Evaluation and propagation of measurement uncertainties

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European Commission

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CIEMAT

JRC-Geel

Directorate for Nuclear Safety and Security

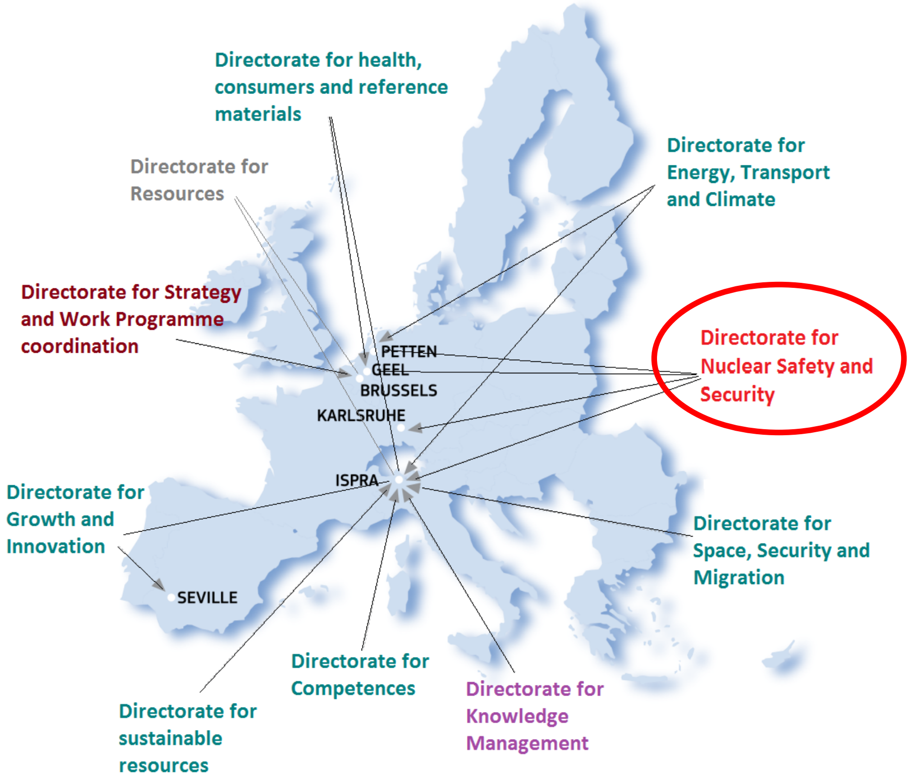
SN3S Unit: standards for nuclear safety, security and safeguards

14
major buildings

Around **230** staff

from **4** JRC Directorates

40 ha

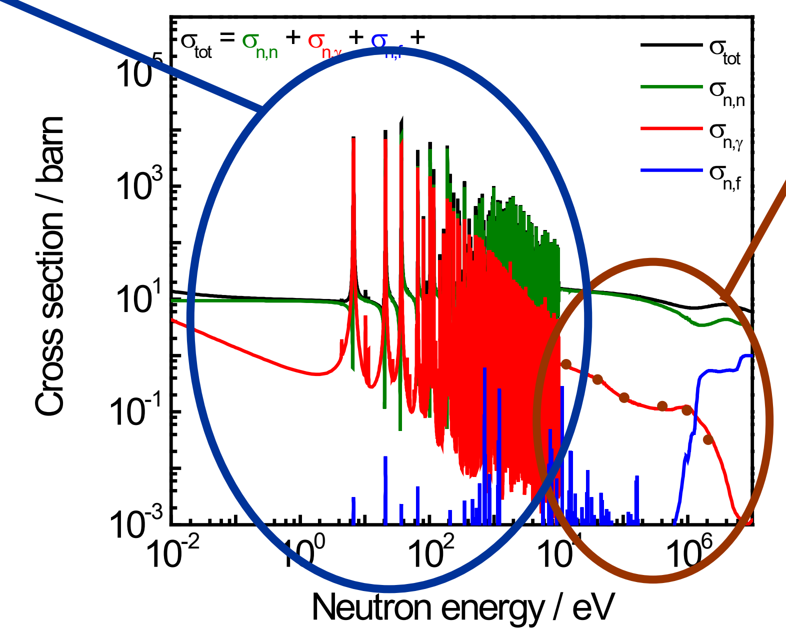


Neutron induced cross section measurements

GELINA

MONNET

^{238}U+n



White neutron source
+
Time-of-flight (TOF)

Mono-energetic neutrons
(cp,n) reactions

Contents

- Measurement uncertainty: definition
- Propagation of uncertainties
  - Properties of normal distribution (Gauss)
  - Propagation of uncertainties of uncorrelated input quantities
  - Propagation of uncertainties of correlated input quantities
  - Application to a simple measurement model: Z = K (Y – B)
- Evaluation of measurement uncertainties
  - Example 1: neutron capture experiment
    - Normalisation
    - Determination of resonance parameters
  - Example 2: determination of activity by alfa-counting (ANOVA)

} Terminology GUM

Basis: Guides in Metrology published by BIPM

https://www.bipm.org/en/publications/guides

- GUM: Guide to the expression of Uncertainty in Measurement
  - Evaluation of measurements data (JCGM 100:2008(E))
  - Supplement 1 – Propagation of distribution using a Monte Carlo method (JCGM 101:2008)
  - Supplement 2 – Extension to any number of quantities (JCGM 102:2011)
  - An introduction to the “GUM” and related documents (JCGM 104:2009)
  - Guide to the expression of uncertainty in measurement Part 6: Developing and using measurement models (JCGM GUM-6:2020)

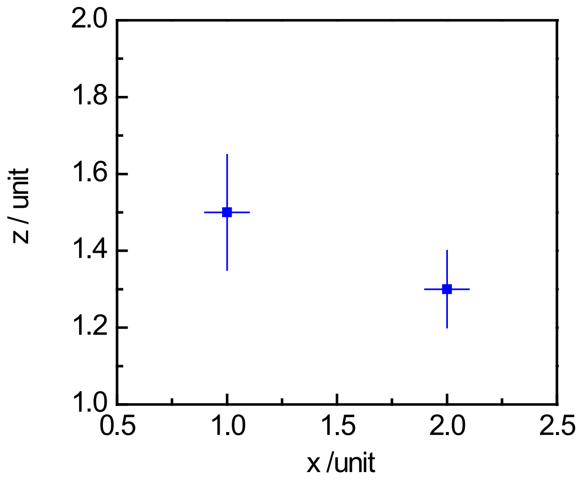
- VIM: International Vocabulary of Metrology
  - VIM – Basic and general concepts and assiociated terms (JCGM 200:2012)

Measurement and uncertainty

- The objective of a **measurement** is to **determine** the **value** of the **measurand** that is, the value of the particular **quantity to be measured**

- **Uncertainty** of a measurement:
  parameter, associated with the result of a measurement, that characterizes the **dispersion** of the **values** that could reasonably be **attributed** to the **measurand**

- **Result** of a **measurement** is only an approximation or **estimate of** the value of the **measurand** and thus is complete only when **accompanied by** a statement of the **uncertainty** of that estimate



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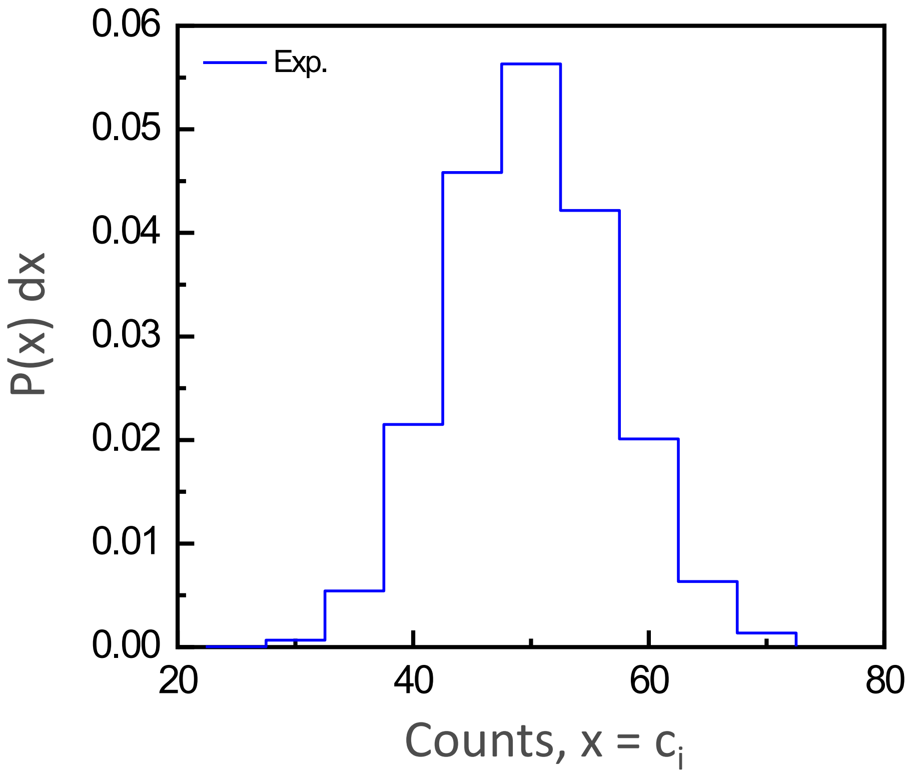
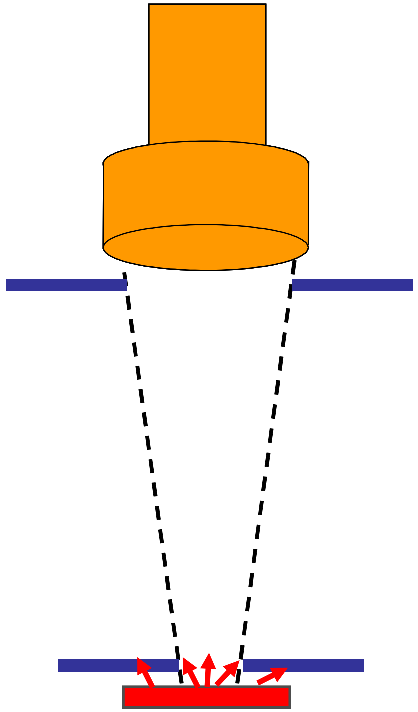
Example: counting experiment

c_i : result of a single counting experiment to estimate the counts C



Standard uncertainty is expressed as the standard deviation



Example: counting experiment

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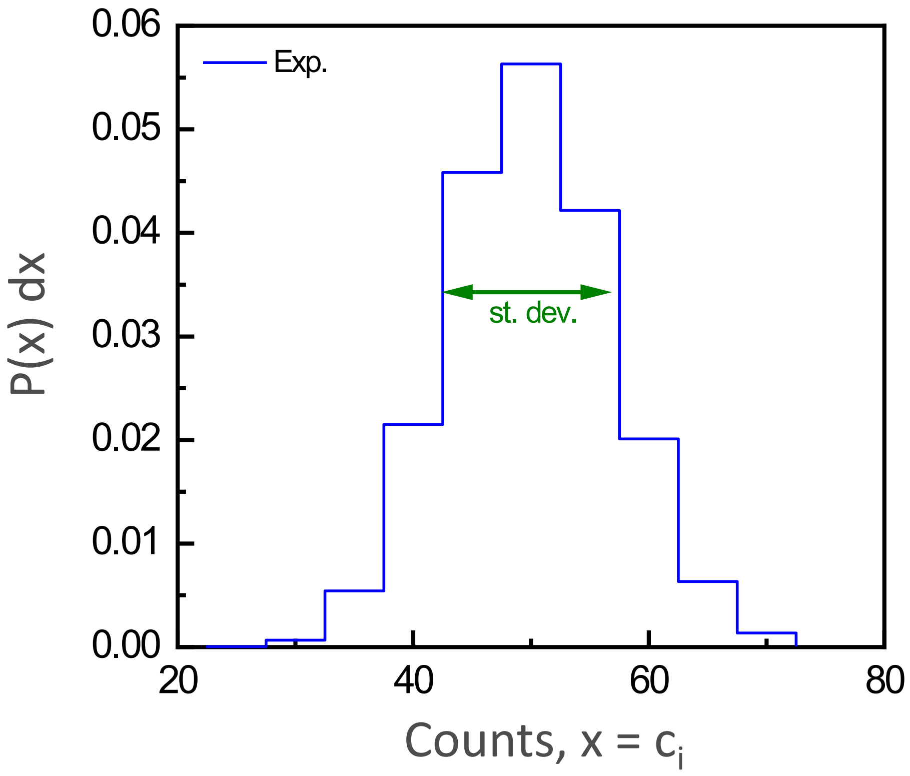


