Modified gravity effects on the primordial power spectrum in warm inflation

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Scheme

Why warm inflation

Why modified gravity

Conclusions and future prospects

Why inflation

- An early period of **inflation** accounts for the inferred spatial flatness of the Universe and also provides a solution to the horizon problem.
- A scalar field ϕ whose potential energy V dominates over other components while on a slow-roll trajectory mimics the equation of state of vacuum and that makes it a viable candidate for an **inflaton** field.
- The process of inflationary expansion must cap off somehow. An almost-exponential expansion leads to the slow-roll conditions $\epsilon, |\eta| \ll 1$, where:

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2; \quad \eta = M_{Pl}^2 \frac{V_{,\phi\phi}}{V}.$$

  These conditions must be satisfied if a prolonged period of exponential expansion is considered.

Why not *cold* inflation

- The η-condition implies a fairly light inflaton field, as $m_\phi \equiv V_{,\phi\phi} \ll H$, that is, well below the Hubble scale.
- Light scalar fields are unrealistic in any promising effective field theory as quantum corrections make their masses severely large.
- The **cold** scenario, despite serving as a useful guide when addressing some of the problems of the old Hot Big Bang model, drags the above drawback (also called the η-*problem*).
- It is noticed that a dominant scalar potential energy may not exclude the presence of other components or degrees of freedom.
- As inflation must end and lead to a viable universe, a later reheating period is needed in the cold scenario, and so is a coupling between the inflaton and other degrees of freedom...

This coupling plays no role during the course of inflation in the cold scenario, but it can be taken into account when extra components are not utterly diluted due to the almost-exponential expansion!

Warm inflation in a nutshell

- In **warm inflation** a radiation bath is sustained during inflation due to the decay of the inflaton, thus the reheating period already takes place.
- Thermal fluctuations come into play as the main source of primordial fluctuations and enter the equation of field perturbations through a noise term.
- The slow-roll conditions get modified in such a way that no extremely flat potential is required:

$$\epsilon, |\eta| \ll 1 + Q;$$

  Q denoting the **dissipative ratio** $Q \equiv \Upsilon / (3H)$, and Υ the **dissipative coefficient**.
- The *strong dissipative regime* (SDR) $Q \gg 1$ nicely gets round the η-problem and produces a nearly scale-invariant spectrum of primordial curvature perturbations.

Realizing warm inflation

- A *consistent* quantum field theory *has proved*[1] to be intricate and elusive.
- Due to the dissipative coefficient, field and radiation fluctuations get coupled, leading to an unwanted enhancement of the amplitude of primordial perturbations, and to the wrong spectral index.
- It is possible to damp that growth via shear effects that stem from the radiation bath's departure from the equilibrium...
- However, only **weakly interacting** radiation fluids can easily have shear viscosities of sufficient magnitude to counterbalance and suppress that growth.
- But the inflaton usually gives a large mass to the particle it couples to! (think of the Higgs field).

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[1] See M. Bastero-Gil, A. Berera, R. O. Ramos and J. G. Rosa. **Towards a reliable effective field theory of inflation**. arXiv:1907.13410.

Modified gravity

- The quantization of both gravity and matter fields in a classic background demand higher order curvature terms and non-minimal couplings to gravity, respectively:

$$f(\phi, R) = M_{Pl}^2 R + \frac{\phi^b R^d}{M^{b+2d-4}}.$$

  M denotes a mass scale that might match the Planck scale. Just a single scalar field has been considered (the inflaton). The corresponding Lagrangian reads:

$$\mathcal{L} = \frac{1}{2} f(\phi, R) - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) + \mathcal{L}_r,$$

  $\mathcal{L}_r$ denoting the Lagrangian of matter (radiation).

- Modified gravity corrections can be treated as an effective fluid:

$$G^{\mu\nu} = \frac{1}{F}\left(T^{\mu\nu}_{f\phi} + T^{\mu\nu}_r\right) \equiv \tilde{T}^{\mu\nu}_{f\phi} + \tilde{T}^{\mu\nu}_r,$$

  where $T^{\mu\nu}_r$ denotes the energy-momentum tensor of radiation (that is traceless).

