The effective fluid approach for Modified Gravity

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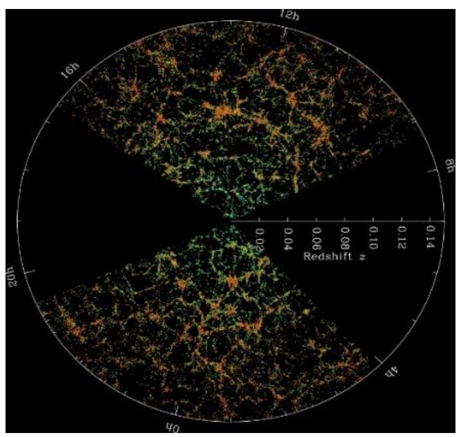
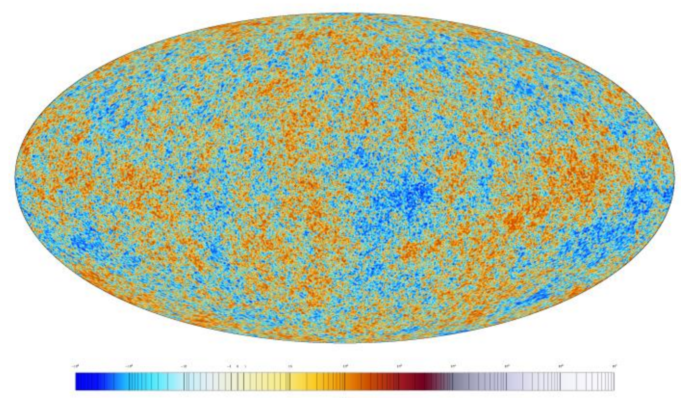
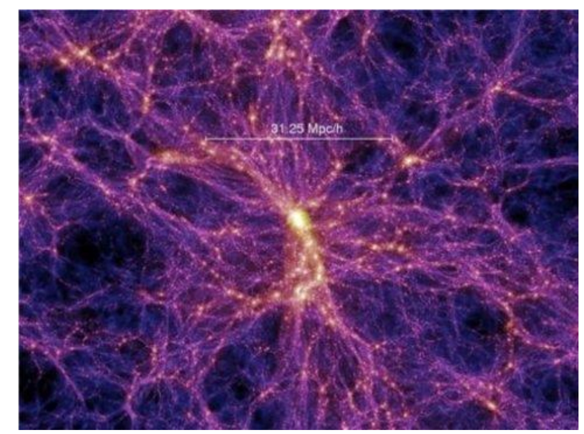
  

9 – 11 September 2019

VII Meeting on Fundamental Cosmology

RA, W.Cardona, S.Nesseris arXiv:1811.02469

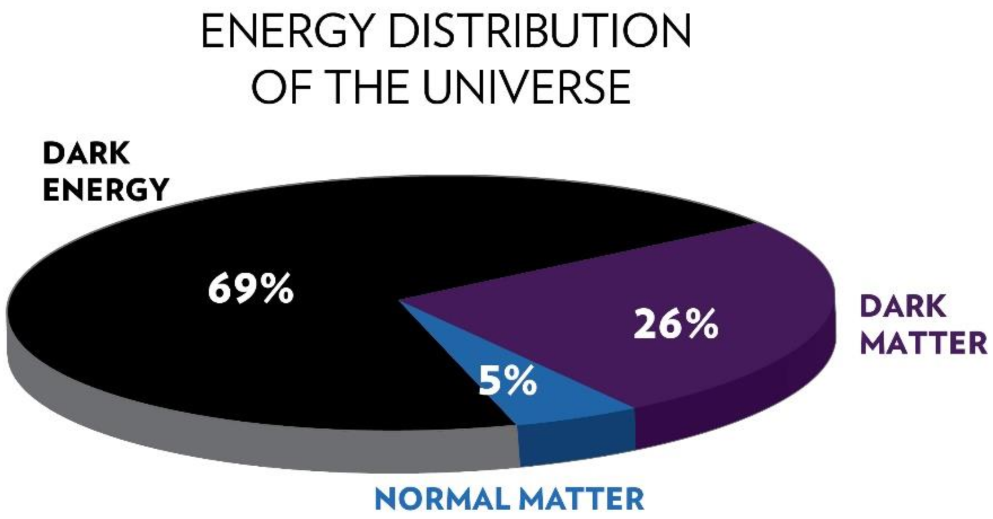
RA, W.Cardona, S.Nesseris arXiv:1904.06294

Outline

- Beyond ΛCDM
- The Effective Fluid Approach
- f(R) theories
- Horndeski theories
- Comparison with Boltzmann codes
- Conclusions

The Standard Cosmological Model (ΛCDM)