Standard uncertainty is expressed as the standard deviation

Mean : $c = \frac{1}{m}\sum_{i=1}^{m} c_i$

Variance of mean : $s_c^2 = \frac{s^2(c_i)}{m}$

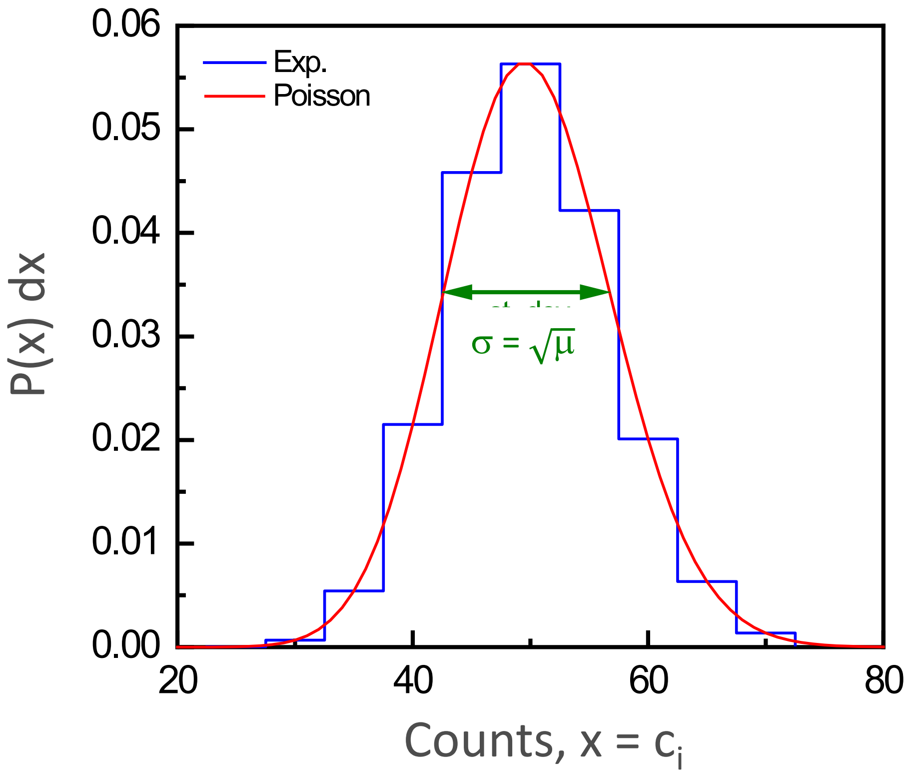
Variance of a single observation : $s_{c_i}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(c_i - c)^2$

Standard uncertainty of c : $u_c = \frac{s_{c_i}}{\sqrt{m}}$

Standard uncertainty of c_i : $u_{c_i} = s_{c_i}$

Example: counting experiment

c_i : result of a single counting experiment to estimate the counts C



Poisson distribution

$$P(c,\mu) = e^{-\mu}\frac{\mu}{c!}$$

Mean : μ

Variance : $\sigma^2 = \mu$

Standard deviation : $\sigma = \sqrt{\mu}$

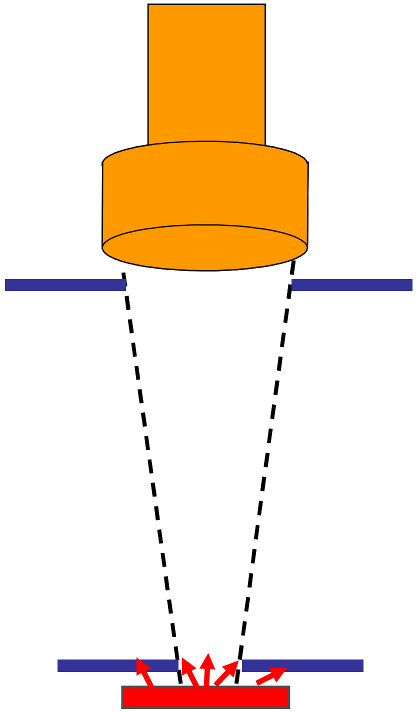
Result of a single counting experiment

Counts : c_i

Uncertainty : $u_{c_i} = \sqrt{c_i}$

$$\frac{u_{c_i}}{c_i} = \frac{1}{\sqrt{c_i}}$$

Uncertainty due to counting statistics



Example: α-activity experiment

Determination of activity A_α based on α-counting

Measurand : A_α (alfa- activity of the sample)

Measurement model : $C_\alpha = \varepsilon_\alpha \Omega P_\alpha A_\alpha$

$A_\alpha = C_\alpha / (\varepsilon_\alpha \Omega P_\alpha)$

- Results of counting experiment: $c_\alpha = c - b$

  sample count rate : c

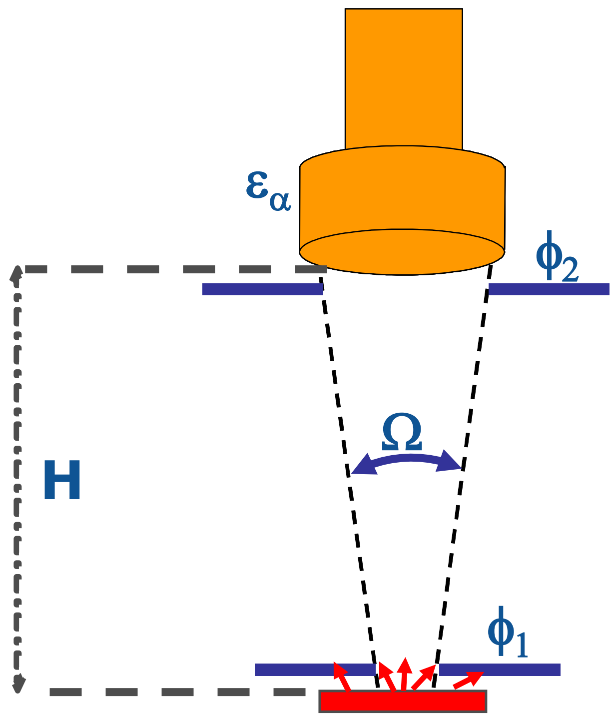
  background count rate : b

- Other input quantities

  P_α : escape probability

  Ω : solid angle depends on (H, ϕ_1, ϕ_2)

  ε_α : detection efficiency



General

Input quantities → **Output quantity (measurand)**

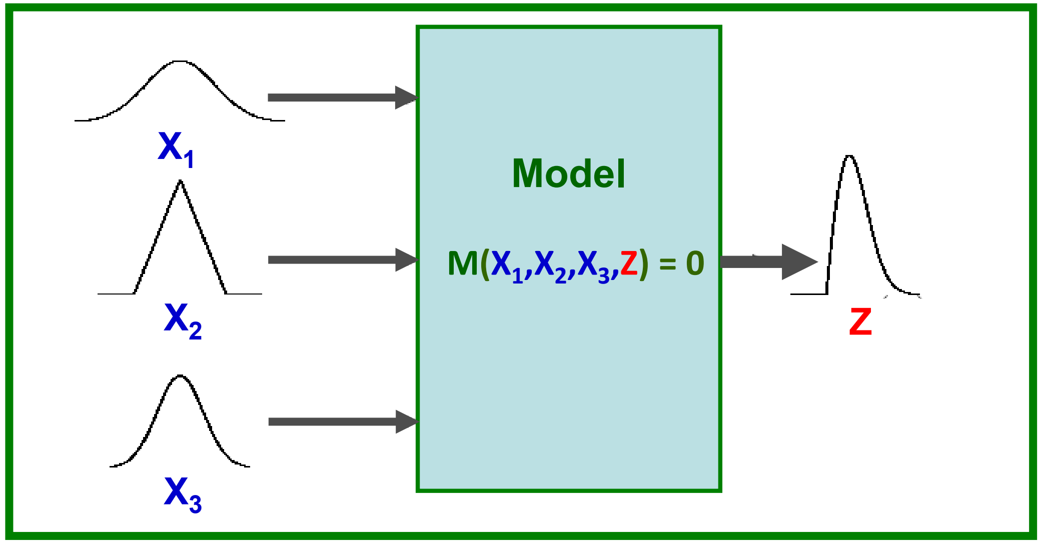
(X_1, X_2, X_3, …) **Model** **Z**

Input

- ***Measurement data***

+

- calibration constants
- influencing quantities
- physics constants, *e.g.* N_A



General

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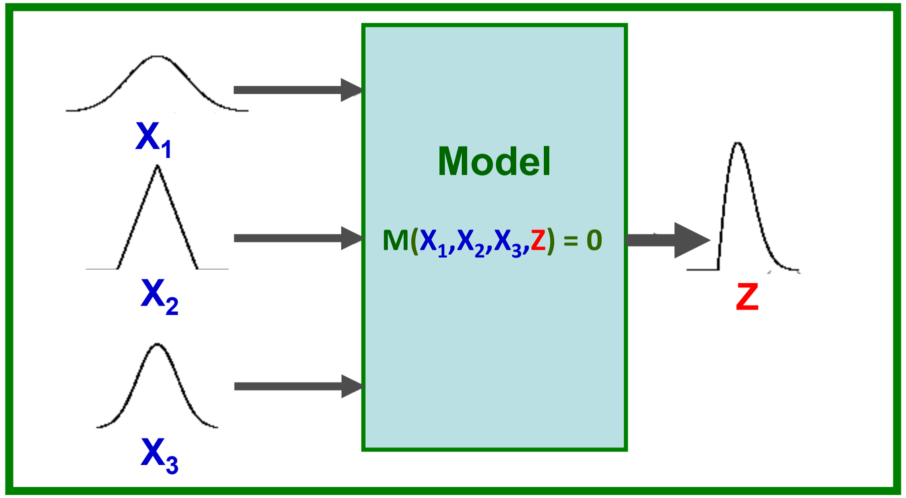


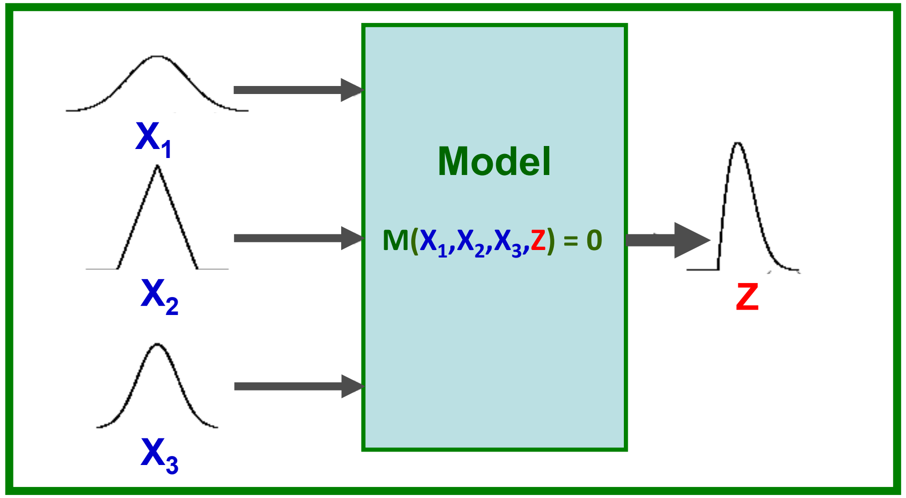
Ideally : define Probability Distribution (PD) of $(X_1,X_2,X_3, \ldots)$ and transform into PD of Z

- Analytically (deterministic, by transformation of variables) $\phi(y)\,dy = \varphi(x)\left|\frac{dx}{dy}\right| dy$ with $y = f(x)$
- Monte Carlo simulations (stochastic)

General

Input quantities → **Output quantity (measurand)**

$(X_1, X_2, X_3, \ldots)$ **Model** **Z**



Common practice: propagate uncertainties by General Law of Uncertainty Propagation (GLUP), based on:

