Horizons and symmetry restoration: the case of QCD.

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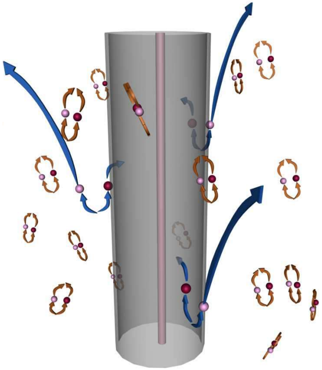
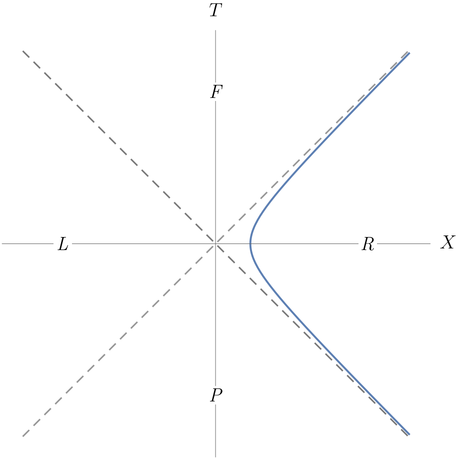
Introduction.

Introduction.

- In the 1970s, **Quantum Field Theory** was generalized to
  - **arbitrary observers** in Minkowski,
  - **curved spacetimes**.
- Several **seminal results**:
  - Hawking: black holes **radiate** until they evaporate;
  - **horizons** have an impact on field quantization.



Accelerated motion in flat spacetime.



Hyperbolic motion:

$$T^2 - X^2 = -\frac{1}{a^2}.$$

Two **branches**: L and R.

Asymptotes:

$$T = \pm X$$

(null lines $\Longrightarrow$ **horizon**).

The Unruh effect.

Thermalization Theorem (Lee, 1986): the restriction of the Minkowski vacuum state $|\Omega_M\rangle$ of *any* quantum field theory to R is given by

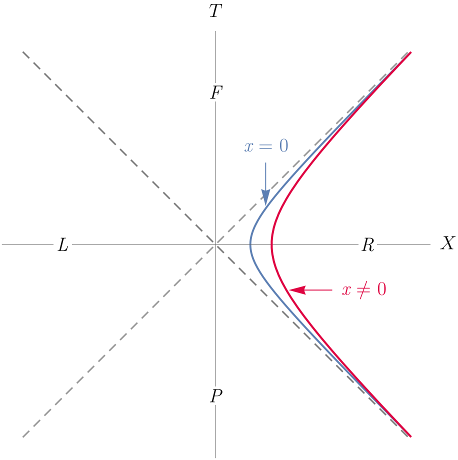
$$\rho_R = \mathrm{tr}_L\, |\Omega_M\rangle\langle\Omega_M| = \frac{\mathrm{e}^{-2\pi H_R/a}}{\mathrm{tr}\ \mathrm{e}^{-2\pi H_R/a}},$$

i.e. it is a thermal state at the Unruh temperature

$$T_{\mathrm{U}} = \frac{a}{2\pi} \qquad (\hbar = c = G = k_{\mathrm{B}} = 1).$$

This is the **generalization of the Unruh effect** to theories of interacting fields of arbitrary spin.

Accelerated motion in flat spacetime.



Comoving coordinates:

$$T = a^{-1} e^{ax} \sinh(at),$$

$$X = a^{-1} e^{ax} \cosh(at),$$

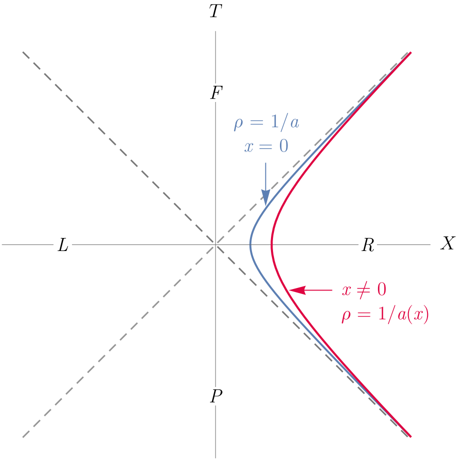
$$X_\perp = x_\perp;$$

$$t, x, y, z \in \mathbb{R}.$$

$$ds^2 = e^{2ax}(dt^2 - dx^2) - dx_\perp^2,$$

$$a(0) = a, \quad a(x) = a\, e^{-ax}.$$

Accelerated motion in flat spacetime.