The Universe is expanding…. but also **accelerating!**

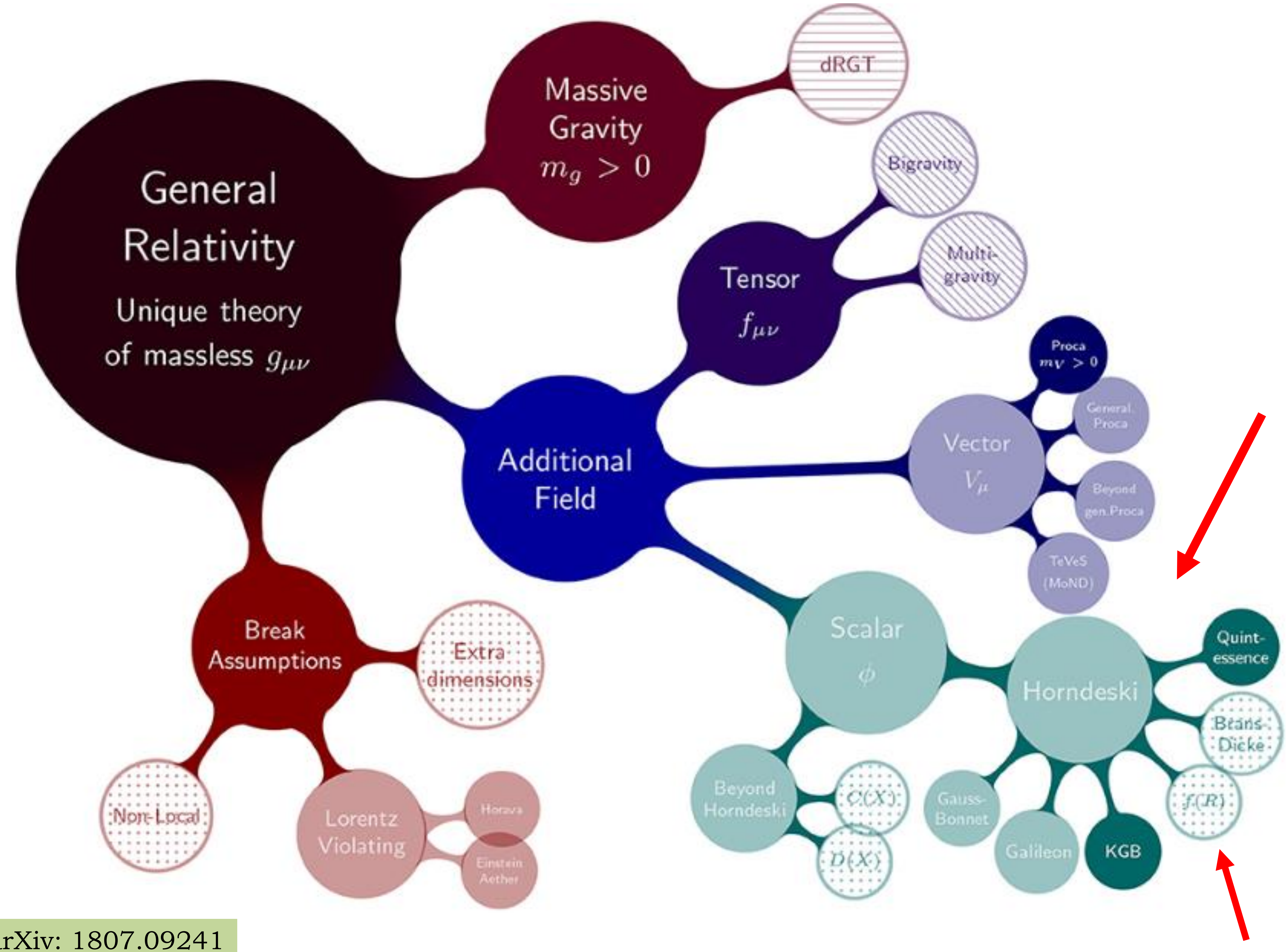


$$G_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \kappa = \frac{8\pi G_N}{c^4}$$

ΛCDM simplest candidate

Fits most data sets. Good phenomenological model

Beyond ΛCDM



arXiv: 1807.09241

Beyond ΛCDM

2 leading approaches

Dark energy

$$G_{\mu\nu} = \kappa T_{\mu\nu} + (\cdots)$$

Keep GR introduce new fields and particles

Modified gravity

$$G_{\mu\nu} + (\cdots) = \kappa T_{\mu\nu}$$

Covariant modifications to GR

Effective Fluid Approach

Departures from GR can be interpreted as an effective fluid contribution

Background

$w(a)$ equation of state

Linear perturbations

$c_s^2(a,k)$ sound speed

$\pi(a,k)$ anisotropic stress

Theoretical framework

Perturbed FRW metric $ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}^2\right]$ scalar

First order of perturbations

$$T^\mu_\nu = Pg^\mu_\nu + (\rho + P)U^\mu U_\nu$$

$$\begin{cases} T^0_0 = -(\bar{\rho} + \delta\rho) \\ T^0_i = (\bar{\rho} + \bar{P})u_i \\ T^i_j = (\bar{P} + \delta P)\delta^i_j + \Sigma^i_j \end{cases}$$

