The fractal geometry of the cosmic web

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PLAN OF THE TALK

1. Nonlinear cosmological dynamics.
2. The Zeldovich approximation and the adhesion model.
3. Fractal geometry of matter clustering.
4. Multifractal analysis of N-body LCDM simulations and of Sloan Digital Sky Survey (SDSS) galaxies.
5. Conclusions.

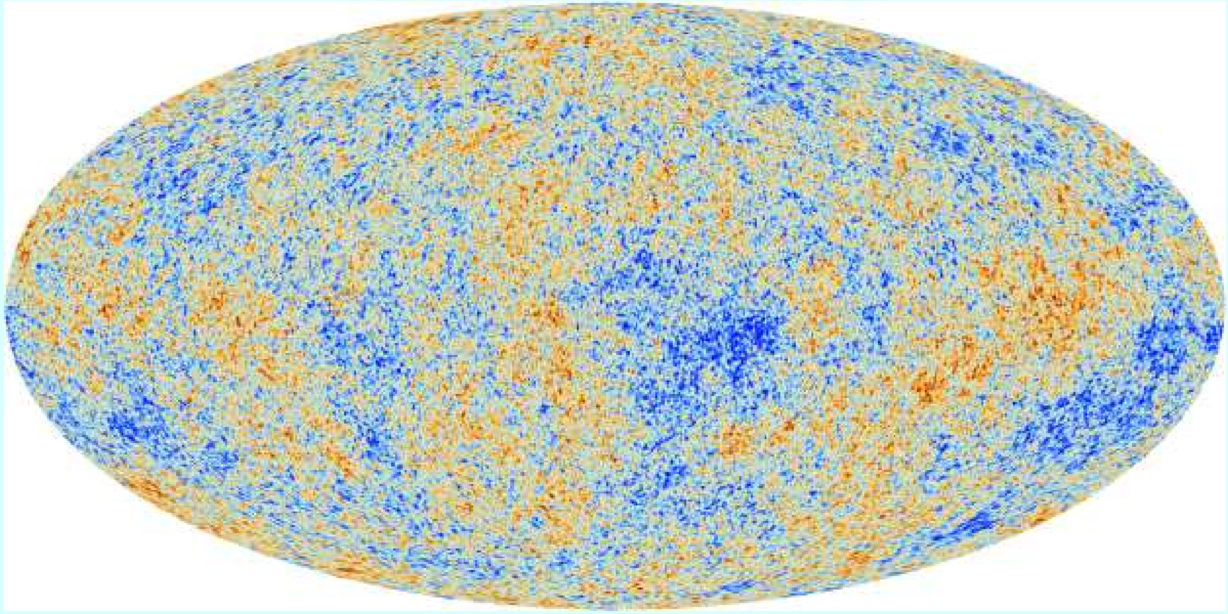
Based on a series of papers in ApJ, EPL, MNRAS, JCAP, AinA, etc.

Perturbations of FLRW

Linear perturbation theory of the FLRW solution (Lifshitz 1946):

- Tensor modes that do not affect the matter $\rightarrow$ gravitational waves.
- Vector modes that do not change the matter density $\rightarrow$ rotational.
- Scalar modes that change the matter density and grow (if P negligible).

Anisotropies of the CMB (Planck) reveal "initial" density fluctuations.



Nonlinear growth

Nonlinear growth of density fluctuations can be studied within **Newtonian gravity**.

- Perturbation theory to higher orders $\rightarrow$ convergence?
- Analytic non-perturbative methods.
  - The Zeldovich approximation.
  - Closure approaches (Peebles, ...).
  - Path integral and instantons.
  - Exact renormalization group.
- N-body cosmological simulations.

Matter fluid motion

Lagrangian map: $\boldsymbol{x}(t, \boldsymbol{x}_0), \ \boldsymbol{x}_0 \in \mathbb{R}^3$ or 3d manifold. ($\boldsymbol{x}_0 \to \boldsymbol{q}$).

It can be singular for some t.