- Properties of Normal Probability Distribution
- Combined with 1st order Taylor development for non-linear problems

GLUP : independent variables

(y, u_y), (b, u_b) and (k, u_k) independent input quantities $\Rightarrow$ estimate of Z

$Z = Y + B \quad \Rightarrow \quad z = y + b \quad u_z^2 = u_y^2 + u_b^2$

$Z = Y - B \quad \Rightarrow \quad z = y - b \quad u_z^2 = u_y^2 + u_b^2$

$Z = KY \quad \Rightarrow \quad z = k\,y \quad u_z^2 = k^2 u_y^2 + y^2 u_k^2 \quad \frac{u_z^2}{z^2} = \frac{u_k^2}{k^2} + \frac{u_y^2}{y^2}$

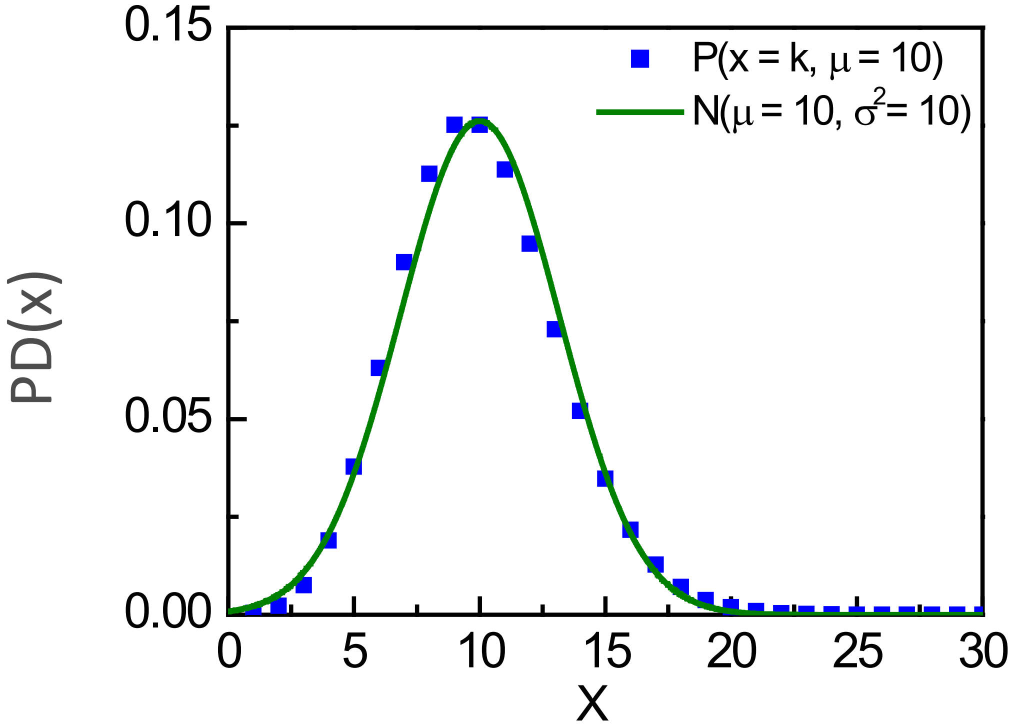
(y, u_y) and constant K $\Rightarrow$ estimate of Z

$Z = KY \quad \Rightarrow \quad z = K\,y \quad u_z^2 = K^2 u_y^2 \quad \frac{u_z^2}{z^2} = \frac{u_y^2}{y^2}$

Probability distributions

(1) Poisson distribution to account for uncertainty due to counting statistics

For large μ the distribution approaches a normal distribution



Poisson : $P(x = k, \mu) = e^{-\mu} \frac{\mu}{k!}$

Normal : $N(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Probability distributions

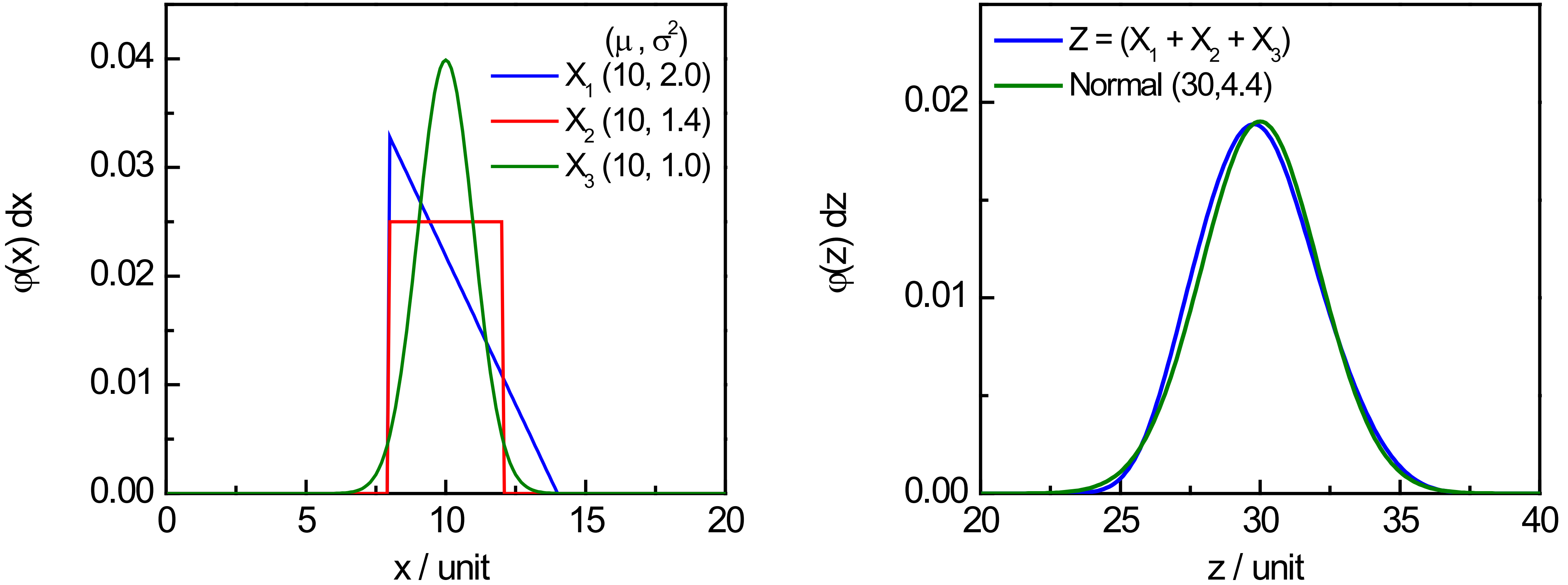
(1) Poisson distribution to account for uncertainty due to counting statistics

For large μ the distribution approaches a normal distribution

(2) Central limit theorem (CLT)

The sum of a large number of independent random variables with a similar distribution (i.e. width) will be approximately normally distributed

Central limit theorem



$Z = \sum_{i=1}^{n} X_i$ normal distribution with $\mu_Z = \sum_{i=1}^{n} \mu_i$ and $\sigma_Z^2 = \sum_{i=1}^{n} \sigma_i^2$

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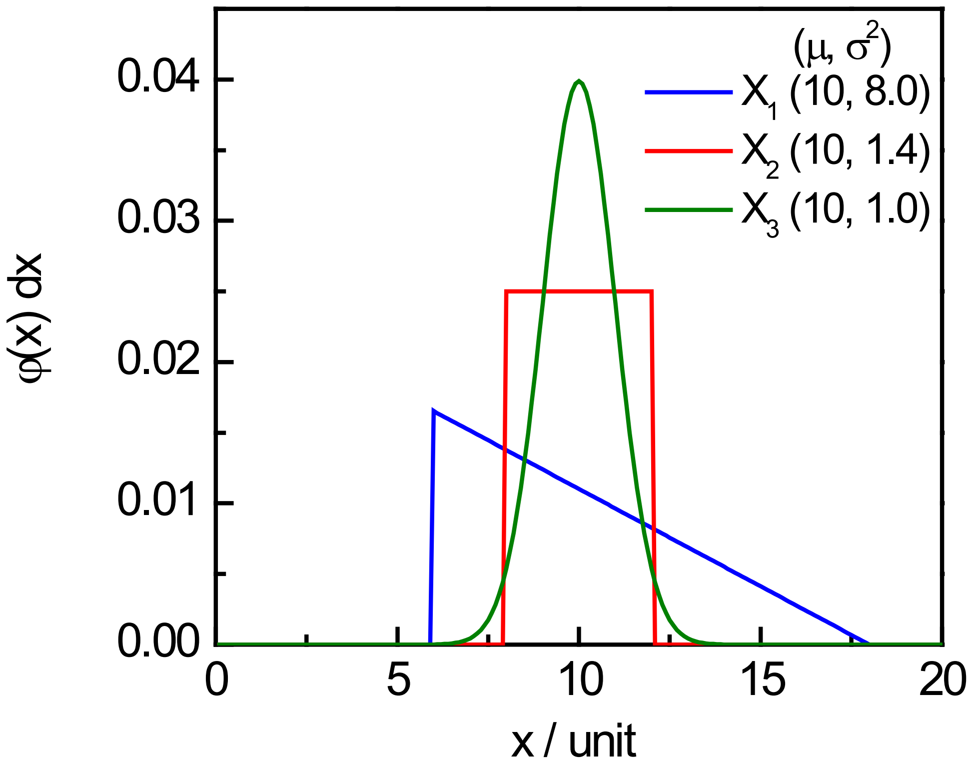
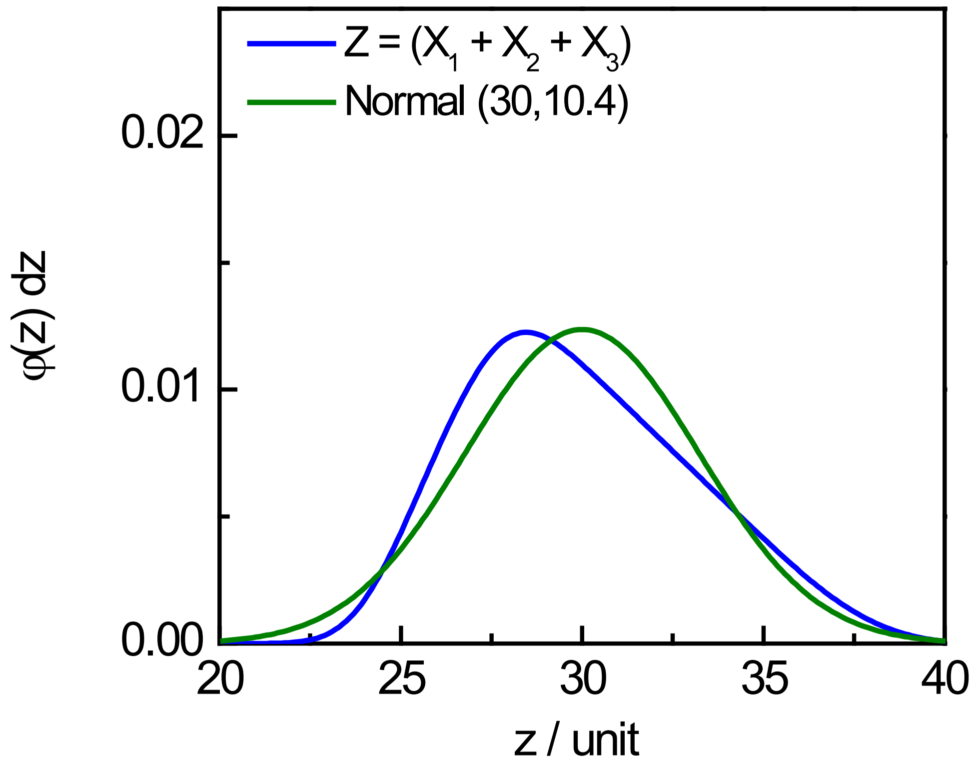
Central limit theorem





$\sigma_1 > (\sigma_2 \text{ and } \sigma_3)$

Probability distributions

(1) Poisson distribution to account for uncertainty due to counting statistics

For large μ the distribution approaches a normal distribution

(2) Central limit theorem (CLT)

The sum of a large number of independent random variables with a similar distribution (i.e. width) will be approximately normally distributed