Slow-roll equations and new slow-roll parameters

- After including modified gravity corrections, we have a slight variation of those slow-roll parameters and a new bunch of them:

$$\epsilon = \frac{1}{2}\left(\frac{W_{,\phi}}{\rho}\right)^2 \frac{F}{1+Q}; \quad \eta = \frac{W_{,\phi\phi}}{\rho}\frac{F}{1+Q};$$

$$\Theta = \frac{\dot{F}}{FH}; \qquad \beta_\Upsilon = \frac{W_{,\phi}\Upsilon_{,\phi}}{\rho\Upsilon}\frac{F}{1+Q}.$$

Slow-roll equations

Plus, slow-roll equations can be written such that:

$$|\dot{\phi}| = \frac{W_{,\phi}}{3H\left(1+Q\right)}; \quad 3H^2 = \frac{V}{F};$$

$$H\dot{F} = \left(1+Q\right)\dot{\phi}^2; \quad 4\rho_r = 3Q\dot{\phi}^2.$$

Fluctuations at linear order

- We shall consider the perturbations at linear order.
- We employ the perturbed Robertson-Walker metric including only scalar perturbations:

$$\mathrm{d}s^2 = -(1+2\alpha)\,\mathrm{d}t^2 - 2a\partial_i\beta\mathrm{d}x^i\mathrm{d}t + a^2\left[\delta_{ij}(1+2\varphi) + 2\partial_i\partial_j\gamma\right]\mathrm{d}x^i\mathrm{d}x^j,$$

  where $K = 0$ (i.e., it is spatially flat).
- From the Einstein field equations at linear order, one obtains:

$$\mathcal{A} = -\frac{\dot{H}}{H^2}\mathcal{R} = \frac{\dot{H}}{H}\frac{1}{\rho+P}\left(\Psi_r^{GI} - \dot{\phi}\delta\phi^{GI} - \delta\dot{F}^{GI} + H\delta F^{GI} + \dot{F}\mathcal{A}\right);$$

$$\frac{k^2}{a^2H^2}\Phi = 3\mathcal{A} + \frac{3}{2}\left(\frac{\delta\rho^{GI}}{\rho} - \frac{\delta F^{GI}}{F}\right);$$

  where one has the gauge invariant combinations:

$$\mathcal{A} \equiv \alpha - \left(\frac{\dot{\varphi}}{H}\right); \quad \Phi \equiv \varphi - H\chi.$$

Shear effects stem from Modified Gravity

- χ denotes the **shear** such that:

$$\chi \equiv a\left(\beta + a\dot{\gamma}\right).$$

- The corresponding perturbed shear viscous tensor when modified gravity correction terms are taken into account has **non-vanishing spatial components**:

$$\tilde{\pi}_{ij}^{f\phi} = -\frac{1}{3}\partial^k\partial_k\sigma\delta_{ij} + \partial_i\partial_j\sigma;$$

where:

$$\sigma \equiv \frac{1}{F}\left(\delta F - \chi\dot{F}\right).$$

We can put it this way too:

$$\pi_{ij}^{f\phi} = -\dot{F}\sigma_{ij} - \frac{1}{3}\partial^k\partial_k\left(\delta F + \Psi\dot{F}\right)\delta_{ij} + \partial_i\partial_j\left(\delta F + \Psi\dot{F}\right);$$

σ_{ij} and Ψ being the shear tensor and the momentum perturbation respectively. $\dot{F}$ may be associated to the **shear viscous coefficient**.