Rindler coordinate:

$$\rho \equiv \frac{1}{a(x)} = \frac{e^{ax}}{a} \in (0, \infty)\,,$$

$$ds^2 = a^2\rho^2 dt^2 - d\rho^2 - dx_\perp^2\,.$$

Euclidization $(t_E = it)$:

$$ds_E^2 = a^2\rho^2 dt_E^2 + d\rho^2 + dx_\perp^2\,.$$

$$t_E \sim t_E + \frac{2\pi}{a} \Rightarrow \textbf{Thermal!}$$

Spherically symmetric spacetimes with horizon.

The second simplest spacetimes with horizon are of the form

$$\mathrm{d}s^2 = f(r)\,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{f(r)} - \mathrm{d}\mathbf{x}_\perp^2,$$

with $f(r)$ having a simple zero at a certain $r = h$:

$$f(r) \underset{r\to h}{\sim} f'(h)\,(r-h) \equiv 2\kappa(r-h).$$

Thus these spacetimes are **spherically symmetric** and have only one bifurcate Killing horizon at $r = h$, with **surface gravity** κ. Examples:

- **Schwarzschild**: $f(r) = 1 - 2M/r$,
- **de Sitter**: $f(r) = 1 - H^2 r^2$.

Spherically symmetric spacetimes with horizon.

Close to the horizon, the **distance to the horizon** is

$$\int_h^r \mathrm{d}r\,\sqrt{g_{rr}} = \int_h^r \frac{\mathrm{d}r}{\sqrt{g_{rr}}} \simeq \int_h^r \frac{\mathrm{d}r}{\sqrt{2\kappa(r-h)}} = \sqrt{\frac{2(r-h)}{\kappa}} \equiv \rho,$$

which behaves as the **Rindler coordinate**:

$$\mathrm{d}s_{\mathrm{E}}^2 \underset{r\to h}{\sim} = \kappa^2\rho^2\mathrm{d}t_{\mathrm{E}}^2 + \mathrm{d}\rho^2 + \mathrm{d}x_{\perp}^2.$$

- Therefore, bifurcate Killing horizons have a thermal character.
- Can non-trivial dynamical effects, such as **phase transitions**, take place close to them?
- We will focus on one particular, highly relevant example.

The QCD phase transition.

QCD and its chiral symmetry.

- **Quantum Chromodynamics** (QCD) is the theory of strong interactions within the Standard Model.
- The Euclidean Lagrangian of two-flavour massless QCD,

$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathrm{i}\bar{q}\gamma_\mu D_\mu q,$$

  is invariant under global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ transformations.
- This **chiral symmetry** is actually **spontaneously broken** (it is *not* a symmetry of the hadronic spectrum).

QCD and its chiral symmetry.

When we consider QCD at **finite temperature** in Minkowski spacetime, chiral symmetry is **restored** for temperatures higher than $T_c = 2f_\pi$:

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = \begin{cases} \sqrt{1 - \dfrac{T^2}{T_c^2}} & \text{if } 0 \leq T < T_c, \\ 0 & \text{if } T \geq T_c. \end{cases}$$

The expectation value $\langle \bar{q}q \rangle$, known as the **quark condensate**, is the order parameter of the **second-order chiral phase transition**.

Chiral symmetry restoration by the Unruh effect.

Chiral symmetry restoration by the Unruh effect.