Perturbed Einstein equations

$$\begin{cases} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} + \frac{\dot{a}}{a}\Psi\right) = 4\pi G_N a^2 \delta T^0_0 & (0,0) \\ k^2\left(\dot{\Phi} + \frac{\dot{a}}{a}\Psi\right) = 4\pi G_N a^2 (\bar{\rho} + \bar{P})\theta & (0,i) \\ k^2(\Phi - \Psi) = 12\pi G_N a^2 (\bar{\rho} + \bar{P})\sigma & (i,j) \end{cases}$$

Evolution equation for the perturbations

$$\nabla_\nu T^{\mu\nu} = 0 \Rightarrow \begin{cases} \delta' = 3(1+w)\Phi' - \frac{V}{a^2 H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right) & \mu=0 \\ V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi & \mu=i \end{cases}$$

arXiv:astro-ph/9506072v1

The Effective Fluid Approach

Rewrite the EOM as the usual Einstein equations plus an effective DE fluid along with the usual matter fields.

Map Modify Gravity as a Dark Energy fluid

$$T^{DE}_{\mu\nu} = M^2_{pl} G_{\mu\nu} - T_{\mu\nu} \Rightarrow$$

$$\delta' = -3(1+w)\Phi' - \frac{V}{a^2 H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right)$$

$$V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi$$

f(R) Models

$$S=\frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R)+\int d^4x\mathcal{L}_M\left(g_{\mu\nu},\Psi_M\right)$$

Field equations

$$FG_{\mu\nu}-\frac{1}{2}\left(f(R)-RF\right)g_{\mu\nu}+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)F=\kappa T^{(m)}_{\mu\nu}$$

$$F=f'(R)$$

Eff. Fluid approach ⇒

$$G_{\mu\nu}=\kappa\left(T^{(m)}_{\mu\nu}+T^{(DE)}_{\mu\nu}\right)$$

$$\kappa T^{(DE)}_{\mu\nu}=(1-F)G_{\mu\nu}+\frac{1}{2}(f(R)-R\ F)g_{\mu\nu}-\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)F$$

$$\nabla^\mu T^{(DE)}_{\mu\nu}=0$$

f(R) Models

$$\kappa T_{\mu\nu}^{(DE)} = (1-F)G_{\mu\nu} + \frac{1}{2}(f(R) - R\ F)g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F$$

Background Eqs.

$$\begin{cases} \mathcal{H}^2 = \frac{\kappa}{3}a^2\left(\bar{\rho}_m + \bar{\rho}_{DE}\right) \\ \dot{\mathcal{H}} = -\frac{\kappa}{6}a^2\left(\left(\bar{\rho}_m + 3\bar{P}_m\right) + \left(\bar{\rho}_{DE} + 3\bar{P}_{DE}\right)\right) \end{cases}$$

Effective DE density and pressure

$$\kappa\bar{P}_{DE} = \frac{f}{2} - \mathcal{H}^2/a^2 - 2F\mathcal{H}^2/a^2 + \mathcal{H}\dot{F}/a^2 - 2\dot{\mathcal{H}}/a^2 - F\dot{\mathcal{H}}/a^2 + \ddot{F}/a^2$$

$$\kappa\bar{\rho}_{DE} = -\frac{f}{2} + 3\mathcal{H}^2/a^2 - 3\mathcal{H}\dot{F}/a^2 + 3F\dot{\mathcal{H}}/a^2$$

DE equation of state

$$w_{DE} = \frac{-a^2 f + 2\left((1+2F)\mathcal{H}^2 - \mathcal{H}\dot{F} + (2+F)\dot{\mathcal{H}} - \ddot{F}\right)}{a^2 f - 6(\mathcal{H}^2 - \mathcal{H}\dot{F} + F\dot{\mathcal{H}})}$$

f(R) Models

Effective pressure, density and velocity perturbations

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3F}\frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F}+3(1+5\frac{k^2}{a^2}\frac{F_{,R}}{F})\ddot{F}k^{-2}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{1}{F}\frac{1-F+\frac{k^2}{a^2}(2-3F)\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$V_{DE} \equiv (1+w_{DE})\theta_{DE} \simeq \frac{\dot{F}}{2F}\frac{1+6\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\pi_{DE} \simeq \frac{1}{F}\frac{\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$c^2_{s,DE} \simeq \frac{1}{3}\frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F}+3(1+5\frac{k^2}{a^2}\frac{F_{,R}}{F})\ddot{F}k^{-2}}{1-F+\frac{k^2}{a^2}(2-3F)\frac{F_{,R}}{F}}$$

ΛCDM ⇒ 0

$$f(R) = R - 2\Lambda$$

RA, W.Cardona, S.Nesseris arXiv:1904.06294

f(R) Models

Designer Models. Background exactly that of the ΛCDM

$$f(R) = R - 2\Lambda + \alpha\, H_0^2 \left(\frac{\Lambda}{R-3\Lambda}\right)^{b_2} {}_2F_1\left(b_2, \frac{3}{2}+b_2, \frac{13}{6}+2b_2, \frac{\Lambda}{R-3\Lambda}\right)$$

$c_0 = \frac{1}{12}\left(-7+\sqrt{73}\right)$ and α is a free dimensionless parameter

T. Multamaki, I.Vilja. arXiv: 0506692
A. de la Cruz-Dombriz, A.Dobado. arXiv:gr-qc/0607118
L. Pogosian, A. Silvestri. arXiv:0709.0296
S. Nesseris. arXiv:1309.1055

Hu & Sawicki model.