Perturbed equations at linear order

Field perturbation

Field perturbations are affected by thermal fluctuations (Gaussian white noise term), and radiation and F perturbations:

$$\delta\phi^{GI''} + 3\left(1 + Q - \Theta\frac{\rho_r}{3\dot{\phi}^2} + \frac{H^{'}}{H}\right)\delta\phi^{GI'} + \left[\frac{k^2}{a^2H^2} + 3\eta\left(1+Q\right)\right]\delta\phi^{GI} =$$

$$= \left(1+3Q\right)^{1/2}\left(2HT\right)^{1/2}\left(Ha\right)^{-3/2}\xi - \frac{\delta\Upsilon^{GI}}{H}\phi^{'} + \frac{1}{2H^2}F_{,\phi}\delta R^{GI} + \phi^{'}\mathcal{A}^{'} +$$

$$+ 2\left[\phi^{''} + \left(3 + \frac{H'}{H}\right)\phi^{'}\right]\mathcal{A} + 3Q\phi^{'}\mathcal{A} - \frac{k^2}{a^2H^2}\Phi\phi^{'} - \frac{\Theta}{\phi^{'}H^2}\delta\rho_r^{GI} -$$

$$- \frac{\rho_r}{F\phi^{'}H^2}\left(\delta F^{GI'} - \Theta\delta F^{GI}\right).$$

- $\delta\Upsilon^{GI}$ and modified gravity corrections introduce couplings between $\delta\phi$ and $\delta\rho_r$!

Perturbed equations at linear order

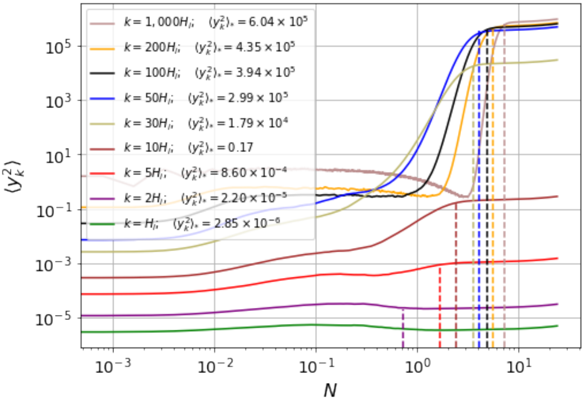
Radiation and its momentum perturbations

Radiation fluctuations get coupled to the radiation momentum perturbation Ψ_r too:

$$\delta\rho_r^{GI'} + 4\left(1 - \frac{\Theta}{4}\right)\delta\rho_r^{GI} = \frac{k^2}{a^2}\left(\frac{\Psi_r^{GI}}{H} - \frac{4}{3}\rho_r\frac{\Phi}{H^2}\right) + \frac{1}{H}\delta\Upsilon^{GI}\phi'^2 +$$
$$+ 6QH^2\phi'\delta\phi^{GI'} - 3QH^2\phi'^2\mathcal{A} + \frac{\rho_r}{F}\left(\delta F^{GI'} - \Theta\delta F^{GI}\right);$$

$$\Psi_r^{GI'} + (3 - \Theta)\Psi_r^{GI} = -\frac{1}{3H}\delta\rho_r^{GI} - 3QH\phi'\delta\phi^{GI} - \frac{4}{3H}\rho_r\mathcal{A} + \frac{1}{3}\rho_r\frac{\delta F^{GI}}{HF}.$$

No modified gravity (no shear)



$\langle y_k^2 \rangle$ vs N ($c = 3$), where all the values at horizon crossing are indicated by vertical dashed lines. With these values at horizon crossing, one obtains a value for n_s that overestimates the observed one by far ($n_s^{\mathrm{obs}} = 0.968$).

Perturbed equations at linear order

F-field perturbation

The new field couples explicitly to the inflaton field perturbation:

$$\delta F^{GI''} + \left(3 + \frac{H^{'}}{H}\right)\delta F^{GI'} + \left[\frac{k^2}{a^2H^2} - 2\left(2 + \frac{H^{'}}{H}\right)\right]\delta F^{GI} - \frac{4}{3}\frac{W_{,\phi}}{H^2}\delta\phi^{GI} +$$
$$+ \frac{2}{3}\phi^{'}\delta\phi^{GI'} + \frac{F}{3H^2}\delta R^{GI} = F^{'}\mathcal{A}^{'} + \left[F^{''} + \left(3 + \frac{H'}{H}\right)F^{'} + \frac{1}{3}\phi^{'2}\right]2\mathcal{A} -$$
$$- \frac{k^2}{a^2H^2}\Phi F^{'}.$$