(3) Principle of maximum entropy (ME)

If only the mean and standard deviation is given, the optimal probability distribution for further inference is the normal distribution

$\Rightarrow$ in most cases normal distribution can be assumed

Reporting of uncertainty (JCGM 100:2008 section 7)

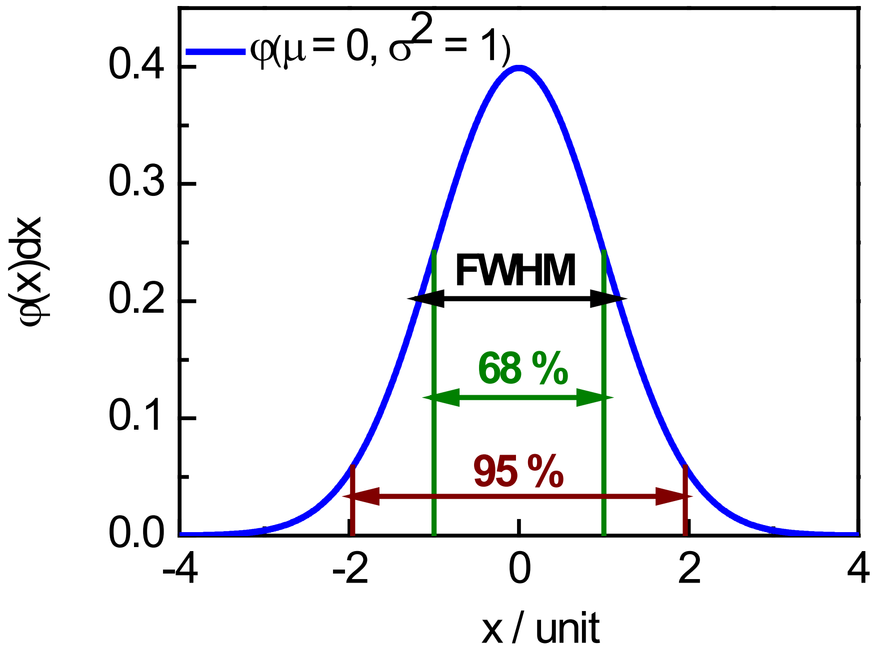
- Standard (k = 1) or expanded (k > 1) uncertainty (x, u_x) with $u_x = k\ s_x$
  - Standard uncertainty

    k = 1 ⇒ 0.68 %
  - Expanded uncertainty

    k > 1

    e.g. k = 1.96 ⇒ 0.95 %

    k = 2.58 ⇒ 0.99 %



- Reporting example: x = 10.21 with u_x = 0.25
  - k = 1 10.21 (25) or 0.21 (0.25)
  - k > 1 10.21 ± 0.25 (specify k)

Note: physicists mostly report a standard uncertainty while chemists mostly report an expanded with k = 2.

Linear function of independent variables

Z : linear function of independent random variables $X_{i = 1,\ldots,n}$ with a normal PD with (μ_i, σ_i^2)

$$z = \sum_{i=1}^{n} c_i \, x_i \qquad\qquad c_i = \left.\frac{\partial f}{\partial x_i}\right|_{\mu_i}$$

$\Rightarrow$ PD of Z = f(X_i; i,…,n) is a normal distribution with

- Mean $$E(Z) = \mu_z = \sum_{i=1}^{n} c_i \, \mu_i$$
- Variance $$V(z) = \sigma_z^2 = \sum_{i=1}^{n} c_i^2 \, \sigma_i^2$$

Linear function of independent variables

Z : non-linear function of independent random variables $X_{i=1,\dots,n}$ with a normal PD with (μ_i, σ_i^2)

1st order Taylor development

$$z \approx f(\mu_1,\dots,\mu_n) + \sum_{i=1}^{n} g_i\,(x_i - \mu_i) \qquad g_i = \left.\frac{\partial f}{\partial x_i}\right|_{\mu_i}$$

⇒ PD of Z = f(X_i; i,…,n) is a normal distribution with

- Mean $$E(Z) = \mu_z \approx f(\mu_1,\dots,\mu_n)$$
- Variance $$V(z) = \sigma_z^2 \approx \sum_{i=1}^{n} g_i^2\,\sigma_i^2$$

$Z = Y - B$ with estimates (y_1, u_{y_1}), (y_2, u_{y_2}) and (b, u_b)

- Experiment

  Independent observables (y_1,y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})

  Background estimate : (b, u_b)

- Determine an estimate of $Z = (Y- B)$

1) Based on input quantities (y_1, u_{y_1}), (y_2, u_{y_2}) and (b, u_b):
   - first define best estimate y of Y
   - $z = y - b$

2) Based on input quantities (z_1, u_{z_1}), (z_2, u_{z_2}) with $z_1 = y_1 - b$ and $z_2 = y_2 - b$

$Z = Y - B$ with estimates (y_1, u_{y_1}), (y_2, u_{y_2}) and (b, u_b)

- Experiment

  Independent observables (y_1,y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})

  Background estimate : (b, u_b)

  Report : (y_1,u_{y_1}), (y_2,u_{y_2}) and (b,u_b)

- Determine an estimate of Z = (Y- B) with input quantities (y_1, u_{y_1}), (y_2, u_{y_2}) and (b,u_b)

  (1) Average of (y_1, y_2) to estimate Y : $y = \frac{y_1+y_2}{2}$ $\quad u_y^2 = \frac{u_{y_1}^2+u_{y_2}^2}{4}$

  (y_1,y_2) independent

  (2) Background subtraction : $z = y - b$ $\quad u_z^2 = u_y^2 + u_b^2$

  (y,b) independent

$$z = \frac{y_1+y_2}{2} - b \qquad u_z^2 = \frac{u_{y_1}^2+u_{y_2}^2}{4} + u_b^2$$

$Z = Y - B$ with estimates (z_1, u_{z_1}), (z_2, u_{z_2})

- Experiment

  Independent observables (y_1, y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})
  Background estimate : (b, u_b)
  Report : (z_1, u_{z_1}), (z_2, u_{z_2})

  $$z_1 = y_1 - b \qquad u_{z_1}^2 = u_{y_1}^2 + u_b^2$$
  $$z_2 = y_2 - b \qquad u_{z_2}^2 = u_{y_2}^2 + u_b^2$$

- Determine an estimate of $Z = (Y - B)$ with input quantities (z_1, u_{z_1}), (z_2, u_{z_2})

  Average of (z_1, z_2) to estimate Z : $z = \frac{z_1 + z_2}{2}$ $\quad u_z^2 = \frac{u_{z_1}^2 + u_{z_2}^2}{4}$

  suppose (z_1, z_2) independent

$$z = \frac{y_1 + y_2}{2} - b \qquad u_z^2 = \frac{u_{y_1}^2 + u_{y_2}^2}{4} + \frac{u_b^2}{2}$$

Z = Y – B with estimates (y_1,u_{y_1}), (y_2,u_{y_2}) and (b,u_b)

- Experiment

  Independent observables (y_1,y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})

  Background estimate : (b, u_b)

- Determine an estimate of Z = (Y- B)

1) Based on input quantities (y_1, u_{y_1}), (y_2, u_{y_2}) and (b, u_b)

$$z = \frac{y_1+y_2}{2} - b \qquad u_z^2 = \frac{u_{y_1}^2+u_{y_2}^2}{4} + u_b^2$$

2) Based on input quantities (z_1, u_{z_1}), (z_2, u_{z_2}) with $z_1 = y_1 - b$ and $z_2 = y_2 - b$ (suppose independent, which is not correct!)

$$z = \frac{y_1+y_2}{2} - b \qquad u_z^2 = \frac{u_{y_1}^2+u_{y_2}^2}{4} + \frac{u_b^2}{2}$$

Contribution due to common uncertainty component is not correct
Underestimation of the uncertainty due to common component

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$(Z_1 = KY_1, Z_2 = KY_2) \Rightarrow R = Z_1 / Z_2$

- Experiment

  Independent observable y_1 of Y1 : (y_1, u_{y_1})

  Independent observable y_2 of Y_2 : (y_2, u_{y_2})

  Normalisation factor K : (k, u_k)

- Determine an estimate of $R = Z_1 / Z_2$

1) Based on input quantities (y_1, u_{y_1}), (y_2, u_{y_2}) and (k, u_k)

$$r = \frac{ky_1}{ky_2} = \frac{y_1}{y_2} \qquad \frac{u_r^2}{r^2} = \frac{u_{y_1}^2}{y_1^2} + \frac{u_{y_2}^2}{y_2^2}$$

2) Based on input quantities (z_1, u_{z_1}), (z_2, u_{z_2}) with $z_1 = ky_1$ and $z_2 = ky_2$ (suppose independent)

$$r = \frac{z_1}{z_2} \qquad \frac{u_r^2}{r^2} = \frac{u_{z_1}^2}{z_1^2} + \frac{u_{z_2}^2}{z_2^2} = \frac{u_{y_1}^2}{y_1^2} + \frac{u_{y_2}^2}{y_2^2} + 2\frac{u_k^2}{k^2}$$

Contribution due to common uncertainty component is not correct

Overestimation of the uncertainty due to common component

Linear function of variables (normal PDF)

z : linear function of random variables $x_{i = 1,\dots,n}$ with $(\mu_i = 1,\dots,n; V_{\vec{x}})$

$$z = f(x_i;\ i = 1,\dots,n) \qquad Z = \sum_{i=1}^{n} c_i X_i \qquad c_i = \left.\frac{\partial f}{\partial x_i}\right|_{\mu_i}$$

	Mean	Variance
• Independent variables :	$\mu_z = \sum_{i=1}^{n} c_i \mu_i$	$\sigma_z^2 = \sum_{i=1}^{n} c_i^2 \sigma_i^2$

Linear function of variables (normal PDF)

z : linear function of random variables $x_{i=1,\dots,n}$ with $(\mu_i = 1,\dots,n; V_{\vec{x}})$

$$z = f(x_i;\ i = 1,\dots,n) \qquad Z = \sum_{i=1}^{n} c_i\, X_i \qquad c_i = \left.\frac{\partial f}{\partial x_i}\right|_{\mu_i}$$

	Mean	**Variance**
• Independent variables :	$\mu_z = \sum_{i=1}^{n} c_i\, \mu_i$	$\sigma_z^2 = \sum_{i=1}^{n} c_i^2\, \sigma_i^2$
• Dependent variables :	$\mu_z = \sum_{i=1}^{n} c_i\, \mu_i$	$\sigma_z^2 = \sum_{i=1}^{n} c_i^2\, \sigma_i^2 + \boxed{\sum_{i \neq} \sum_{j} c_i\, c_j\, v_{ij}}$

GLUP : matrix notation, z scalar (1 dim.)

