Constructing new non-singular and ghost-free theories of gravity

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Sciama (1962) and Kibble (1961)

- Poincaré gauge invariance $\longrightarrow$ Naturally defined spin fields
- More degrees of freedom $\longrightarrow$ New possibilities to explain open problems in Cosmology

Introduction
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General connections

An arbitrary connection has D^3 degrees of freedom in D dimensions:

- $\frac{D^2(D-1)}{2}$ in the antisymmetric part (*torsion*):

$$T^{\rho}_{\mu\nu} \equiv \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} \,.$$

- $\frac{D^2(D+1)}{2}$ in the *non-metricity* tensor:

$$Q_{\rho\mu\nu} = \nabla_{\rho} g_{\mu\nu} \,.$$

- Remark: A different connection does not always lead to different phenomenology (e.g. Teleparallel Gravity).

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Properties of the theories with torsion

- Gauge theory of the Poincaré Group.
- Metricity

$$\nabla_\rho g_{\mu\nu} = 0$$

- Non symmetric connection

$$T^{\rho}_{\mu\nu} \equiv \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} \neq 0$$

- Relation between the connection and the Levi-Civita one

$$\mathring{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + K^{\rho}_{\mu\nu}$$

where

$$K^{\rho}_{\mu\nu} = \frac{1}{2}\left(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}_{\mu\nu}\right)$$

Decomposition of the torsion tensor

$$T_{\mu\nu\rho} = \frac{1}{3}\left(T_\nu g_{\mu\rho} - T_\mu g_{\mu\nu}\right) - \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma}S^\sigma + q_{\mu\nu\rho},$$

where

$$\underbrace{T_\mu = T^{\nu}_{\ \mu\nu}}_{\text{Trace}},\ \underbrace{S^\mu = \varepsilon^{\rho\sigma\nu\mu}T_{\rho\sigma\nu}}_{\text{Axial vector}},\ \underbrace{q^{\mu}_{\ \nu\rho}\ s.t.\ q^{\nu}_{\ \mu\nu} = 0\ and\ \varepsilon^{\rho\sigma\nu\mu}q_{\rho\sigma\nu} = 0}_{\text{Tensor}}$$

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Why infinite derivative theories?

- Modification at the UV level $\longrightarrow$ Compatible with current experimental data.
- Infinite derivatives $\longrightarrow$ Can be made ghost and singularity free.

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Most general IDG quadratic action with torsion

A. Cruz-Dombriz, FJMT, A. Mazumdar: 1812.04037. Phys. Rev. D, April 2019.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\widetilde{R}}{2} + \widetilde{R}_{\mu_1\nu_1\rho_1\sigma_1}\mathscr{O}^{\mu_1\nu_1\rho_1\sigma_1}_{\mu_2\nu_2\rho_2\sigma_2}\widetilde{R}^{\mu_2\nu_2\rho_2\sigma_2} \right.$$
$$\left. + \widetilde{R}_{\mu_1\nu_1\rho_1\sigma_1}\mathscr{O}^{\mu_1\nu_1\rho_1\sigma_1}_{\mu_2\nu_2\rho_2}K^{\mu_2\nu_2\rho_2} + K_{\mu_1\nu_1\rho_1}\mathscr{O}^{\mu_1\nu_1\rho_1}_{\mu_2\nu_2\rho_2}K^{\mu_2\nu_2\rho_2}\right]$$

Physical motivation

- Non-local terms $\longrightarrow$ Effective theory with quantum effects..
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Linearised action

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + K^{\rho}_{\mu\nu},$$

where $K \sim \mathscr{O}(h)$. Then

$$S_q = -\int \mathrm{d}^4x\sqrt{-g}\,(\mathscr{L}_M + \mathscr{L}_{MT} + \mathscr{L}_T) = S_M + S_{MT} + S_T$$

