Constructing new non-singular and ghost-free theories of gravity

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National Research Foundation



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- General aspects of the theories with torsion
- Motivation for infinite derivative gravity (IDG)

- 3 Linear IDG with axial torsion
 - Ghost-free conditions
 - Solving the torsion singularities

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Introduction Why torsion theories?

Sciama (1962) and Kibble (1961)

 $\bullet\,$ Poincaré gauge invariance \longrightarrow Naturally defined spin fields

 More degrees of freedom —> New possibilities to explain open problems in Cosmology

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General connections

An arbitrary connection has D^3 degrees of freedom in D dimensions:

• $\frac{D^2(D-1)}{2}$ in the antisymmetric part (*torsion*):

$$T^{\rho}_{\mu\nu} \equiv \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} \; . \label{eq:T_multiple}$$

• $\frac{D^2(D+1)}{2}$ in the *non-metricity* tensor:

$$Q_{\rho\mu\nu} = \nabla_{\rho}g_{\mu\nu}$$
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Properties of the theories with torsion

- Gauge theory of the Poincaré Group.
- Metricity

$$\nabla_{\rho}g_{\mu\nu}=0$$

Non symmetric connection

$$T^{\rho}_{\mu\nu} \equiv \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} \neq 0$$

• Relation between the connection and the Levi-Civita one

$$\mathring{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + K^{\rho}_{\mu\nu}$$

where

$$\mathcal{K}^{\rho}_{\mu\nu} = \frac{1}{2} \left(T^{\rho}_{\mu\nu} + T^{\rho}_{\nu\mu} - T^{\rho}_{\mu\nu} \right)$$

Decomposition of the torsion tensor

$$T_{\mu\nu\rho} = \frac{1}{3} \left(T_{\nu} g_{\mu\rho} - T_{\mu} g_{\mu\nu} \right) - \frac{1}{6} \varepsilon_{\mu\nu\rho\sigma} S^{\sigma} + \frac{q_{\mu\nu\rho}}{q_{\mu\nu\rho}},$$

where

$$\underbrace{T_{\mu} = T_{\mu\nu}^{\nu}}_{\text{Trace}}, \underbrace{S^{\mu} = \varepsilon^{\rho\sigma\nu\mu}T_{\rho\sigma\nu}}_{\text{Axial vector}}, \underbrace{q^{\mu}_{\nu\rho} \text{ s.t. } q^{\nu}_{\mu\nu} = 0 \text{ and } \varepsilon^{\rho\sigma\nu\mu}q_{\rho\sigma\nu} = 0}_{\text{Tensor}}$$

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Why infinite derivative theories?

- Modification at the UV level → Compatible with current experimental data.
- Infinite derivatives —> Can be made ghost and singularity free.

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Most general IDG quadratic action with torsion

A. Cruz-Dombriz, FJMT, A. Mazumdar: 1812.04037. Phys. Rev. D, April 2019.

$$S = \int d^{4}x \sqrt{-g} \left[\frac{\widetilde{R}}{2} + \widetilde{R}_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \mathscr{O}_{\mu_{2}\nu_{2}\rho_{2}\sigma_{2}}^{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \widetilde{R}^{\mu_{2}\nu_{2}\rho_{2}\sigma_{2}} \right. \\ \left. + \widetilde{R}_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \mathscr{O}_{\mu_{2}\nu_{2}\rho_{2}}^{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \mathcal{K}^{\mu_{2}\nu_{2}\rho_{2}} + \mathcal{K}_{\mu_{1}\nu_{1}\rho_{1}} \mathscr{O}_{\mu_{2}\nu_{2}\rho_{2}}^{\mu_{1}\nu_{1}\rho_{1}} \mathcal{K}^{\mu_{2}\nu_{2}\rho_{2}} \right]$$

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Physical motivation

\bullet Non-local terms \longrightarrow Effective theory with quantum effects..

• Torsion \longrightarrow From the Palatini formalism.

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Physical motivation

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Linearised action

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},$$

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + K^{\rho}_{\mu\nu},$$

where $K \sim \mathscr{O}(h)$. Then

$$S_q = -\int d^4x \sqrt{-g} \left(\mathscr{L}_M + \mathscr{L}_{MT} + \mathscr{L}_T \right) = S_M + S_{MT} + S_T$$