We can compute the Euclidean partition function of the **lowest-order effective description of low-energy QCD** in Rindler space,

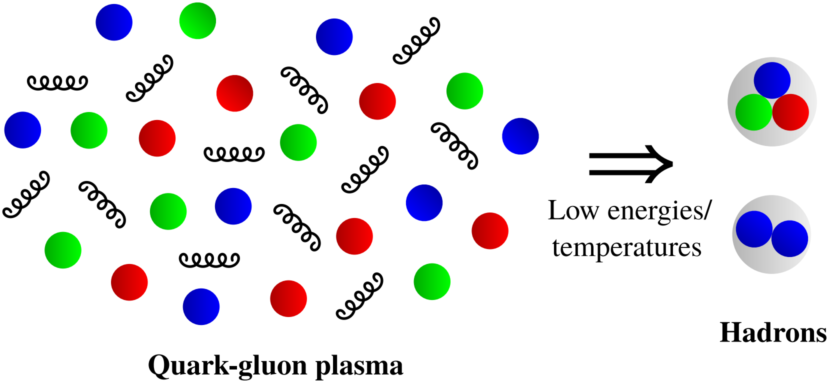
$$Z_{\mathrm{NL}\sigma\mathrm{M}} = \int [\mathrm{d}\pi^a][\mathrm{d}\sigma][\mathrm{d}\lambda]\, \mathrm{e}^{-\Gamma[\pi^a,\sigma,\lambda]},$$

where the effective action in the exponent is

$$\Gamma[\pi^a,\sigma,\lambda] = \int \mathrm{d}^4x\, \sqrt{g}\, \Big(-\frac{1}{2}\pi^a \Box \pi^a - \\ -\frac{1}{2}\sigma\Box\sigma + \frac{\lambda}{2}(\pi^a\pi^a + \sigma^2 - f_\pi^2) \Big),$$

with $\pi^a\pi^a + \sigma^2 = f_\pi^2$ and $\langle\sigma\rangle \propto \langle\bar{q}q\rangle$.

Why an effective description? The two phases of hadronic matter.



The functional integral over the pion fields is Gaussian. ✓

What about the remaining integrals in σ, λ?

$\Longrightarrow$ **Saddle-point approximation.**

- Large-N limit: $f_\pi^2 \equiv NF^2$, $F \neq F(N)$.
- The fields are expanded around $(\bar{\sigma}, \bar{\lambda})$, with

$$\left.\frac{\delta\Gamma[\sigma,\lambda]}{\delta\sigma}\right|_{\sigma=\bar{\sigma}} = 0, \qquad \left.\frac{\delta\Gamma[\sigma,\lambda]}{\delta\lambda}\right|_{\lambda=\bar{\lambda}} = 0.$$

$\Longrightarrow$ $\bar{\sigma}^2(x) = \langle\sigma(x)\rangle_a^2 = \langle\sigma^2(x)\rangle_a.$

Chiral symmetry restoration by the Unruh effect.

It is easy to check that $\bar{\sigma}$ and $\bar{\lambda}$ are then the solutions of

$$\begin{aligned} \frac{\delta\Gamma}{\delta\sigma(x)} &= -\Box\sigma + \lambda\sigma = 0, \\ \frac{\delta\Gamma}{\delta\lambda(x)} &= \frac{1}{2}(\sigma^2 - f_\pi^2) + \frac{N}{2}G(x,x;\lambda) = 0, \end{aligned}$$

where the Euclidean Green function $G(x, x'; \lambda)$ satisfies

$$(-\Box + \lambda)_x \, G(x, x'; \lambda) = \frac{\delta^4(x - x')}{\sqrt{g}}.$$

This system of equations can be solved in the limit $\lambda \simeq 0$.

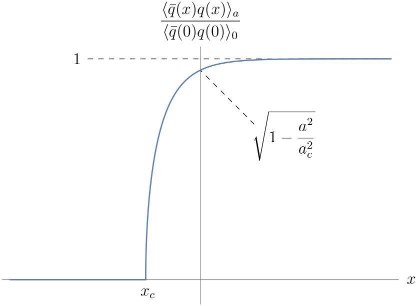
Chiral symmetry restoration by the Unruh effect.

