frac{1}{6}(-2a + b - 3\lambda)T_{\mu\nu\rho}T^{\nu\rho\mu} + \frac{1}{3}(-a + 2c - 3\lambda)T^{\lambda}_{\cdot\mu\lambda}T_{\rho}^{\cdot\mu\rho} + 2t\widehat{R}_{\mu\nu}\widehat{R}^{[\mu\nu]} + \mathcal{L}_M$$

with $t < 0$, $c + 3\lambda > 0$ and $b + 3\lambda > 0$.