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Linearised action components

$$\mathscr{L}_{M} = \frac{1}{2} h_{\mu\nu} \Box a(\Box) h^{\mu\nu} + h^{\alpha}_{\mu} b(\Box) \partial_{\alpha} \partial_{\sigma} h^{\sigma\mu} + hc(\Box) \partial_{\mu} \partial_{\nu} h^{\mu\nu} + \frac{1}{2} h \Box d(\Box) h + h^{\lambda\sigma} \frac{f(\Box)}{\Box} \partial_{\sigma} \partial_{\lambda} \partial_{\mu} \partial_{\nu} h^{\mu\nu},$$

$$\begin{aligned} \mathscr{L}_{MT} &= h \Box u (\Box) \partial_{\rho} K^{\rho\sigma}_{\sigma} + h_{\mu\nu} v_{1} (\Box) \partial^{\mu} \partial^{\nu} \partial_{\rho} K^{\rho\sigma}_{\sigma} \\ &+ h_{\mu\nu} v_{2} (\Box) \partial^{\nu} \partial_{\sigma} \partial_{\rho} K^{\mu\sigma\rho} + h_{\mu\nu} \Box w (\Box) \partial_{\rho} K^{\rho\mu\nu}, \end{aligned}$$

$$\begin{aligned} \mathscr{L}_{\mathcal{T}} &= \ \mathcal{K}^{\mu\sigma\lambda}p_{1}(\Box)\,\mathcal{K}_{\mu\sigma\lambda} + \mathcal{K}^{\mu\sigma\lambda}p_{2}(\Box)\,\mathcal{K}_{\mu\lambda\sigma} + \mathcal{K}^{\rho}_{\mu\rho}p_{3}(\Box)\,\mathcal{K}^{\mu\sigma}_{\sigma} \\ &+ \ \mathcal{K}^{\mu}_{\nu\rho}q_{1}(\Box)\,\partial_{\mu}\partial_{\sigma}\mathcal{K}^{\sigma\nu\rho} + \mathcal{K}^{\mu}_{\nu\rho}q_{2}(\Box)\,\partial_{\mu}\partial_{\sigma}\mathcal{K}^{\sigma\rho\nu} \\ &+ \ \mathcal{K}^{\rho}_{\mu\nu}q_{3}(\Box)\,\partial_{\rho}\partial_{\sigma}\mathcal{K}^{\mu\nu\sigma} + \mathcal{K}^{\rho}_{\mu\nu}q_{4}(\Box)\,\partial_{\rho}\partial_{\sigma}\mathcal{K}^{\mu\sigma\nu} \\ &+ \ \mathcal{K}^{\mu\rho}_{\rho}q_{5}(\Box)\,\partial_{\mu}\partial_{\nu}\mathcal{K}^{\nu\sigma}_{\sigma} + \mathcal{K}^{\lambda}_{\lambda\sigma}q_{6}(\Box)\,\partial_{\mu}\partial_{\alpha}\mathcal{K}^{\sigma\mu\alpha} \\ &+ \ \mathcal{K}^{\nu\rho}_{\mu}s(\Box)\,\partial_{\nu}\partial_{\rho}\partial_{\alpha}\partial_{\sigma}\mathcal{K}^{\mu\alpha\sigma}, \end{aligned}$$

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Ghost-free conditions Solving the torsion singularities

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Simplified action

$$\mathscr{L}_{MT} = 0,$$

$$\mathscr{L}_{T} = S_{\mu} \Box \Lambda (\Box) S^{\mu} - S_{\mu} \Sigma (\Box) \partial^{\mu} \partial_{\nu} S^{\nu},$$

where
$$\Sigma(\Box) = q_1(\Box) - q_2(\Box) - q_3(\Box) + q_4(\Box)$$
, and $\Lambda(\Box) = 3(p_1(\Box) + p_2(\Box)) + \Sigma(\Box)$.

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Ghost-free conditions for the torsion sector

Performing variations with respect to the axial torsion

$$\Box \Lambda(\Box) S^{\mu} - \Sigma(\Box) \partial^{\mu} \partial_{\nu} S^{\nu} = 0.$$

Calculating the propagator and eliminating the scalar mode we find

$$\Lambda(\Box) = \Sigma(\Box) = e^{\beta(\Box)}$$

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Solving the field equations for a delta ring source

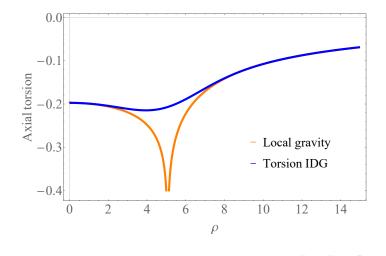
$$\Box e^{\beta(\Box)}S^{\mu} = A^{\mu}\delta(z)\delta(x^{2}+y^{2}-R^{2}).$$

Assuming $eta(\Box)=-\Box/M_s^2$ one can see that

$$S^{\mu}(\rho) = -\frac{1}{4}A^{\mu}\int_{0}^{\infty} \mathrm{d}\xi J_{0}\left(-R\xi\right)J_{0}\left(-\xi\rho\right)\mathrm{Erfc}\left(\xi/M_{s}\right)$$

Ghost-free conditions Solving the torsion singularities

Resolution of the singularity



Conclusions

- Presentation of a new theory of gravity, as an ultra violet extension of Poincaré Gauge Gravity.
- It is possible to find solutions.
- These solutions can be made ghost and singularity free in the torsion sector.

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