Defining the **critical acceleration** as

$$a_c^2 \equiv \frac{48\pi^2 f_\pi^2}{N} \neq a_c^2(N),$$

we obtain, for the **quark condensate**,

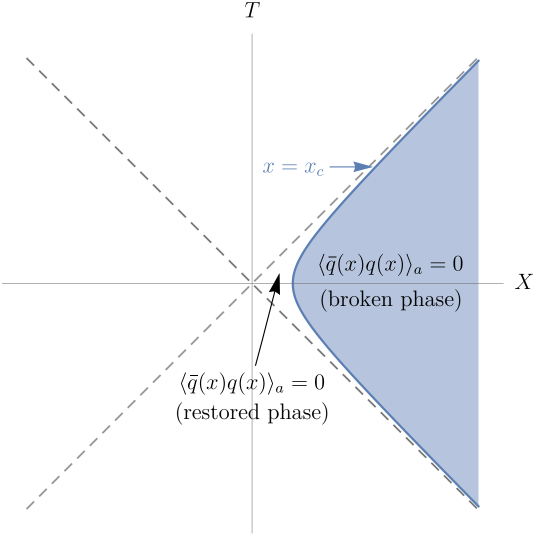
$$\frac{\langle \bar{q}(x)q(x)\rangle_a}{\langle \bar{q}(0)q(0)\rangle_0} = \sqrt{1 - \frac{a^2}{a_c^2}\,\mathrm{e}^{-2ax}}.$$



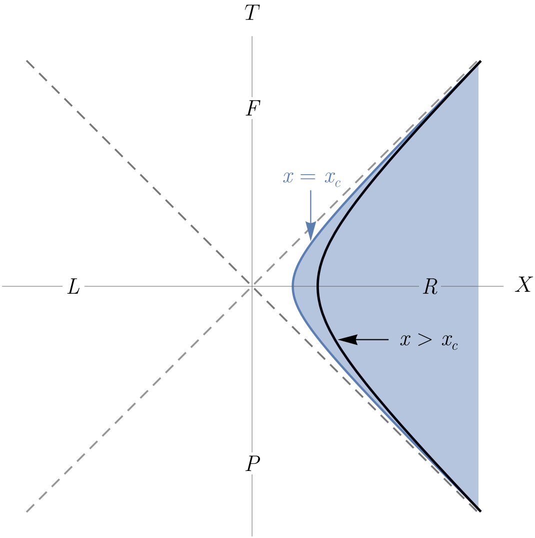
The condensate **vanishes** if $\begin{cases} x = 0,\ \ a \geq a_c\ \ (\textbf{thermal-like!}),\ \text{or} \\ x \leq x_c \equiv \dfrac{1}{a}\ln\left(\dfrac{a}{a_c}\right) < 0. \end{cases}$

*[A.C.-T., A. Dobado, Phys. Rev. D **99** 125018 (2019), arXiv:1905.11179]*

Chiral symmetry restoration by the Unruh effect.



Chiral symmetry restoration by the Unruh effect.



Chiral symmetry restoration in cosmological spacetimes.

Remember: The immediate surroundings of a horizon look like Rindler spacetime!

Thus, we also expect a restoration of chiral symmetry close to the horizons of **Schwarzschild**, **de Sitter**, and other sperically symmetric spacetimes with bifurcate Killing horizons:

$$\frac{\langle \bar{q}(\rho) q(\rho) \rangle}{\langle \bar{q}(0) q(0) \rangle} = \sqrt{1 - \frac{1}{a_c^2 \rho^2}}, \qquad a_c^2 \equiv \frac{48\pi^2 f_\pi^2}{N}.$$

Other symmetries should also be restored in this way.

Conclusions.

- Horizons play an important role in field quantization.
- The region close to any horizon resembles Rindler spacetime, where Lee's **Thermalization Theorem** holds.
- This allows us to study the triggering of **phase transitions** through the Unruh effect.
- In particular, we have found that **chiral symmetry is restored close to any horizon**, with the results being equivalent to the inertial, thermal case: $a_c = 4\pi f_\pi = 2\pi T_c$ (for $N = 3$ pions).
- At least in principle, our results may have **applications** in Cosmology and Astrophysics (and also heavy-ion collisions).

Thank you!

¡Muchas gracias!