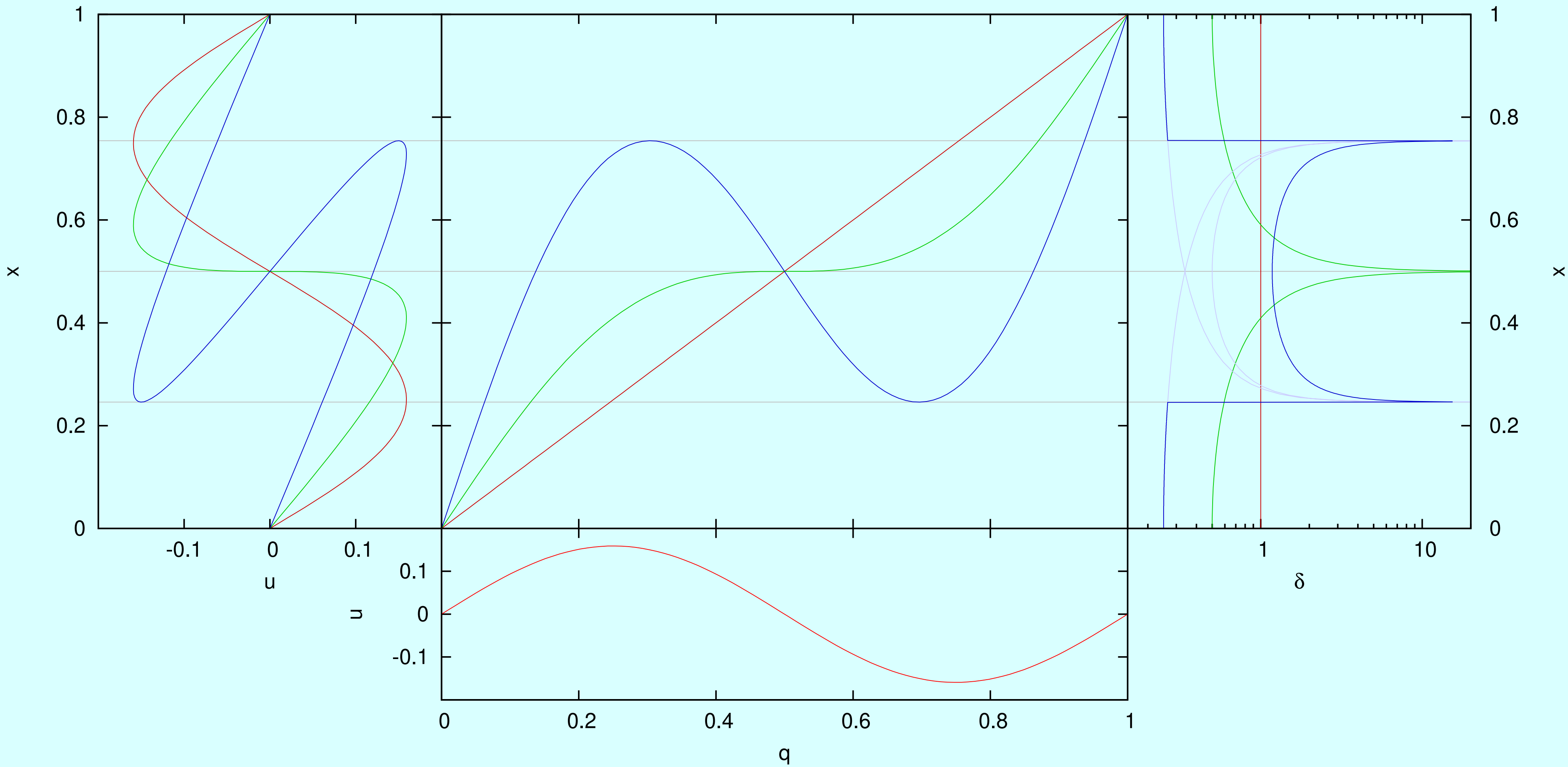
Lifshitz, Khalatnikov and Sudakov (1961): caustics in a family of geodesics $\perp$ Cauchy hypersurface $\to$ density singularities for dust matter.

Classification of singularities (or *catastrophes*) is part of *differential topology*. Singularities of Lagrangian maps classified by Arnold (1972):

- Cusp $A_3^{\pm}$ (creation or annihilation).
- Swallowtail A_4.
- Umbilic $D_4^{\pm}$ (hyperbolic or elliptic).
- $\dots$