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{1 + c_2 (R/m^2)^n}$$

After some algebraic manipulations

$$f(R) = R - \frac{2\Lambda}{1+\left(\frac{b\Lambda}{R}\right)^n}$$

$$\lim_{b\to 0} f(R) = R - 2\Lambda$$
$$\lim_{b\to\infty} f(R) = R$$

Small perturbation around ΛCDM

Horndeski theories

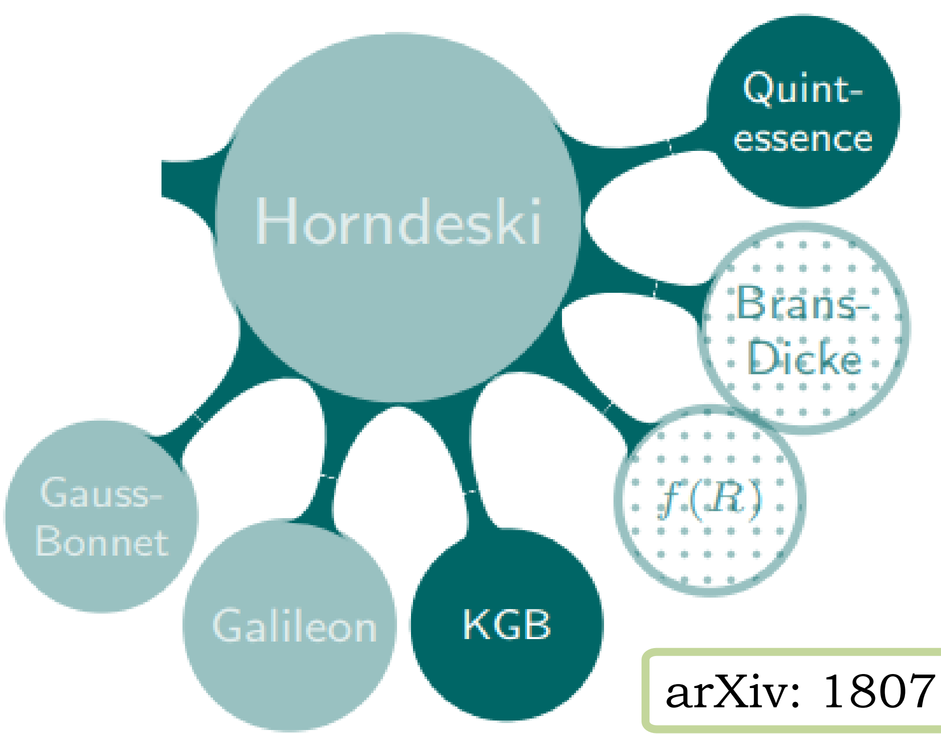
Most general **scalar-tensor theory** whose equations of motion contain derivatives up to **second order**

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\sum_{i=2}^{5} \mathcal{L}_i \left[g_{\mu\nu}, \phi\right] + \mathcal{L}_m \left[g_{\mu\nu}, \psi_M\right] \right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X) \Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} \right] \\
\mathcal{L}_5 &= G_5(\phi, X) G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6} G_{5X}(\phi, X) \left[(\Box\phi)^3 + 2\phi_{;\mu}^{\nu}\phi_{;\nu}^{\alpha}\phi_{;\alpha}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi \right]
\end{aligned}$$

ϕ

Scalar field



$X \equiv -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

Kinetic term

arXiv: 1807.09241

Horndeski after GW170817

GRB170817A+GW170817

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$

arXiv: 1710.05901

$$c_g = 1 + \alpha_T$$

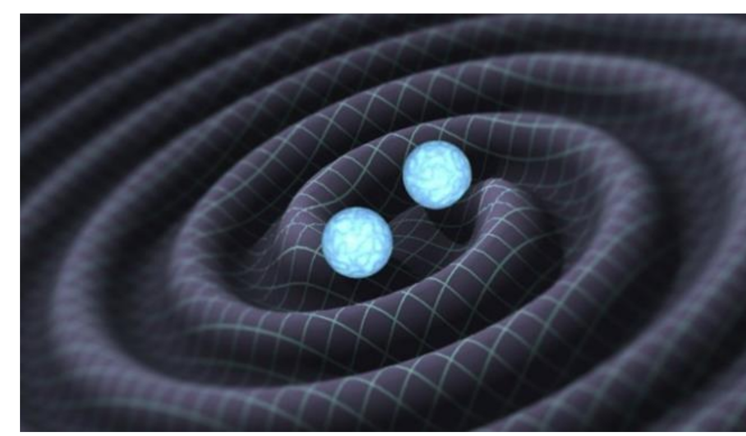
c_g: sound speed tensor perturb.