	Linear	Non - linear
	$z = C\,\vec{x}$	$z \approx f(\vec{\mu}_x) + G_{\vec{x}}(\vec{x} - \vec{\mu}_x)$
	C : dim (1 x n)	$G_{\vec{x}}$: gradient matrix of f Jacobian matrix
	$c_k = \frac{\partial f}{\partial x_k}$	$g_k = \frac{\partial f}{\partial x_k}$
• **Mean**	$\mu_z = C\;\vec{\mu}_X$	$\mu_z \approx f(\vec{\mu}_x)$
• **Covariance matrix**	$V_z = C\,V_{\vec{x}}\;C^T$	$V_z \approx G_{\vec{x}}\;V_{\vec{x}}\;G_{\vec{x}}^T$

$\Rightarrow$ basis of General Law of Uncertainty Propagation (GLUP)

(sandwich formula, V_z = G V_x G^T)

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b) \Rightarrow z$

1) Experiment

Independent observables (y_1, y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})

Background estimate : (b, u_b)

Report estimates of Z = Y- B : (z_1, z_2) and $V_{z_1, z_2,}$

2) Evaluator determines best estimate of Z

Input observables : (z_1, z_2) and $V_{z_1, z_2,}$

Result : (z, u_z)

GLUP : $(y_1,y_2,b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b)$

$$\vec{z} = C\,\vec{x} \qquad V_{\vec{z}} = G\ V_{\vec{x}}\ G^T$$

$\vec{z} = (z_1,z_2)$ $\qquad$ $\vec{x} = (y_1,y_2,b)$

1) $V_{y_1,y_2,b}$: experimental input quantities, determined by experimental conditions

independent observables

$\Rightarrow$ uncorrelated uncertainties

$$V_{y_1,y_2,b} = \begin{bmatrix} u_{y_1}^2 & 0 & 0 \\ 0 & u_{y_2}^2 & 0 \\ 0 & 0 & u_b^2 \end{bmatrix}$$

2) Matrix G : determined by model Z = Y - B

model : $z_1 = y_1 - b$ and $z_2 = y_2 - b$

$$G = \begin{bmatrix} \frac{\partial z_1}{\partial y_1} & \frac{\partial z_1}{\partial y_2} & \frac{\partial z_1}{\partial k} \\ \frac{\partial z}{\partial y_1} & \frac{\partial z_2}{\partial y_2} & \frac{\partial z_2}{\partial k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

3) $V_{z_1,z_2} = G\ V_{y_1,y_2,b}\ G^T$

GLUP : $(y_1,y_2,b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b)$

1) $V_{y_1,y_2,b}$: experimental input quantities,

Independent observables, uncorrelated uncertainties

$$V_{y_1,y_2,b} = \begin{bmatrix} u_{y_1}^2 & 0 & 0 \\ 0 & u_{y_2}^2 & 0 \\ 0 & 0 & u_b^2 \end{bmatrix}$$

2) Matrix G : determined by model Z = Y - B

Model : $z_1 = y_1 - b$ and $z_2 = y_2 - b$

$$G = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

3) $V_{z_1,z_2} = G\, V_{y_1,y_2,b}\, G^T$

$$V_{z_1,z_2} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_{y_1}^2 & 0 & 0 \\ 0 & u_{y_2}^2 & 0 \\ 0 & 0 & u_b^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_{y_1}^2 & 0 \\ 0 & u_{y_2}^2 \\ -u_b^2 & -u_b^2 \end{bmatrix} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix}$$

GLUP : $(y_1,y_2,b) \Rightarrow (z_1 = y_1 - b,\ z_2 = y_2 - b)$

1) $V_{y_1,y_2,b}$: experimental input quantities,

$$V_{y_1,y_2,b} = \begin{bmatrix} u_{y_1}^2 & 0 & 0 \\ 0 & u_{y_2}^2 & 0 \\ 0 & 0 & u_b^2 \end{bmatrix}$$

Independent observables, uncorrelated uncertainties

2) Matrix G : determined by model Z = Y - B

$$G = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

Model : $z_1 = y_1 - b$ and $z_2 = y_2 - b$

3) $V_{z_1,z_2} = G\, V_{y_1,y_2,b}\, G^T$

$$V_{z_1,z_2} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix}$$

The covariance matrix contains a non-zero non-diagonal element, which reflects the contribution of a common uncertainty component

GLUP : $(y_1,y_2,b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b) \Rightarrow z = (z_1+z_2)/2$

Step (1) : Determine $(z_1,z_2) = (y_1 – b, y_2 – b)$ and covariance matrix $V_{z1,z2}$

$$(z_1,z_2) = (y_1 - b, y_2 - b) \qquad V_{z_1,z_2} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix}$$

Step (2) : Determine $z = (z_1+z_2)/2$ and covariance matrix V_z

$$z = \frac{z_1 + z_2}{2} \qquad V_z = G\, V_{z_1,z_2}\, G^T \qquad G = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$V_z = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{y_1}^2 + u_B^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{u_{y_1}^2 + u_{y_2}^2}{4} + u_b^2$$

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GLUP : $(y_1,y_2,b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b) \Rightarrow z = (z_1+z_2)/2$

Experimental data (y_1,y_2) and b : independent

(1) Based on reporting full experimental details (y_1,u_{y_1}), (y_2,u_{y_2}) and (b,u_b)

$$z = \frac{y_1 + y_2}{2} - b$$

$$u_z^2 = \frac{u_{y_1}^2 + u_{y_2}^2}{4} + u_b^2$$

(2) Based on reporting of (z_1, z_2) and V_{z_1,z_2}

$$z = \frac{z_1 + z_2}{2}$$

Full covariance

$$u_z^2 = \frac{u_{y_1}^2 + u_{y_2}^2}{4} + u_b^2$$

Only diagonal terms

$$u_z^2 = \frac{u_{y_1}^2 + u_{y_2}^2}{4} + \frac{u_b^2}{2}$$

European Commission

Exercise

- Experiment: Independent observables (y_1, u_{y_1}), (y_1, u_{y_1}) and (b, u_b)

- Model: $(y_1, y_2, b) \rightarrow (z_1, z_2) = (y_1 - b, y_2 - b) \rightarrow (v_1, v_2) = (z_1+z_2, z_1-z_2) \rightarrow (v_1+v_2) = 2z_1$

  - Full covariance : $u^2_{2z_1} = 4u^2_{2z_1} = 4\,(u^2_{y_1} + u^2_b)$

  - Only diagonal terms : $u^2_{2z_1} = 2u^2_{z_1} + 2u^2_{z_2} = 2\,u^2_{y_1} + 2\,u^2_{y_2} + 4\,u^2_b$

GLUP : $(y_1, \ldots, y_n, b) \Rightarrow Z = Y - B$

(y_1,u_{y_1}) , (y_2,u_{y_2}), …, (y_n,u_{y_n}) and $(b,u_b) \rightarrow (z_1,u_{z_1})$, (z_1,u_{z_2}), …, $(z_n,u_{z_n}) \rightarrow (z,u_z)$

$(y_1,y_2,\ldots,y_n,b)$: independent

$z_i = y_i - b$

$(z_1,z_2,\ldots,z_n) + V_{\vec{z}}$ $\Rightarrow$

$$z = \frac{1}{m}\sum_{j=1}^{m} y_j - b$$

$$u_z^2 = \frac{1}{m^2}\sum_{j=1}^{m} u_{y_j}^2 + u_b^2$$

$$V_{\vec{z}} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 & \cdot\ \cdot\ \cdot & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 & \cdot\ \cdot\ \cdot & u_b^2 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ u_b^2 & u_b^2 & \cdot\ \cdot\ \cdot & u_{y_n}^2 + u_b^2 \end{bmatrix}$$

European Commission

GLUP : $(y_1,\ldots, y_n, k) \Rightarrow Z = K\,Y$

(y_1,u_{y_1}) , (y_2,u_{y_2}), …, (y_n,u_{y_n}) and (k,u_k) $\rightarrow$ (z_1,u_{z_1}), (z_1,u_{z_2}), …, (z_n,u_{z_n}) $\rightarrow$ (z,u_z)

$(y_1,y_2,\ldots,y_n,k)$: independent

$z_i = k\,y_i$

$$(z_1,z_2,\ldots,z_n)\ + V_{\vec{z}} \quad \Rightarrow \quad z = k\frac{1}{m}\sum_{j=1}^{m} y_j$$

$$V_{\vec{z}} = \begin{bmatrix} k^2u_{y_1}^2 + y_1^2u_k^2 & y_1y_2u_k^2 & \cdot\ \cdot\ \cdot & y_1y_nu_k^2 \\ y_2y_1u_k^2 & k^2u_{y_2}^2 + y_2^2u_k^2 & \cdot\ \cdot\ \cdot & y_2y_nu_{Kk}^2 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ k^2u_{y_{n-1}}^2 & y_{n-1}y_nu_k^2 \\ y_ny_1u_k^2 & y_ny_2u_k^2 & \cdot\ \cdot\ y_ny_{n-1}u_k^2 & k^2u_{y_n}^2 + y_n^2u_k^2 \end{bmatrix}$$

$$u_z^2 = \frac{k^2}{m^2}\sum_{j=1}^{m} u_{y_j}^2 + y^2u_k^2 \qquad y = \frac{\sum_{j=1}^{m} y_j}{m}$$

$$\frac{u_z^2}{z^2} = \frac{1}{m^2}\frac{\sum_{j=1}^{m} u_{y_j}^2}{y^2} + \frac{u_k^2}{k^2}$$

Error and uncertainty (JCGM 100:2008 section 3.2)

- Measurement error : **difference** between **two values**

  *“**result** of a measurement **minus** a **true value** of the measurand ”*

  can be + or -

- Measurement uncertainty : **dispersion** of a distribution

  *“**non-negative parameter** characterizing the **dispersion** of the values being attributed to the measurand”*

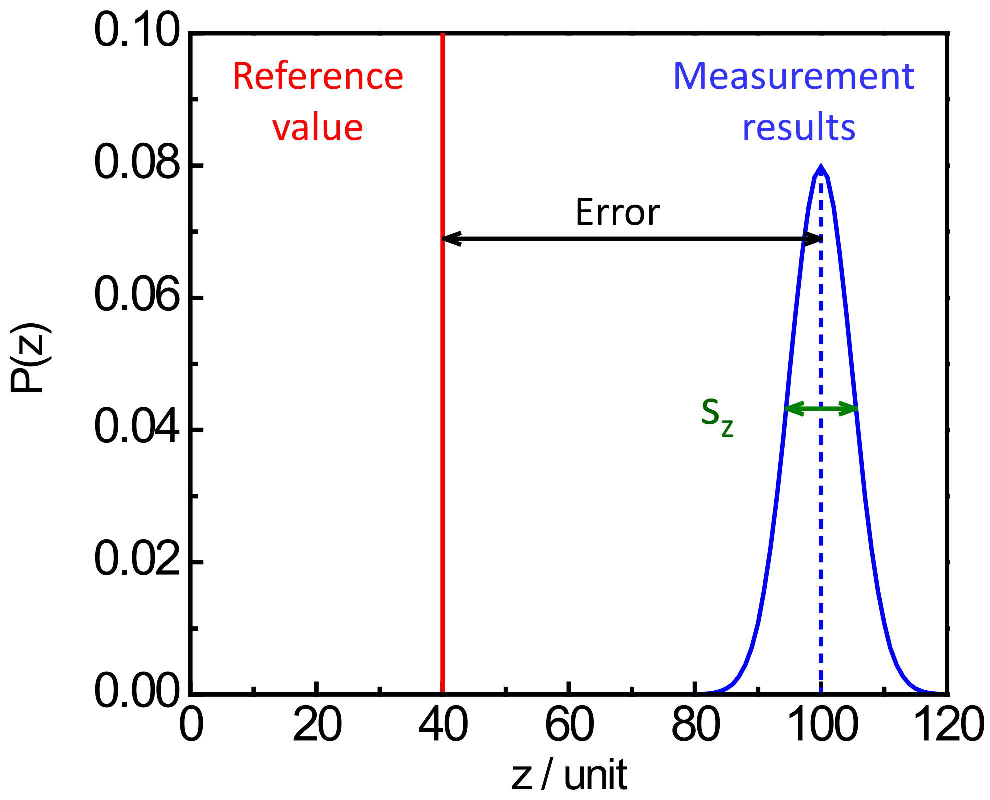
  always > 0

  determined by the width of the PD of the error component(s)

error $\neq$ uncertainty

Measurement error and uncertainty

z : result of a measurement to estimate the value of quantity Z



- Measurement error

  Difference between values
  + or -

- Uncertainty

  Derived from the standard deviation
  (width) of a distribution
  > 0

Measurement error

- Systematic error (bias)

  “Mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand.”