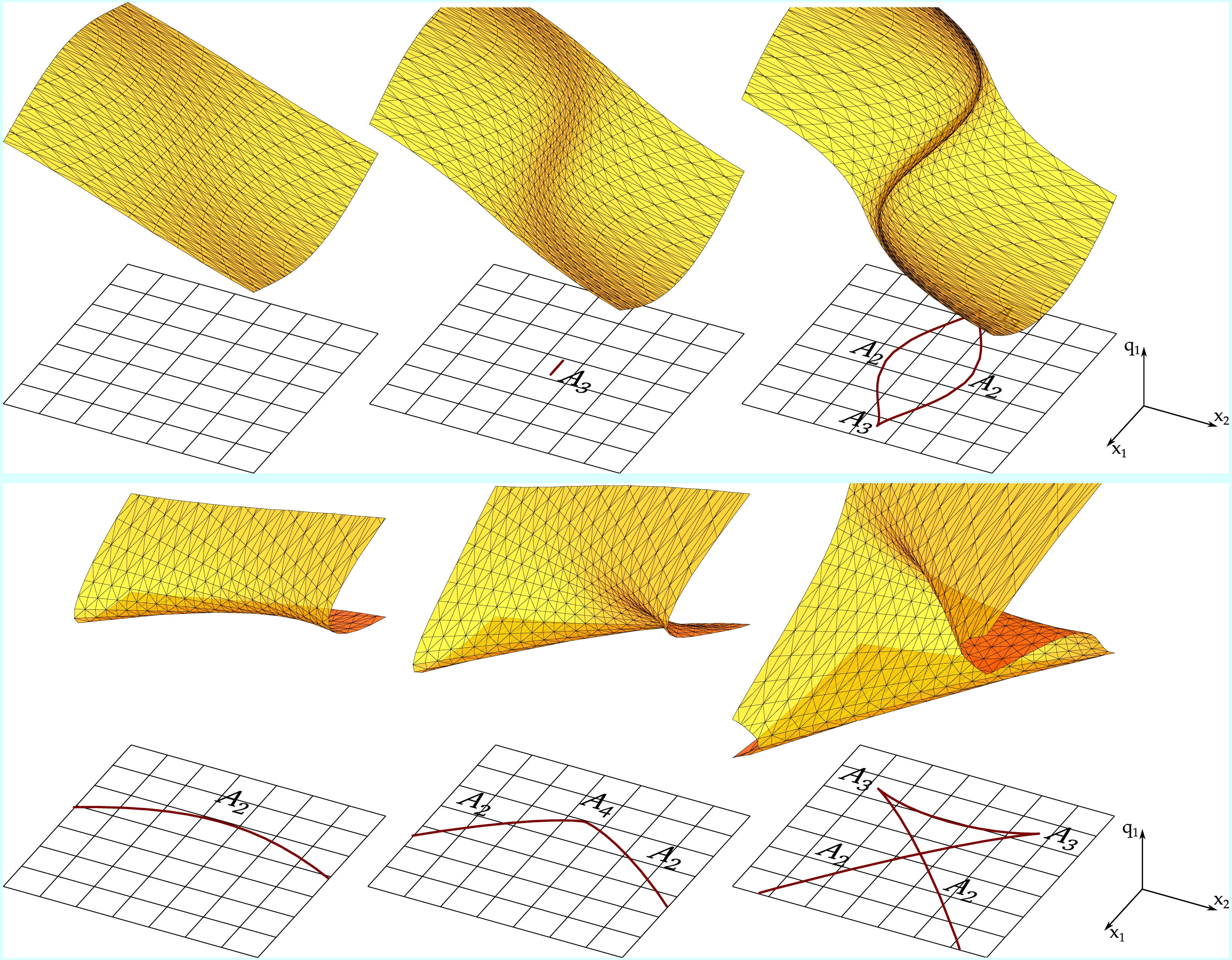
Matter fluid motion

Three-stream flow in 1d: *cusp* singularity $A_3 \rightarrow$ 2 *fold* A_2 singularities.



Velocity $u(x)$, Lagrangian map $x(q)$, and density $\delta(x)$, for given $u(q)$ and $\delta(q) = 1$. Three subsequent times.

Matter fluid motion



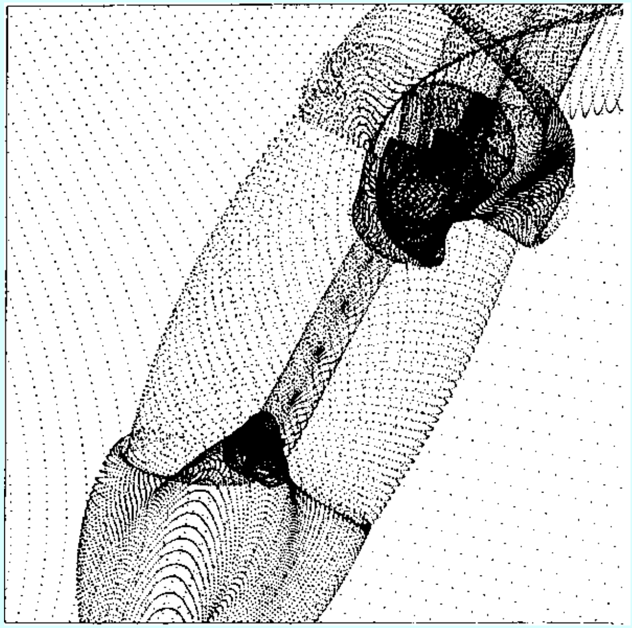
The Zeldovich approximation

- Newton's equations of motion in comoving coordinates $\boldsymbol{x} = \boldsymbol{r}/a$, such that the peculiar velocity $\boldsymbol{u} = a\,\dot{\boldsymbol{x}} = \boldsymbol{v} - H\boldsymbol{r}$:

$$\frac{d\boldsymbol{u}}{dt} + H\boldsymbol{u} = \boldsymbol{g}, \quad \boldsymbol{g} = \boldsymbol{g}_T - \boldsymbol{g}_b, \quad \boldsymbol{g}_b = \dot{H}\boldsymbol{r} + H^2\boldsymbol{r}.$$

- Zeldovich (1970) prolongs linear solution $\boldsymbol{x}(t, \boldsymbol{x}_0) = \boldsymbol{x}_0 + b(t)\,\boldsymbol{g}(\boldsymbol{x}_0)$ into the nonlinear regime.

  $\tau := b(t) \Rightarrow$ free motion with velocity $\widetilde{\boldsymbol{u}} = \frac{d\boldsymbol{x}}{d\tau} = \boldsymbol{g}(\boldsymbol{x}_0)$.



The adhesion model

- Zeldovich approximation fails after caustics form (locally).
- Instead of $\widetilde{\boldsymbol{u}} = \text{cst}$,

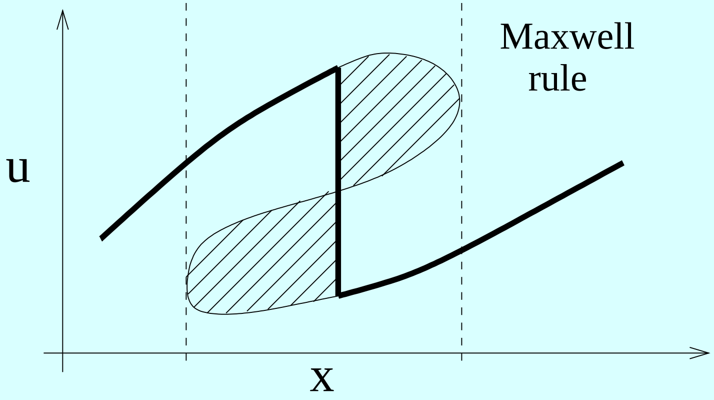
$$\frac{d\widetilde{\boldsymbol{u}}}{d\tau} = \frac{\partial \widetilde{\boldsymbol{u}}}{\partial \tau} + \widetilde{\boldsymbol{u}} \cdot \nabla \widetilde{\boldsymbol{u}} = \nu \nabla^2 \widetilde{\boldsymbol{u}}, \quad \nu \to 0,$$

  plus $\nabla \times \boldsymbol{g}(\boldsymbol{x}_0) = 0 \Rightarrow \nabla \times \widetilde{\boldsymbol{u}} = 0$ (potential flow).