α_T: tensor speed excess

$$\ddot{h}_{ij} + (3 + \alpha_M)\, H \dot{h}_{ij} + (1 + \alpha_T)\, k^2 h_{ij} = 0$$

propagation eq. of GW scalar-tensor gravity

$$G_{4X} \approx 0, \quad G_5 \approx \text{constant}$$



$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\, \Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right] \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6} G_{5X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}^{\nu}\phi_{;\nu}^{\alpha}\phi_{;\alpha}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]
\end{aligned}$$

The Effective Fluid Approach

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} = (...)\delta\phi + (...)\dot{\delta\phi} + (...)\ddot{\delta\phi} + (...)\Psi + (...)\dot{\Psi} + (...)\Phi + (...)\dot{\Phi} + (...)\ddot{\Phi}$$

$$\delta_{DE} = (...)\delta\phi + (...)\dot{\delta\phi} + (...)\Psi + (...)\Phi + (...)\dot{\Phi}$$

$$V_{DE} \equiv (1+w_{DE})\theta_{DE} = (...)\delta\phi + (...)\dot{\delta\phi} + (...)\Psi + (...)\Phi + (...)\dot{\Phi}$$

Subhorizon and Quasistatic approximation

Horndeski models with DE anisotropic stress

$$\Phi + \Psi = \frac{G_{4\phi}}{G_4}\delta\phi \qquad \pi_{DE} = \frac{\frac{k^2}{a^2}(\Phi+\Psi)}{\kappa\,\bar{\rho}_{DE}} \simeq \frac{\frac{k^4}{a^4}\mathcal{F}_4^2 B_7(B_7 - A_6)}{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}\delta_{DE}$$

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3\mathcal{F}_4}\frac{\frac{k^4}{a^4}\mathcal{F}_1 + \frac{k^2}{a^2}\mathcal{F}_2 + \mathcal{F}_3}{\frac{k^4}{a^4}\mathcal{F}_5 + \frac{k^2}{a^2}\mathcal{F}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}{\frac{k^4}{a^4}\mathcal{F}_5 + \frac{k^2}{a^2}\mathcal{F}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$V_{DE} \simeq a\frac{\frac{k^2}{a^2}\mathcal{F}_{10} + \mathcal{F}_{11}}{\frac{k^2}{a^2}\mathcal{F}_5 + \mathcal{F}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$c_{s,DE}^2 \equiv \frac{\delta P_{DE}}{\delta\rho_{DE}} = \frac{1}{3}\frac{\frac{k^4}{a^4}\mathcal{F}_1 + \frac{k^2}{a^2}\mathcal{F}_2 + \mathcal{F}_3}{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}$$

f(R)

Horndeski models with NON DE anisotropic stress

$$\Phi = -\Psi \qquad \pi_{DE} = 0$$

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3}\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{\frac{k^4}{a^4}\hat{\mathcal{F}}_7 + \frac{k^2}{a^2}\hat{\mathcal{F}}_8 + \hat{\mathcal{F}}_9}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$V_{DE} \simeq a\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_{10} + \hat{\mathcal{F}}_{11}}{\frac{k^2}{a^2}\hat{\mathcal{F}}_5 + \hat{\mathcal{F}}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$c_{s,DE}^2 = \frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_7 + \frac{k^2}{a^2}\hat{\mathcal{F}}_8 + \hat{\mathcal{F}}_9}$$

Quintessence, K-essence
Kinetic Gravity Braiding
Designer Model (HDES)

Designer model (HDES)

Background exactly equal to that of ΛCDM model but perturbations given by the Horndeski theory

Modified Friedmann Equation

$$-H(a)^2 - \frac{K(X)}{3} + H_0^2\Omega_m(a) + 2\sqrt{2}X^{3/2}H(a)G_{3X} + \frac{2}{3}XK_X = 0$$

Scalar Field Conservation Equation

$$\frac{J_c}{a^3} - 6XH(a)G_{3X} - \sqrt{2}\sqrt{X}K_X = 0$$

$$\phi \to \phi + c$$

Family of Designer Models

$$K(X) = \frac{\sqrt{2}J_c c_0^{2/n} X^{\frac{1}{2}-\frac{2}{n}}}{H_0^2\Omega_{m,0}} - 3H_0^2\Omega_{\Lambda,0} - \frac{\sqrt{2}J_c\sqrt{X}\Omega_{\Lambda,0}}{\Omega_{m,0}}$$

$$G_3(X) = -\frac{2J_c c_0^{1/n} X^{-1/n}}{3H_0^2\Omega_{m,0}}$$

HDES

$$X = \frac{c_0}{H(a)^n}$$

RA, W.Cardona, S.Nesseris arXiv:1904.06294

HDES: Modifications to CLASS

EFCLASS
$$\begin{cases} V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi \\ \pi_{DE} = 0 \end{cases}$$

Using
$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3}\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\boxed{V_{DE} \simeq \left(-\frac{14\sqrt{2}}{3}\Omega_{m,0}^{-3/4}\tilde{J}_c\, H_0\, a^{1/4}\right)\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m}$$