- δR^{GI} is given by the Einstein field equations:

$$\delta R^{GI} = 2\left[\left(\frac{k^2}{a^2} - R\right)\mathcal{A} - 3H\dot{\mathcal{A}} + \left(4H^2 + \dot{H}\right)\frac{k^2}{a^2H^2}\Phi + H\dot{\left(\frac{k^2}{a^2H^2}\Phi\right)}\right];$$

or $\delta R^{GI} = \frac{1}{F_{,R}}\delta F^{GI} - \frac{F_{,\phi}}{F_{,R}}\delta\phi^{GI}$ if $F_{,R} \neq 0$ (i.e., $d \neq 1$).

Primordial power spectrum

- The expression for the **comoving curvature perturbation** is the same as in General Relativity.
- It measures the **spatial curvature of comoving hypersurfaces** and is gauge invariant:

$$\mathcal{R} = -\frac{H}{\rho + P}\left(\Psi_r^{GI} - \dot{\phi}\delta\phi^{GI} - \delta\dot{F}^{GI} + H\delta F^{GI} + \dot{F}\mathcal{A}\right);$$

or:

$$\mathcal{R} = -\frac{H}{(1+\Theta/2)(\rho + P)}\left(\Psi_r^{GI} - \dot{\phi}\delta\phi^{GI} - \delta\dot{F}^{GI} + H\delta F^{GI}\right).$$

- Both, the scale dependence of primordial perturbations and the relative amount of tensor perturbations (primordial gravitational waves) are extracted from the **primordial power spectrum**,

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\langle|\mathcal{R}_k|^2\rangle,$$

and the predictions are set in terms of these features.

Background variables

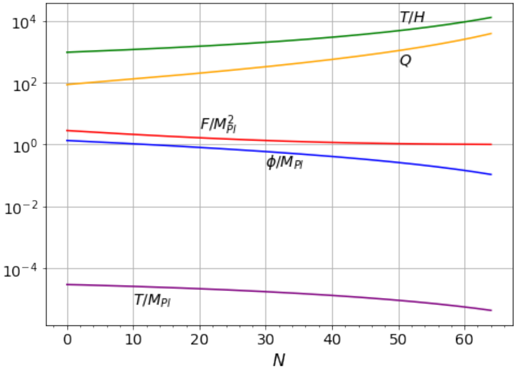
- We can calculate how the background variables evolve w.r.t. N till 60 e-folds. We take $f(\phi, R) = \left(M_{Pl}^2 + \phi^2\right) R$.

$\phi_*/M_{Pl} = 1.2$

$F_*/M_{Pl}^2 = 2.5$

$(T/H)_* = 1.1 \times 10^3$

$Q_* = 104.9$



- We first try to solve the perturbed equations by considering the slow-roll approximation **explicitly**; i.e., the background variables are almost constant and slow-roll suppressed terms are taken away.

Discarding some models

With these new dimensionless variables:

$$y_k = \frac{k^{3/2}}{\left[2\left(\Upsilon + H\right) T\right]^{1/2}} \delta\phi^{GI}; \quad w_k = \frac{k^{3/2}}{\left[2\left(\Upsilon + H\right) T\right]^{1/2}} \frac{\delta\rho_r^{GI}}{\Upsilon\dot{\phi}};$$

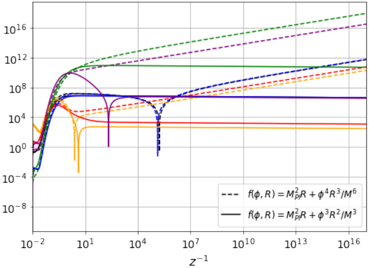
$$u_k = \frac{k^{3/2}}{\left[2\left(\Upsilon + H\right) T\right]^{1/2}} \frac{\Psi_r^{GI}}{\dot{\phi}}; \quad s_k = \frac{k^{3/2}}{\left[2\left(\Upsilon + H\right) T\right]^{1/2}} \frac{H\delta F^{GI}}{\dot{\phi}},$$