  The viscosity gives rise to adhesion of matter and models gravity (Gurbatov & Saichev, 1984).

  Burgers equations for pressureless turbulence $\longrightarrow$ shocks.

- Hopf-Cole solution $\longrightarrow$ Maxwell's equal area rule.

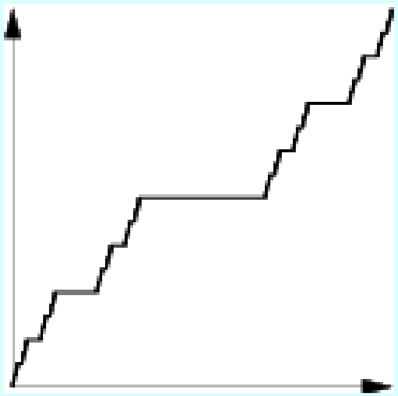


The adhesion model in 1d

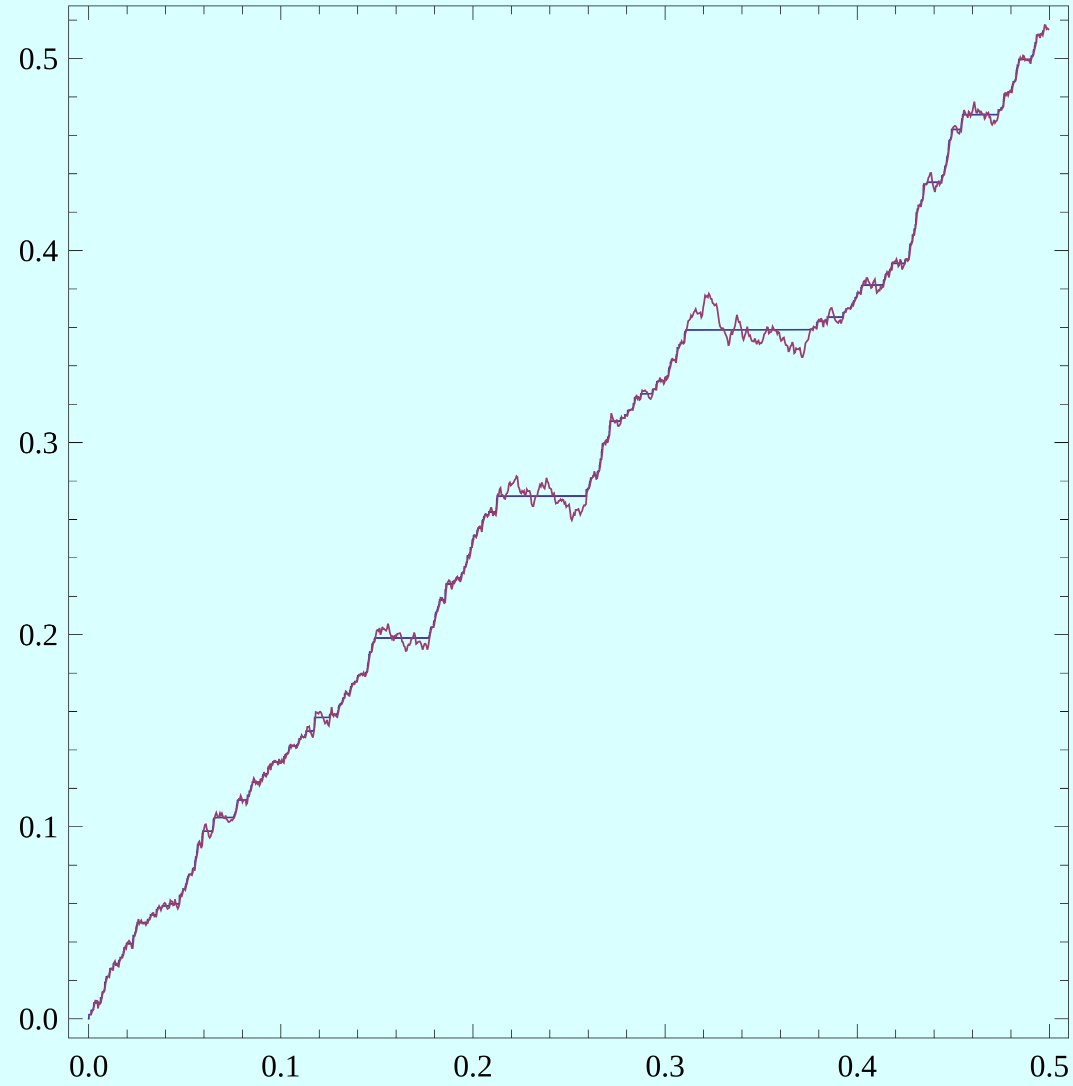
Initial density and velocity fluctuations are non-smooth $\Rightarrow$ non-isolated singularities.