$\tilde{J}_c = J_c/H_0$ and $\tilde{c}_0 = c_0/H_0^{n+2} = 1$

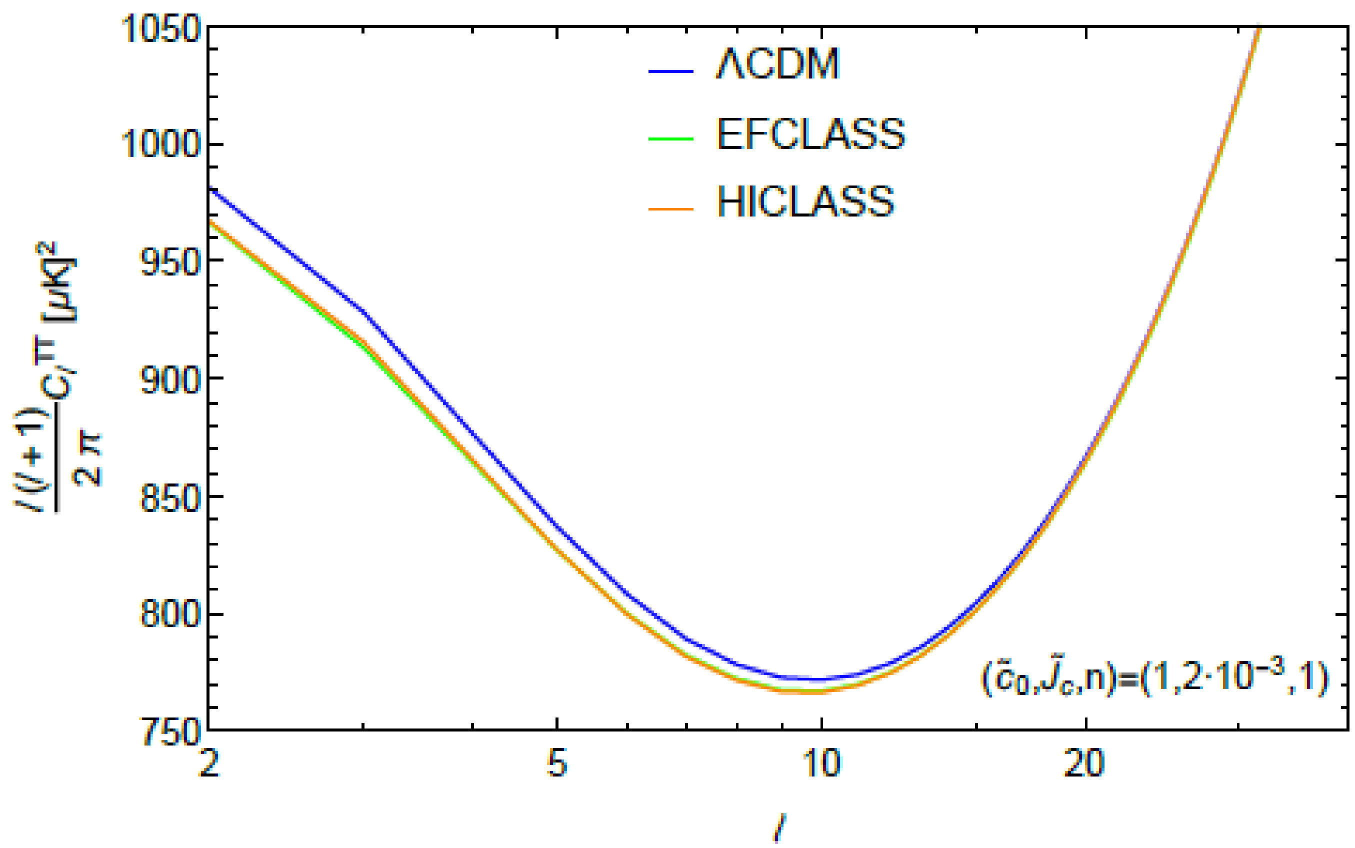
$$G_2,\ G_3,\ G_4,\ G_5 \quad \phi(t_0),\ \dot{\phi}(t_0) \ \Big\} \longrightarrow \Big\{ \ \text{Kineticity } \alpha_K,\ \text{Braiding } \alpha_B,\ M_p \text{ running } \alpha_M,\ \text{Tensor speed } \alpha_T$$