Linearised action components

$$\mathscr{L}_M = \frac{1}{2}h_{\mu\nu}\Box a(\Box)h^{\mu\nu} + h_\mu^\alpha b(\Box)\partial_\alpha\partial_\sigma h^{\sigma\mu} + hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}h\Box d(\Box)h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu},$$

$$\mathscr{L}_{MT} = h\Box u(\Box)\partial_\rho K^{\rho\sigma}_{\ \ \sigma} + h_{\mu\nu}v_1(\Box)\partial^\mu\partial^\nu\partial_\rho K^{\rho\sigma}_{\ \ \sigma} + h_{\mu\nu}v_2(\Box)\partial^\nu\partial_\sigma\partial_\rho K^{\mu\sigma\rho} + h_{\mu\nu}\Box w(\Box)\partial_\rho K^{\rho\mu\nu},$$

$$\begin{aligned}\mathscr{L}_T = {} & K^{\mu\sigma\lambda}p_1(\Box)K_{\mu\sigma\lambda} + K^{\mu\sigma\lambda}p_2(\Box)K_{\mu\lambda\sigma} + K_{\mu\ \rho}^{\ \rho}p_3(\Box)K^{\mu\sigma}_{\ \ \sigma} \\ & + K^\mu_{\ \nu\rho}q_1(\Box)\partial_\mu\partial_\sigma K^{\sigma\nu\rho} + K^\mu_{\ \nu\rho}q_2(\Box)\partial_\mu\partial_\sigma K^{\sigma\rho\nu} \\ & + K_\mu^{\ \rho}{}_\nu q_3(\Box)\partial_\rho\partial_\sigma K^{\mu\nu\sigma} + K_\mu^{\ \rho}{}_\nu q_4(\Box)\partial_\rho\partial_\sigma K^{\mu\sigma\nu} \\ & + K^{\mu\rho}_{\ \ \rho}q_5(\Box)\partial_\mu\partial_\nu K^{\nu\sigma}_{\ \ \sigma} + K^\lambda_{\ \lambda\sigma}q_6(\Box)\partial_\mu\partial_\alpha K^{\sigma\mu\alpha} \\ & + K_\mu^{\ \nu\rho}s(\Box)\partial_\nu\partial_\rho\partial_\alpha\partial_\sigma K^{\mu\alpha\sigma},\end{aligned}$$

Simplified action

$$\mathscr{L}_{MT} = 0,$$

$$\mathscr{L}_T = S_\mu \Box \Lambda(\Box) S^\mu - S_\mu \Sigma(\Box) \partial^\mu \partial_\nu S^\nu,$$

where $\Sigma(\Box) = q_1(\Box) - q_2(\Box) - q_3(\Box) + q_4(\Box)$, and $\Lambda(\Box) = 3(p_1(\Box) + p_2(\Box)) + \Sigma(\Box)$.

Outline

Ghost-free conditions for the torsion sector

Performing variations with respect to the axial torsion

$$\Box\Lambda(\Box)S^{\mu}-\Sigma(\Box)\partial^{\mu}\partial_{\nu}S^{\nu}=0.$$

Calculating the propagator and eliminating the scalar mode we find

$$\Lambda(\Box)=\Sigma(\Box)=e^{\beta(\Box)}$$

Outline

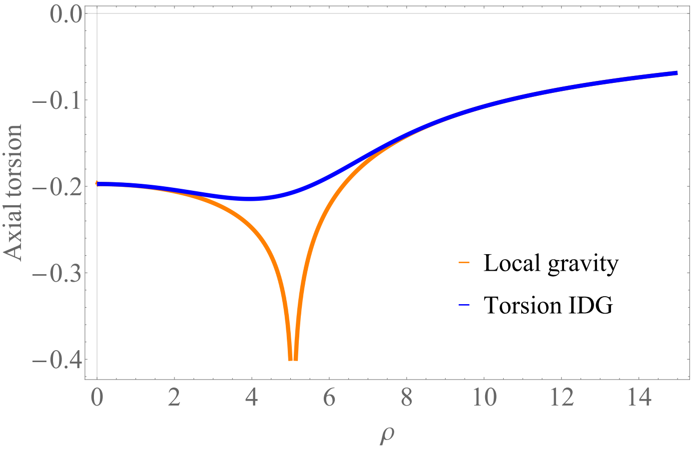
Solving the field equations for a delta ring source

$$\Box e^{\beta(\Box)} S^\mu = A^\mu \delta(z) \delta\left(x^2 + y^2 - R^2\right).$$

Assuming $\beta(\Box) = -\Box/M_s^2$ one can see that

$$S^\mu(\rho) = -\frac{1}{4} A^\mu \int_0^\infty \mathrm{d}\xi \, \mathrm{J}_0(-R\xi) \, \mathrm{J}_0(-\xi\rho) \, \mathrm{Erfc}(\xi/M_s)$$

Resolution of the singularity



Conclusions

- Presentation of a new theory of gravity, as an ultra violet extension of Poincaré Gauge Gravity.
- It is possible to find solutions.
- These solutions can be made ghost and singularity free in the torsion sector.