Lagrangian map $q \to x$.

Multi-streaming $\to$ random *Devil's staircase*.



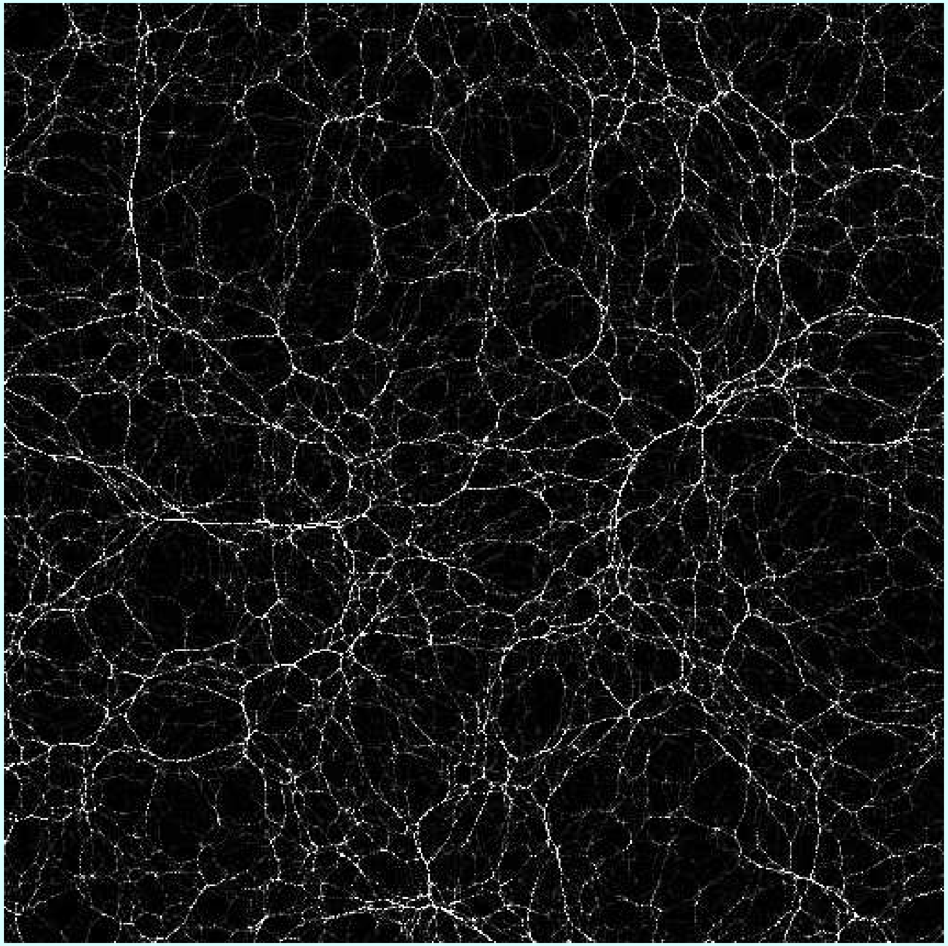
q-axis = mass.



Cosmic web structure

In 2d, mass adhesion $\longrightarrow$ web of filaments.

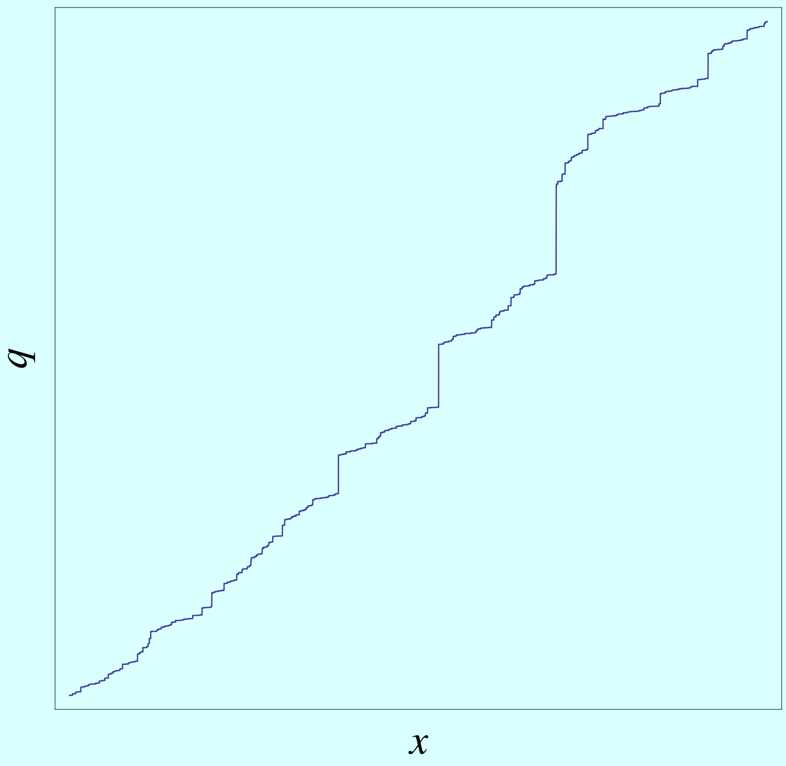
Filaments surround voids, which are not empty, because they contain weaker filaments.