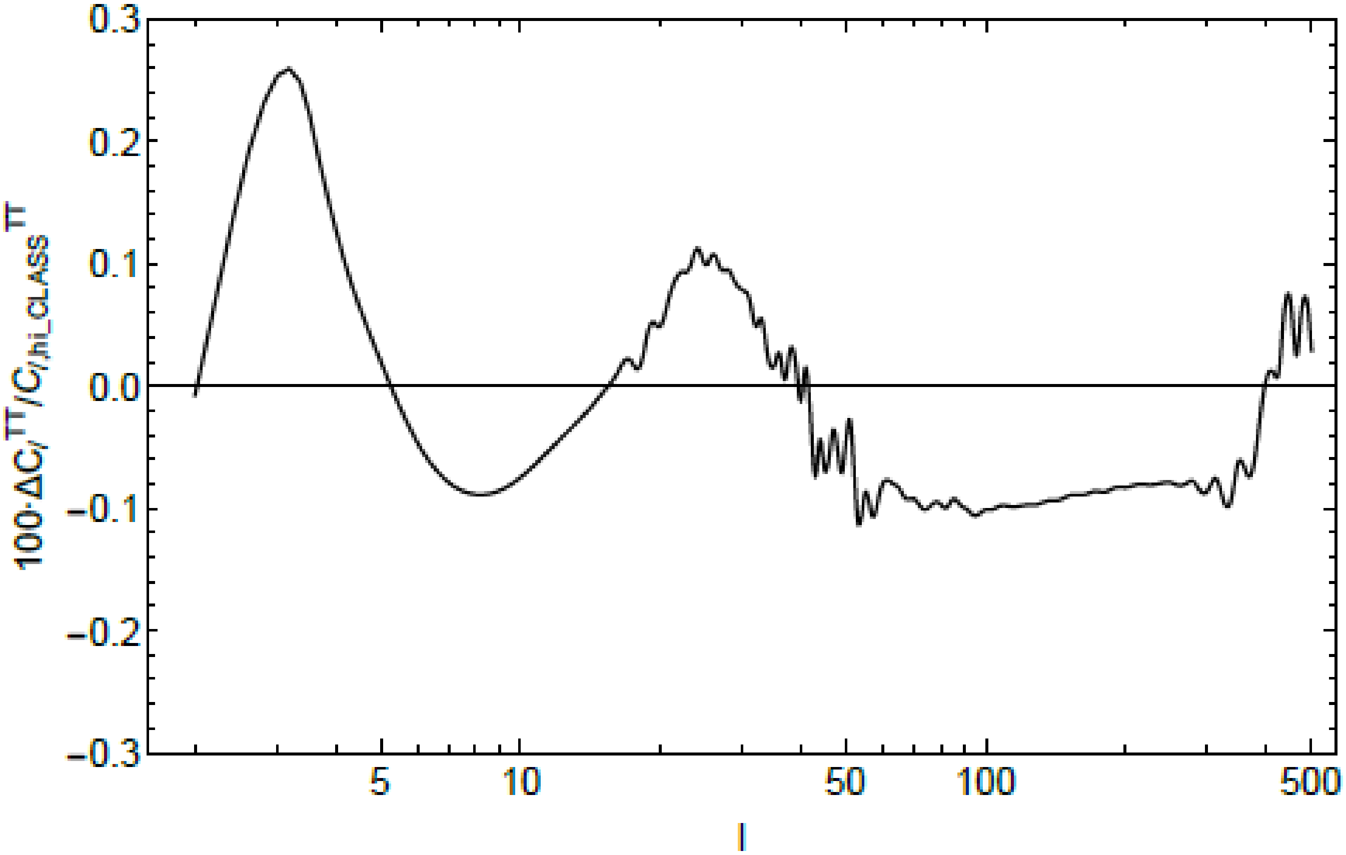
$$\begin{aligned} M_*^2 &\equiv 1 \\ \alpha_M &\equiv \frac{d\ln M_*^2}{d\ln a} = 0 \\ \alpha_K &\equiv -\frac{4\sqrt{2}\sqrt{c_0}J_c(n-2)H(a)^{-\frac{n}{2}}}{H_0^2 n^2 \Omega_{m,0}} \\ \alpha_B &\equiv \frac{4\sqrt{2}\sqrt{c_0}J_c H(a)^{-\frac{n}{2}}}{3H_0^2 n \Omega_{m,0}} \\ \alpha_T &\equiv 0 \end{aligned}$$

hi_class implements Horndeski's theory in the modern **Cosmic Linear Anisotropy Solving System**

Comparison with Boltzmann codes

Comparison of our Horndeski eff. Fluid code (EFCLASS) with hi_CLASS





$$\Omega_{m,0} = 0.3,\ n_s = 1,\ A_s = 2.3\cdot 10^{-9}\ , h = 0.7 \text{ and } (\tilde{c_0}, \tilde{J}_c, n) = (1, 2\cdot 10^{-3}, 1)$$

Conclusions

- Described the **Effective Fluid approach**
- **Theoretical expressions** for the effective dark energy pressure, velocity and sound speed (Effective Fluid Approach).
- Presented **Designer Horndeski** models (HDES).
- Our **EFCLASS** modification is accurate to the level of ~0.1%.