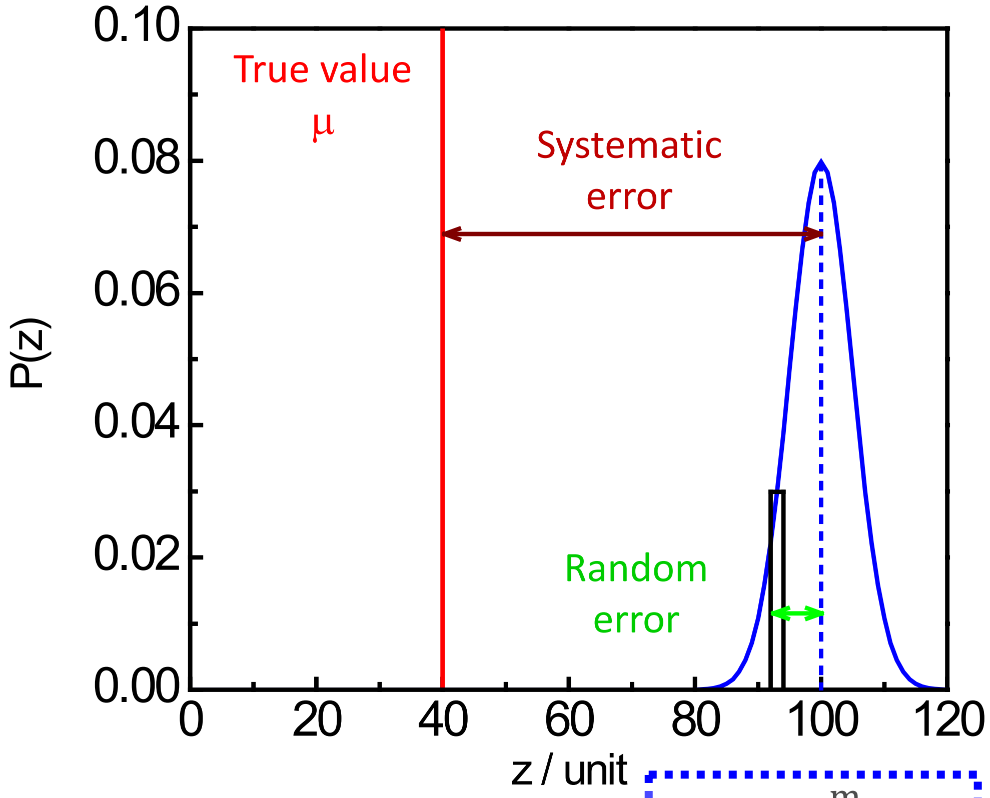
- Random error

  “Result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions.”

Random and systematic error

z_j : result of a single measurement to estimate the value of quantity Z

$z_i = \mu + \beta_s + \varepsilon_{r,i}$



$$z = \frac{1}{m}\sum_{j=1}^{m} z_j$$

- Systematic error : β_s

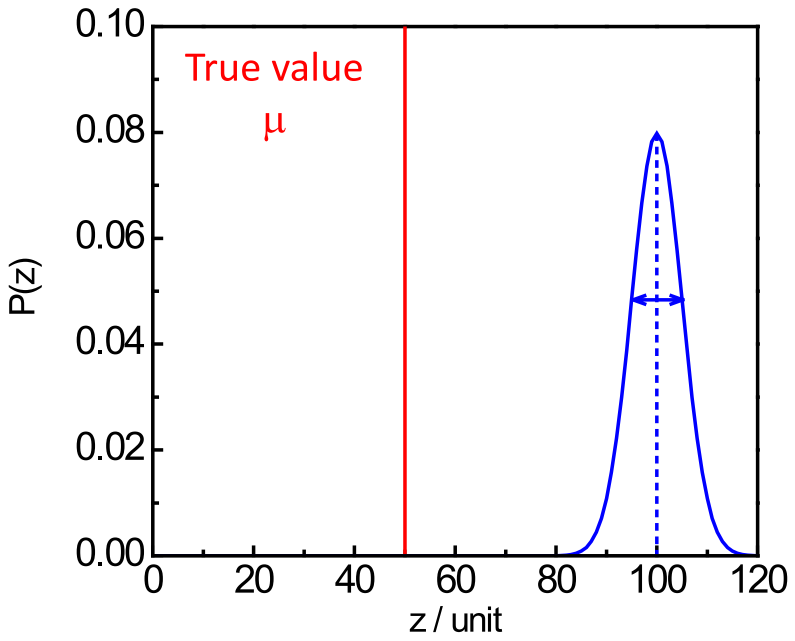
  $\beta_s = z - \mu$

- Random error : $\varepsilon_{r,i}$

  $\varepsilon_{r,i} = z_i - z$

$(y_1, \ldots, y_n, b) \Rightarrow$ estimate of Z = Y - B

Z	: measurand, i.e. quantity of interest, with true value μ
y_i	: result of a single experiment to estimate true value of Z
	all measurements in same conditions (e.g. measurement time,…)
b	: correction for background, z = y - b



$$y = \frac{1}{m}\sum_{j=1}^{m} y_j \qquad u_y = \frac{s_{y_j}}{\sqrt{m}}$$

$$s_{y_j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_j - y)^2$$

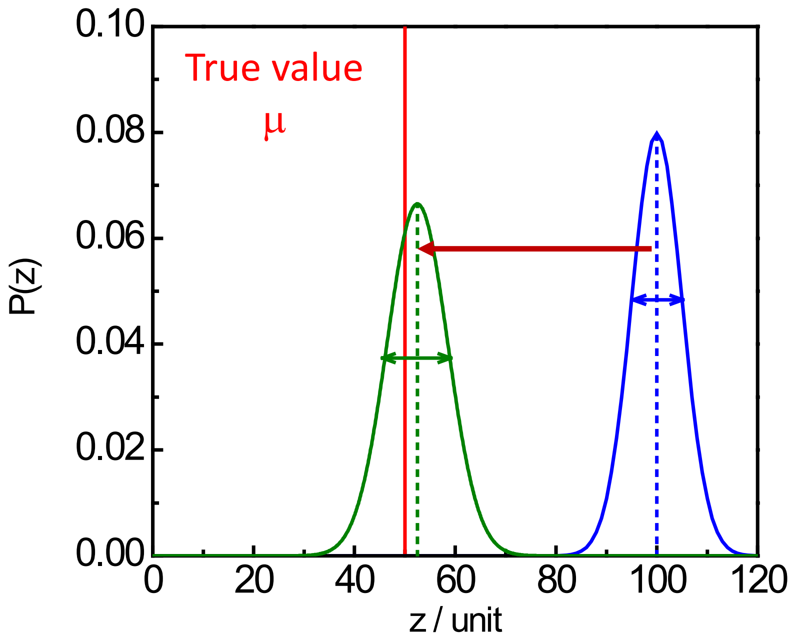
$(y_1, \ldots, y_n, b) \Rightarrow$ estimate of Z = Y - B

Z : measurand, i.e. quantity of interest, with true value μ

y_i : result of a single experiment to estimate true value of Z

all measurements in same conditions (e.g. measurement time,…)

b : correction for background, z = y - b



$$y = \frac{1}{m}\sum_{j=1}^{m} y_j \qquad u_y = \frac{s_{y_j}}{\sqrt{m}}$$

$$s_{y_j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_j - y)^2$$

$\Rightarrow$ Not correcting for the background results in a systematic error b

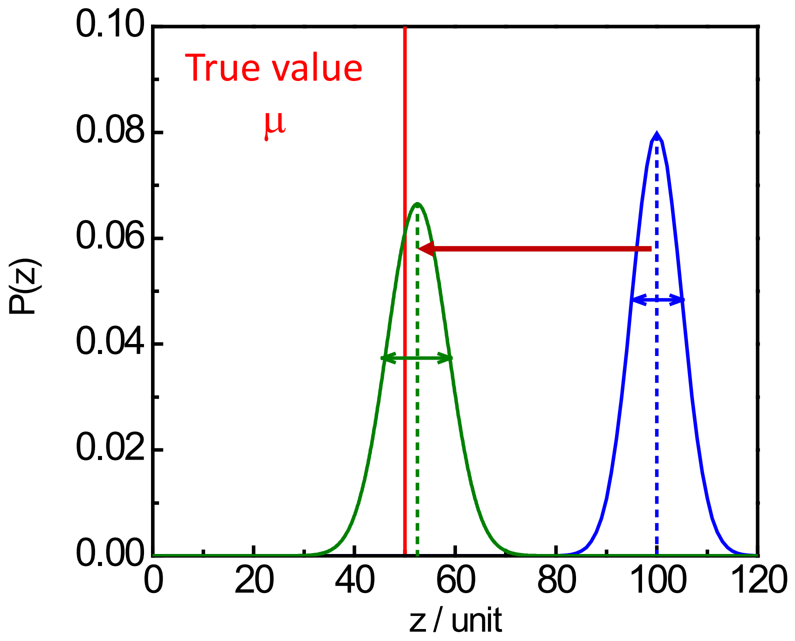
$(y_1, \dots, y_n, b) \Rightarrow$ estimate of Z = Y - B

Z : measurand, i.e. quantity of interest, with true value μ

y_i : result of a single experiment to estimate true value of Z

all measurements in same conditions (e.g. measurement time,…)

b : correction for background, z = y - b



$$y = \frac{1}{m}\sum_{j=1}^{m} y_j \qquad u_y = \frac{s_{y_j}}{\sqrt{m}}$$

$$s_{y_j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_j - y)^2$$

$$z = y - b \qquad u_z = \sqrt{\frac{s_{y_j}^2}{m} + u_b^2}$$

$\Rightarrow$ Not correcting for the background results in a systematic error b

$\Rightarrow$ Additional uncertainty u_b due to a systematic effect (background)

Measurement precision and accuracy

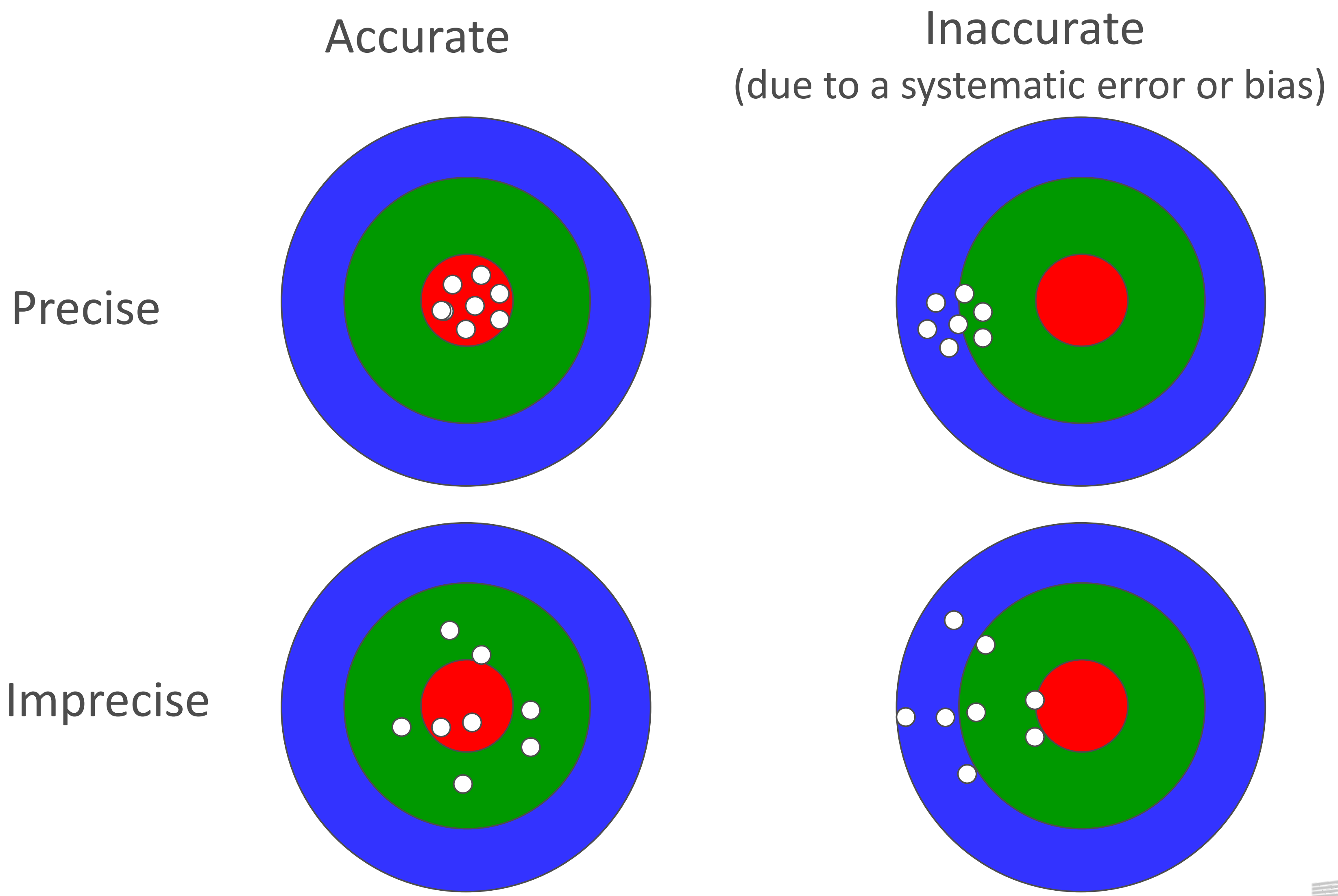
- **Measurement precision**

  *“**Closeness of agreement between** indications or **measured quantity values** obtained by replicate measurements on the same or similar objects under specified conditions”*

- **Measurement accuracy**

  *“**Closeness of agreement between** a **measured quantity value** and a **true quantity value** of a measurand”*

Precision ⇔ Accuracy



Probability distribution and statistical theory (2 dim.)