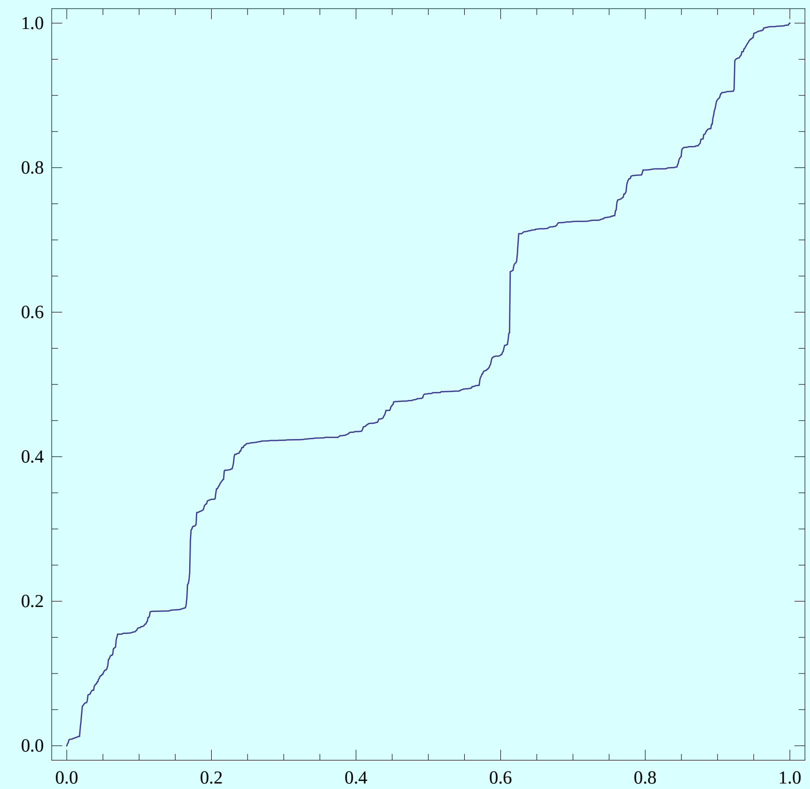


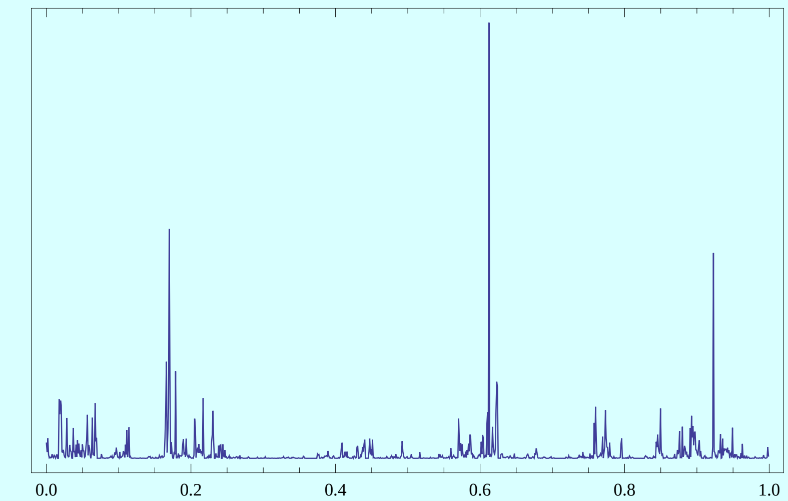
Adhesion dynamics $\longrightarrow$ movie 1.

N-body **gravitational** dynamics $\longrightarrow$ movie 2.

Random mass distributions (1d)







Inverse Devil's staircase and random **continuous** mass distribution.

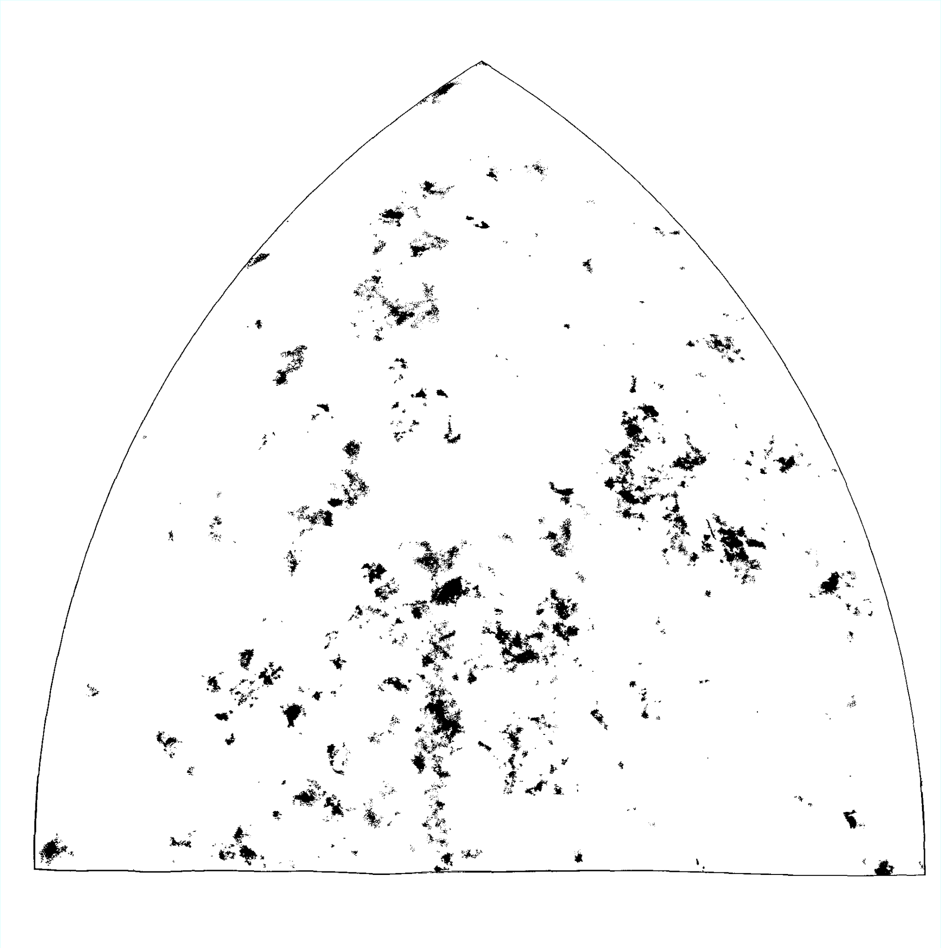
Both are strictly singular.