RA, W.Cardona, S.Nesseris arXiv:1811.02469

RA, W.Cardona, S.Nesseris arXiv:1904.06294

Back-up slides

C L A S S

the Cosmic Linear Anisotropy Solving System

The purpose of CLASS is to simulate the evolution of linear perturbations in the universe and to compute CMB and large scale structure observables.

$$c_g^2 = \frac{G_4 - XG_{5\phi} - XG_{5X}\ddot{\phi}}{G_4 - 2XG_{4X} - X\left(G_{5X}\dot{\phi}H - G_{5\phi}\right)}$$

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$

$$c_g = 1 + \alpha_T$$

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [47] Brans-Dicke/$f(R)$ [48, 49] Kinetic Gravity Braiding [51]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [50] $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [5], $f(\phi)$·Gauss-Bonnet [53]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

Ezquiaga et al. 1710.05901

Full f(R) solution

Background eqs FRW metric

$$3FH^2 = (FR - f)/2 - 3H\dot{F} + 8\pi G\rho_m \qquad (2.3)$$
$$-2F\dot{H} = \ddot{F} - H\dot{F} + 8\pi G(\rho_m + P_m)$$

The ΛCDM model satisfies

$$H(a)^2 = H_0^2\left(\Omega_m a^{-3} + 1 - \Omega_m\right), \qquad (2.6)$$

while for the FRW metric we can express the Ricci scalar as

$$R(a) = 6(2H(a)^2 + aH(a)H'(a)). \qquad (2.7)$$

From these two equations we can express the Hubble parameter as

$$H(R)^2 = \frac{1}{3}\left(R - 9(1-\Omega_m)H_0^2\right) \qquad (2.8)$$

and reexpress Eq. (2.3) in terms of R

$$f(R) + (R - 6\Lambda)f'(R) - 2(R - 4\Lambda)\left(1 + 3(R - 3\Lambda)f''(R)\right) = 0, \qquad (2.9)$$

where we have set $\Lambda = 3H_0^2(1 - \Omega_m)$. This differential equation describes all the Lagrangians $f(R)$ that have as a background the ΛCDM model. The general solution can be found to be

$$f(R) = R - 2\Lambda + \alpha\, H_0^2\left(\frac{\Lambda}{R - 3\Lambda}\right)^{b_2} {}_2F_1\left(b_2, \frac{3}{2} + b_2, \frac{13}{6} + 2b_2, \frac{\Lambda}{R - 3\Lambda}\right)$$

Theoretical framework

Perturbed FRW metric $ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}^2\right]$ scalar

First order of perturbations

$$T^\mu_\nu = P g^\mu_\nu + (\rho + P)U^\mu U_\nu$$

$$\left\{\begin{array}{lcl} T^0_0 & = & -(\bar\rho + \delta\rho) \\ T^0_i & = & (\bar\rho + \bar P)u_i \\ T^i_j & = & (\bar P + \delta P)\delta^i_j + \Sigma^i_j \end{array}\right.$$

Perturbed Einstein equations

$$\left\{\begin{array}{rcl} k^2\Phi + 3\frac{\dot a}{a}\left(\dot\Phi + \frac{\dot a}{a}\Psi\right) & = & 4\pi G_N a^2 \delta T^0_0 \quad (0,0) \\ k^2\left(\dot\Phi + \frac{\dot a}{a}\Psi\right) & = & 4\pi G_N a^2(\bar\rho + \bar P)\theta \quad (0,i) \\ k^2(\Phi - \Psi) & = & 12\pi G_N a^2(\bar\rho + \bar P)\sigma \quad (i,j) \end{array}\right.$$

Evolution equation for the perturbations

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Longrightarrow$$

$$\left\{\begin{array}{rcl} \dot\delta & = & -(1+w)(\theta - 3\dot\Phi) - 3\frac{\dot a}{a}\left(c_s^2 - w\right)\delta \quad \mu=0 \\ \dot\theta & = & -\frac{\dot a}{a}(1-3w)\theta - \frac{\dot w}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\Psi \quad \mu=i \end{array}\right.$$

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Theoretical framework

Evolution equation for the perturbations

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Rightarrow \quad \begin{cases} \dot{\delta} = -(1+w)(\theta - 3\dot{\Phi}) - 3\frac{\dot{a}}{a}\left(c_s^2 - w\right)\delta & \mu=0 \\ \dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\Psi & \mu=i \end{cases}$$

Scalar velocity perturbation $V \equiv (1+w)\theta$

Anisotropic stress parameter $\pi = \frac{3}{2}(1+w)\sigma$

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Rightarrow \quad \begin{cases} \delta' = 3(1+w)\Phi' - \frac{V}{a^2 H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right) \\ V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi \end{cases}$$

Horndeski theories

- **f(R) theories**

$$K = -\frac{Rf_{,R} - f}{2\kappa} \qquad G_4 = \frac{\phi}{2\sqrt{\kappa}} \qquad \text{where } \phi \equiv \frac{f_{,R}}{\sqrt{\kappa}}$$

- **Kinetic gravity braiding**

$$K = K(X) \qquad G_3 = G_3(X) \qquad G_4 = \frac{1}{2\kappa}$$

- **Non-minimal coupling (NMC) model**

$$K = \omega(\phi)X - V(\phi) \quad G_4 = \left(\frac{1}{2\kappa} - \frac{\zeta\phi^2}{2}\right) \quad G_3 = 0$$

Higgs inflation $\omega(\phi) = 1,\ V(\phi) = \lambda\left(\phi^2 - \nu^2\right)^2/4.$

Numerical solution of the evolution equations