P(x,y) theoretical probability density function of (x,y)

– Mean

$$\mu_x = \int\int x\, P(x,y)\, dxdy \qquad \mu_y = \int\int y\, P(x,y)\, dxdy$$

– Variance

$$\sigma_x^2 = \int\int (x-\mu_x)^2\, P(x,y)\, dxdy \qquad \sigma_y^2 = \int\int (y-\mu_y)^2\, P(x,y)\, dxdy$$

– Covariance matrix

$$V_{x,y} = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y}^2 \\ \sigma_{x,y}^2 & \sigma_y^2 \end{bmatrix}$$

$$\sigma_{xy}^2 = \langle (x-\mu_x)(y-\mu_y) \rangle = \int (x-\mu_x)(y-\mu_y) P(x,y)\, dxdy$$

– Correlation matrix

$$\rho_{x,y} = \begin{bmatrix} 1 & \rho(x,y) \\ \rho(x,y) & 1 \end{bmatrix}$$

$$\rho(x,y) = \frac{\sigma_{xy}^2}{\sigma_x\, \sigma_y}$$

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b)$

- Experiment

  Independent observables (y_1,y_2) of Y : (y_1, u_{y_1}) and (y_1, u_{y_1})

  Background estimate : (b, u_b)

  Report estimates of Z = Y- B : (z_1, z_2) and V_{z_1, z_2}

$$V_{z_1,z_2} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix}$$

- Covariance matrix V_{z_1,z_2} of (z_1,z_2) consists of
  - **Uncorrelated** components (u_{y_1},u_{y_2}) : contribute only to diagonal elements
  - **Correlated** component u_b : contributes also to off-diagonal elements

$\Rightarrow$ Systematic effect (background) contributes to non-diagonal terms

Note: correlated uncertainty component u_b can be due to counting statistics!

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b, z_2 = y_2 - b)$

$z_1 = y_1 - b$

$z_2 = y_2 - b$

$$V_{z_1, z_2} = \begin{bmatrix} u_{y_1}^2 + u_b^2 & u_b^2 \\ u_b^2 & u_{y_2}^2 + u_b^2 \end{bmatrix}$$

$$\downarrow$$

$$\rho_{x,y} = \begin{bmatrix} 1 & \rho(x,y) \\ \rho(x,y) & 1 \end{bmatrix}$$

$$\rho(x,y) = \frac{u_{xy}^2}{u_x\, u_y}$$

$$\rho(z_1, z_2) = \frac{u_b^2}{\sqrt{(u_{y_1}^2 + u_b^2)(u_{y_2}^2 + u_b^2)}}$$

$b \searrow \quad \Rightarrow z_1 \nearrow$ and $z_2 \nearrow$

$b \nearrow \quad \Rightarrow z_1 \searrow$ and $z_2 \searrow$

$u_b = 0 \quad \Rightarrow \rho(z_1, z_2) = 0$

Note: correlation is due to the uncertainty on b

It is different from a correlation due to physics phenomena, e.g.

- linear correlation between position FE peak and γ-ray energy in a Ge-detector
- correlation between altitude and atmospheric pressure

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b,\ z_2 = y_2 - b)$

$(y_1, u_{y_1}) = (100, 1)$

$(y_2, u_{y_2}) = (98, 2)$

$(b, u_b) = (10.0, 0.1)$

$(z_1, z_2) = (90, 88)$

$$V_{z_1,z_2} = \begin{bmatrix} 1.01 & 0.01 \\ 0.01 & 4.01 \end{bmatrix}$$

$$\rho_{z_1,z_2} = \begin{bmatrix} 1 & 0.005 \\ 0.005 & 1 \end{bmatrix}$$

Note: correlation is due to the uncertainty on b

It is different from a correlation due to physics phenomena

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b,\ z_2 = y_2 - b)$

$(y_1, u_{y_1}) = (100, 1)$

$(y_2, u_{y_2}) = (98, 2)$

$(b, u_b) = (10.0, 1.0)$

$(z_1, z_2) = (90, 88)$

$$V_{z_1,z_2} = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$$

$$\rho_{z_1,z_2} = \begin{bmatrix} 1 & 0.32 \\ 0.32 & 1 \end{bmatrix}$$

Note: correlation is due to the uncertainty on b

It is different from a correlation due to physics phenomena

Example: $(y_1, y_2, b) \Rightarrow (z_1 = y_1 - b,\ z_2 = y_2 - b)$

$(y_1, u_{y_1}) = (100, 1)$

$(y_2, u_{y_2}) = (98, 2)$

$(b, u_b) = (10.0, 5.0)$

$(z_1, z_2) = (90, 88)$

$$V_{z_1, z_2} = \begin{bmatrix} 26 & 25 \\ 25 & 29 \end{bmatrix}$$

$$\rho_{z_1, z_2} = \begin{bmatrix} 1 & 0.91 \\ 0.91 & 1 \end{bmatrix}$$

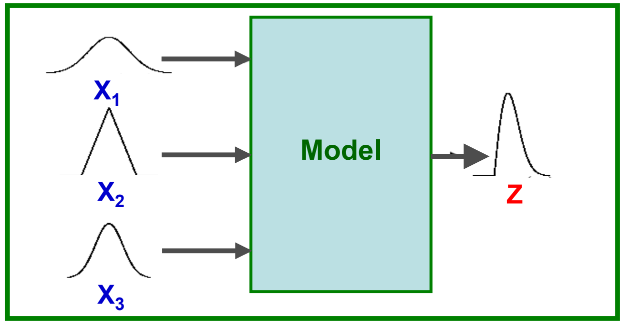
Note: correlation is due to the uncertainty on b

It is different from a correlation due to physics phenomena

Final result of a measurement

Input quantities → Output quantity (measurand)

(X_1, X_2, X_3, …) Model Z



- Input quantities (x_1, …, x_n) with V_x
- Model $z = f(x_1, \ldots, x_n)$

→ $(z, \mathbf{V_z})$

$$V_z = G\,V_x\,G^T$$

$$g_k = \frac{\partial f}{\partial x_k}$$

How to estimate input covariance V_X?

Determination of input covariance matrix V_x

$$V_z = G\, V_x\, G^T$$

- Define **measurement model:** $z = f(x_1,\ldots,x_n)$
  - **Identify** basic **metrological parameters** of the measurement **process**
  - Define **a model f** that starts from **independent input quantities**

- Example: $Z = K\,(Y-B)$, e.g. $A_\alpha = K\,(C - B)$ $\quad A_\alpha = \frac{1}{\varepsilon_\alpha\, \Omega\, P_\alpha}(C - B)$

1) define independent input quantities

- Experimental observable (sample measurement) (y, u_y)
- Background (independent background measurement) (b, u_b)
- Normalisation (independent measurement with reference sample) (k, u_k)

$$V_{(y,b,k)} = \begin{bmatrix} u_y^2 & 0 & 0 \\ 0 & u_b^2 & 0 \\ 0 & 0 & u_k^2 \end{bmatrix}$$

2) estimate uncertainties of independent input quantities

(see additional slides to estimate uncertainty due to systematic effects for α-activity measurements)

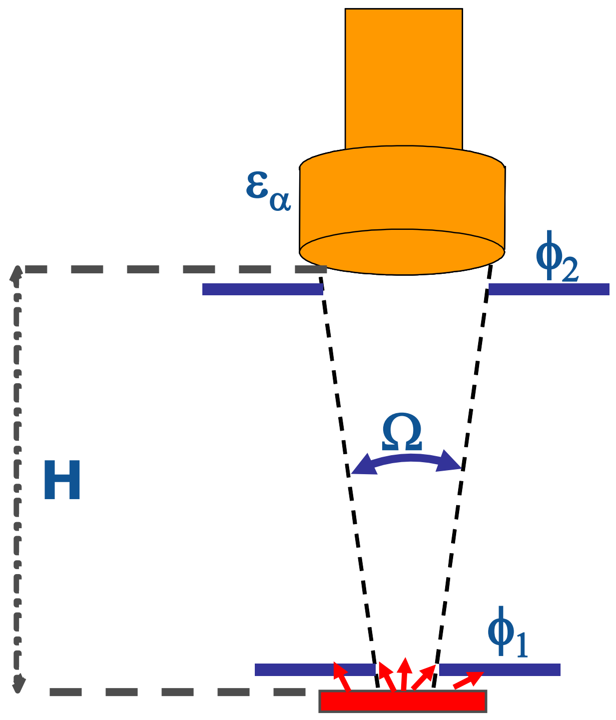
Example: α-activity experiment

Determination of activity A_α based on α-counting

Measurand : A_α (alfa- activity of the sample)

Measurement model : $c_\alpha = \varepsilon_\alpha \Omega P_\alpha A_\alpha$

$A_\alpha = c_\alpha / (\varepsilon_\alpha \Omega P_\alpha)$



- Results of counting experiment: $c_\alpha = c - b$
  - sample count rate : c
  - background count rate : b
- Other input quantities
  - P_α : escape probability
  - Ω : solid angle depends on (H, ϕ_1, ϕ_2)
  - ε_α : detection efficiency

Example: α-activity experiment

- Evaluate the impact of influencing components
  - Target – detector geometry (distance target-detector, target position, collimation)
  - High Voltage
  - Long term stability of detector/electronics chain
- Perform a set of replicate measurements under the “same” conditions

  Experiment 1

  (1) position sample, produce vacuum, high voltage on, determine distance

  (2) replicate measurements

  (3) high voltage off, brake vacuum, take sample out

  Experiment 2

  Repeat (1), (2), (3)

  …

  Experiment k

  Repeat (1), (2), (3)
- Statistical analysis: ANalysis Of VAriance (ANOVA)

Results: α-activity measurements

number of groups : $k = 7$
number of measurements in each group : $n = n_j = 24$
total number of measurements : $N = 168$

	j = 1	2	3	4	5	6	7
i = 1	29691	29413	29570	29770	29717	29873	29695
2	29728	29474	29885	29871	30059	29926	29671
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
24	29775	29802	29723	29797	29685	29386	30007
x_j =	29693.21	29769.17	29719.79	29784.00	29730.33	29666.13	29087.46
$S_{r,j}$ =	24674.08	28609.10	32532.70	32102.61	23401.71	39917.33	22539.56

$$x_j = \frac{1}{n_j}\sum_{i=1}^{n_j} x_{ij}$$

$$S_{r,j}^2 = \sum_{i=1}^{n_j} (x_{ij} - x_j)^2$$

ANOVA table

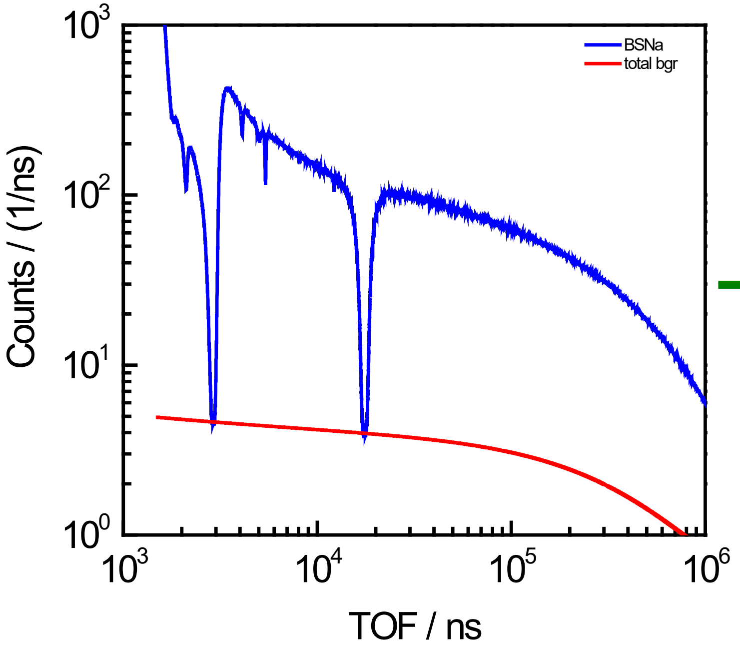
Sum of squares	χ^2	ν	χ^2/ν Expectation value	
$S_g^2 = \sum_{j=1}^{k} n_j (x_j - x)^2$	$= 371330$	6	$u_\varepsilon^2 + n\, u_\beta^2$	= 61888.3
$S_r^2 = \sum_{j=1}^{k} \sum_{i=1}^{n} (x_{ij} - x_j)^2$	$= 4686873$	167	u_ε^2	= 29111.0

$$x = \frac{1}{k}\sum_{j=1}^{k} x_j = \frac{1}{k}\sum_{j=1}^{k}\frac{1}{n_j}\sum_{i=1}^{n_j} x_{ij}$$