Fractal geometry

Mandelbrot's uni-fractal (1980), with dimension $D \simeq 1.23$.

Integral of cond. 2-point correl. function

$$M(r) \propto r^D.$$



Generic mass distributions:

$$M(r) \propto r^\alpha,$$

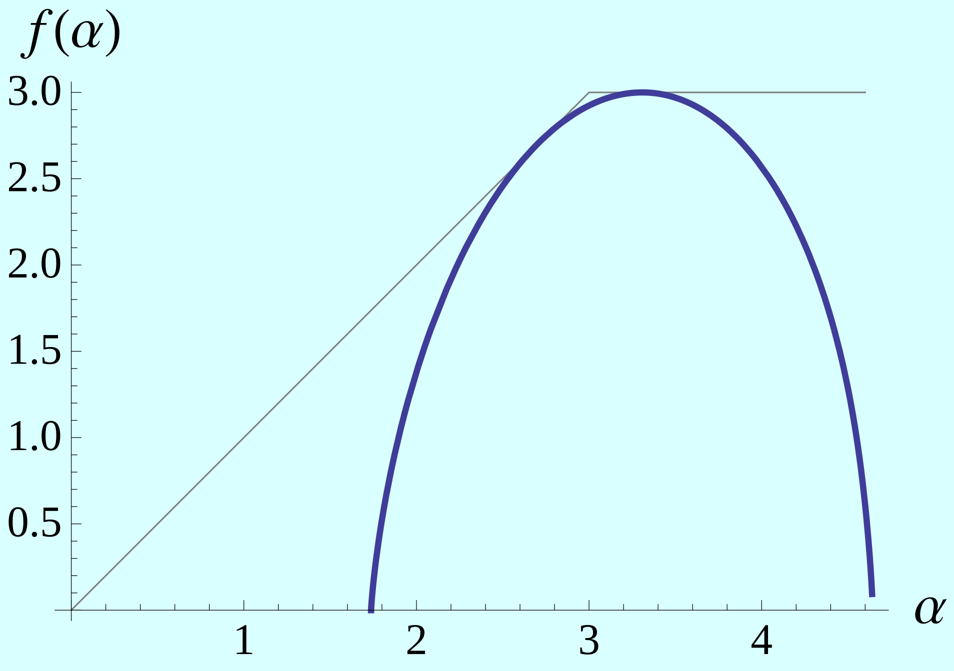
with a range of α.

- Singular mass concentrations: $\alpha < 3 \Rightarrow$ infinite density.
- Mass depletions: $\alpha > 3 \Rightarrow$ vanishing density.

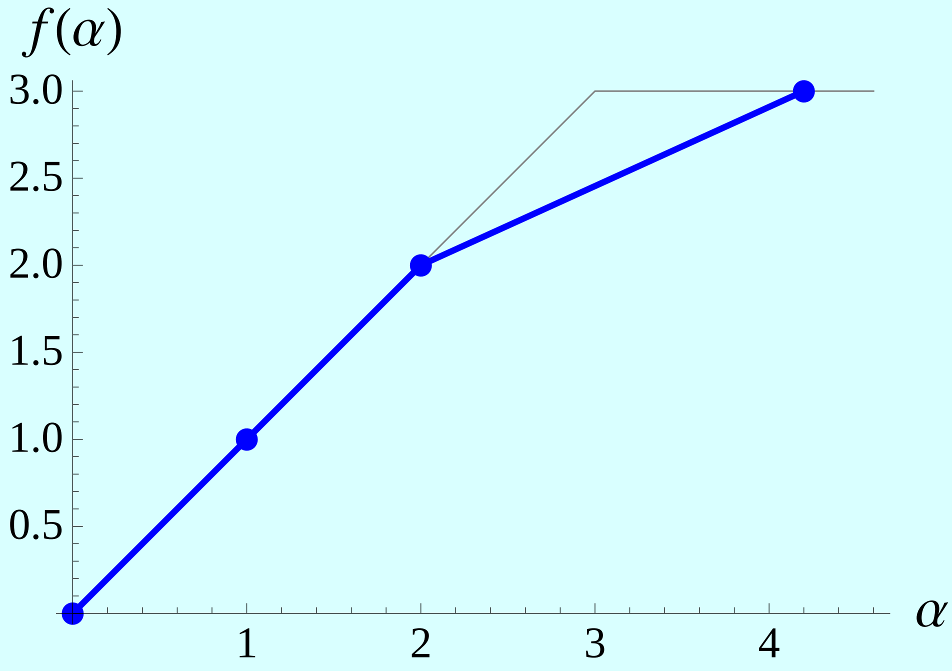
Multifractal analysis

Sizes of sets of mass concentrations or depletions: given α, we have an infinite set of points, measured by its fractal dimension $f(\alpha)$.