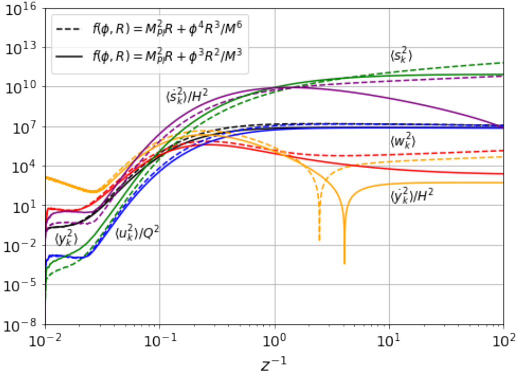
we solve the perturbation equations with the explicit slow-roll approximation for some models.

- Solving those equations allow us to remove some models according to the resulting super-horizon perturbations
- It can be checked that only $\phi^b R^2$ models give a constant amplitude when field perturbations cross the horizon (for $F_{,R} \neq 0$).

Discarding some models

For $\phi^4 R^3$ and $\phi^3 R^2$, we have:

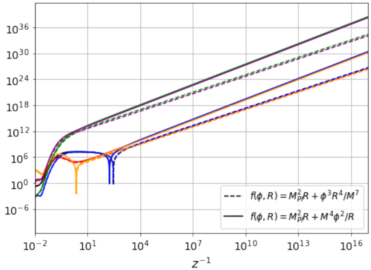


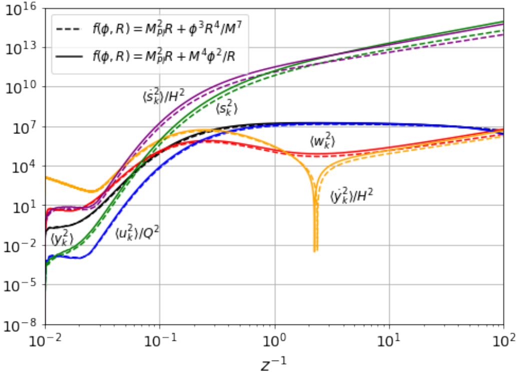


For $\phi^3 R^2$ we have set $M = 3 \times 10^{-5} M_{Pl}$, so $Q = 106.7$, belonging to the strong dissipative regime. We have $\phi = 1.4 M_{Pl}$ and $F = 3 M_{Pl}^2$ too. For $\phi^4 R^3$, $Q = 115.7$, $\phi = 1.3 M_{Pl}$ and $F = 2.5 M_{Pl}^2$ (and same M).

Discarding some models

For $\phi^3 R^4$ and ϕ^2/R, we have:





For $\phi^3 R^4$ we have set $M = 3 \times 10^{-6} M_{Pl}$, getting $Q = 110.2$, $\phi = 2.7 M_{Pl}$ and $F = 9 M_{Pl}^2$. For ϕ^2/R, $Q = 116.9$, $\phi = 0.6 M_{Pl}$ and $F = 0.3 M_{Pl}^2$ (and $M = 8 \times 10^{-8} M_{Pl}$).

Conclusions and future prospects

Conclusions

1. Warm inflation successfully deals with the η-problem in the SDR.
2. Shear effects also stem from modified gravity and may damp the unwanted growth of field perturbations when $\Upsilon \propto T^3$.
3. Only $\phi^b R^2$ models lead to a constant amplitude of superhorizon field modes when slow-roll approximation is explicitly assumed ($F_{,R} \neq 0$).

Prospects

1. Starting with a well-known modified gravity model, such as $\phi^2 R$, we shall solve the perturbation equations and check the different amplitudes of superhorizon modes.
2. These must not differ from a red-tilted, almost scale-invariant power spectrum.