Types of multifractal spectrum $f(\alpha)$ (in 3d):



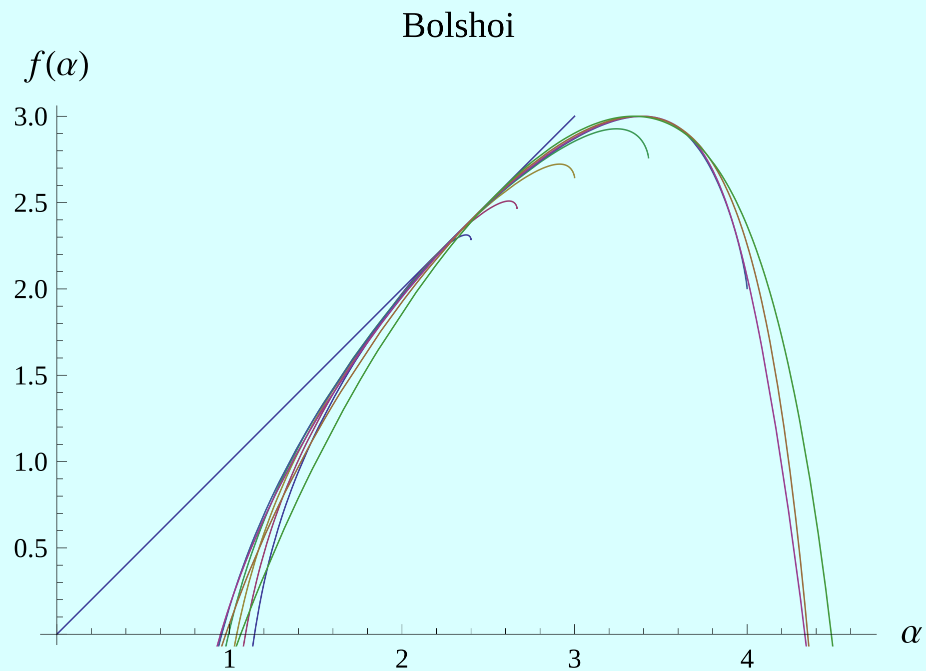
Typical self-similar mass dist. (lognormal like density).



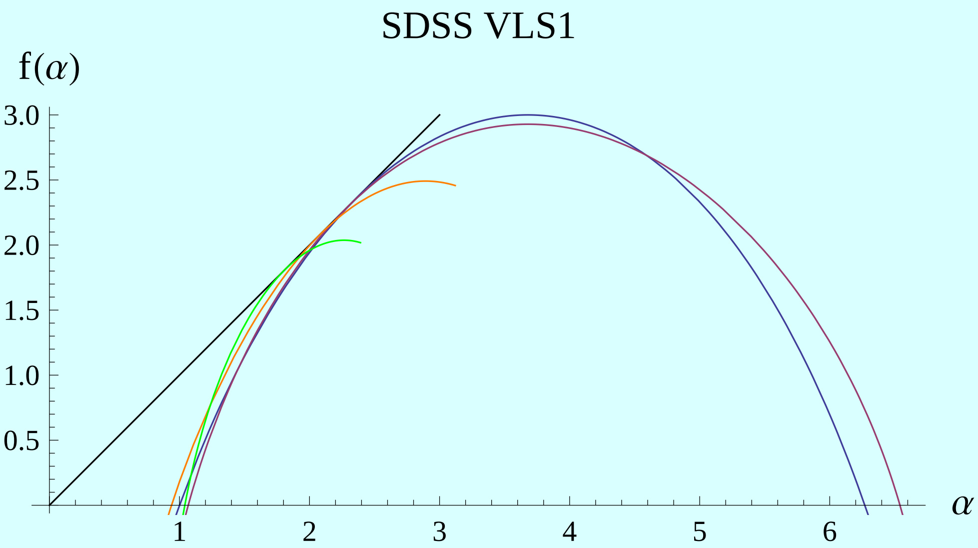
3d adhesion model (delta-function singularities).

N-body LCDM simulations and SDSS galaxies

Coarse-grained multifractal spectra for dark matter and stellar mass distributions:



MF spectra of Bolshoi sim.



Stellar mass MF spectra of SDSS sample.

They coincide for mass concentrations, $\alpha < 3$.

CONCLUSIONS

- Structure formation in the nonlinear regime described qualitatively by the adhesion model = Burgers turbulence.
- Strictly singular mass distribution with mass concentrations (clusters) and mass depletions (cosmic voids).
- Multifractal spectrum of the mass distribution obtained from data differs from the adhesion model prediction:
  - $\alpha_{\rm min} = 1$ instead of $0 \to$ Newton's gravitational energy diverges for $\alpha < 1$.
  - $\max f = 3 \to$ no empty voids, like in the adhesion model.