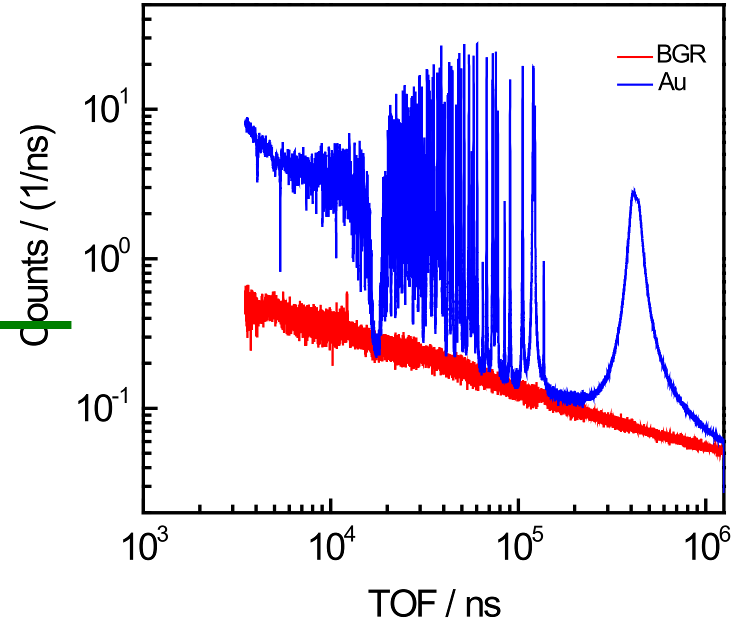
x = 29738.58

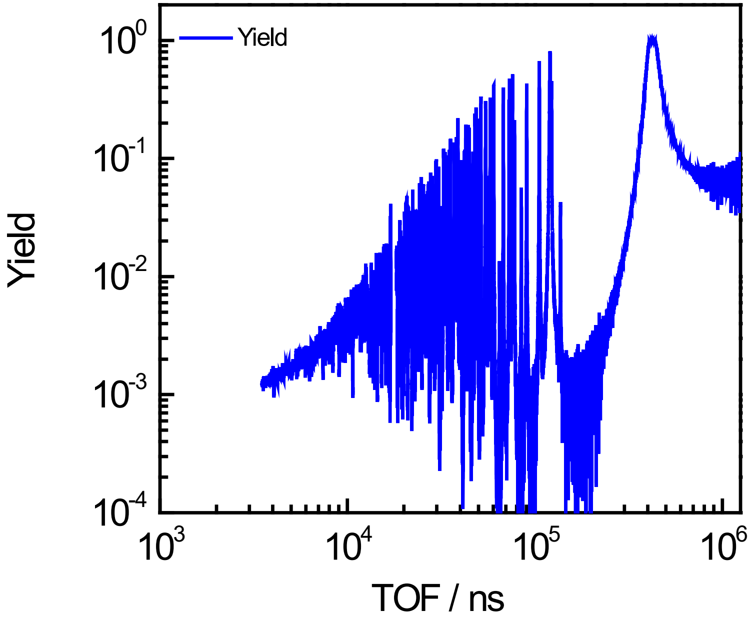
$\Rightarrow u_\beta = 37$ (0.12 %)

Example: Y_{exp} for $^{197}Au(n,\gamma)$

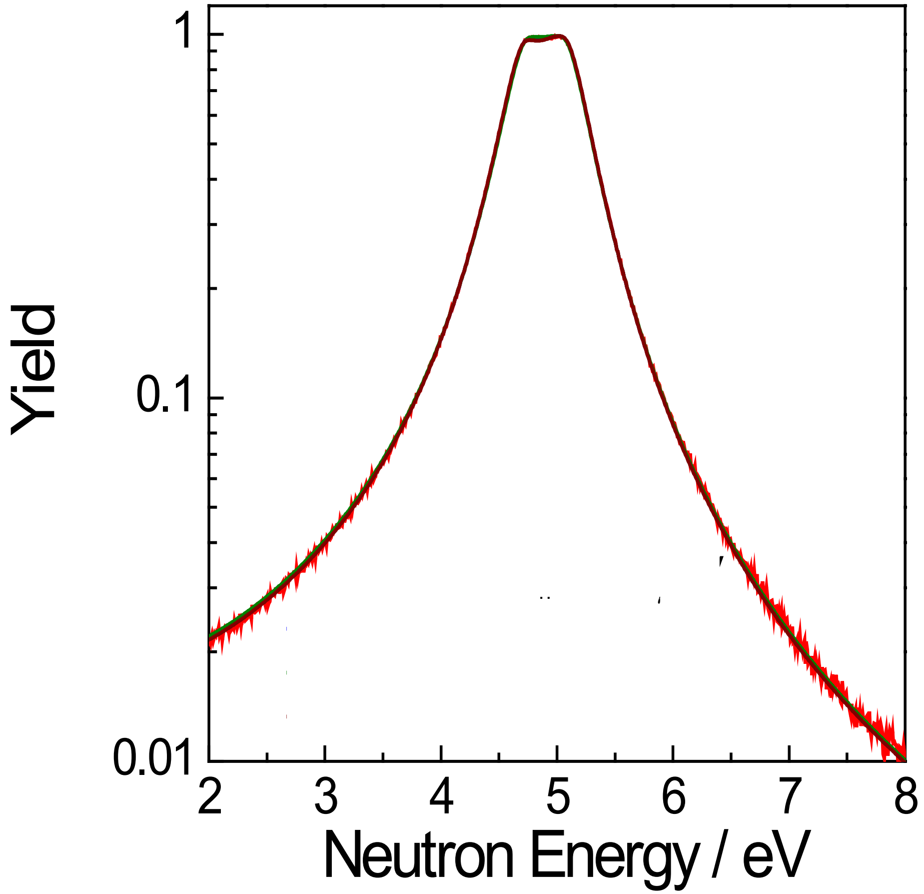


$$Y_{exp} = \mathbf{N}\,\frac{C_w - B_w}{C_\varphi - B_\varphi}\,Y_\varphi$$





Normalisation at saturated resonance profile



$$Y_\gamma \cong \frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}\left(1 - e^{-n\bar{\sigma}_{tot}}\right) + \ldots$$

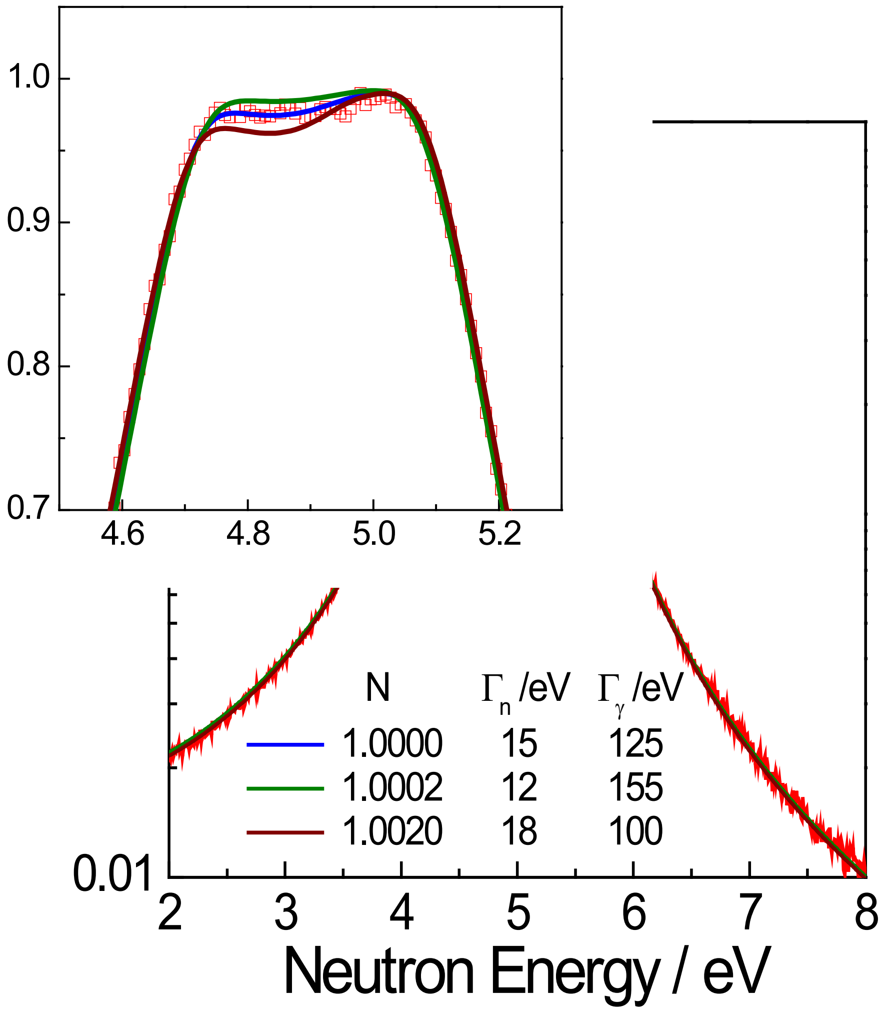
$n\sigma_{tot} >> 1$ and $\sigma_\gamma \approx \sigma_{tot}$

$$Y_\gamma \cong 1$$

$$Y_{exp} = N \frac{C_w - B_w}{C_\varphi - B_\varphi} Y_\varphi$$

$$\Rightarrow N \cong \frac{C_\varphi - B_\varphi}{C_w - B_w} \frac{1}{Y_\varphi}$$

Normalisation at saturated resonance profile



$$Y_\gamma \cong \frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}\left(1 - e^{-n\bar{\sigma}_{tot}}\right) + \ldots$$

$n\sigma_{tot} >> 1$ and $\sigma_\gamma \approx \sigma_{tot}$

$$Y_\gamma \cong 1$$

$$Y_{exp} = N \frac{C_w - B_w}{C_\varphi - B_\varphi} Y_\varphi$$

$$\Rightarrow N \cong \frac{C_\varphi - B_\varphi}{C_w - B_w} \frac{1}{Y_\varphi}$$

N is independent of :

- sample thickness
- nuclear data

Requires special procedures (weighting function)

Normalisation uncertainty: experimental evaluation

- Perform dedicated experiments to uncertainty due sample properties
  (systematic effect: β)
  - experiments using samples with different characteristics
  - experiments with similar contribution of random error component (counting statistics) : ε
- Statistical analysis of the data: $u_N^2 = u_\varepsilon^2 + u_\beta^2 \quad \Rightarrow u_\beta \leq 0.003$

Sample	Diameter	Thickness	N	
Au	80 mm	0.05 mm	1.002	(2)
Au	80 mm	0.11 mm	1.001	(2)
Au	80 mm	1.02 mm	0.997	(2)
		Mean	1.000	
		Std	0.003	

Sample	Diameter	Thickness	N	
Ag	60 mm	0.08 mm	1.005	(4)
Ag	60 mm	0.18 mm	0.993	(4)
^{nat}PbAg	60 mm	1.07 mm	0.999	(4)
^{206}PbAg	60 mm	1.15 mm	1.003	(4)
		Mean	1.000	
		Std.	0.005	

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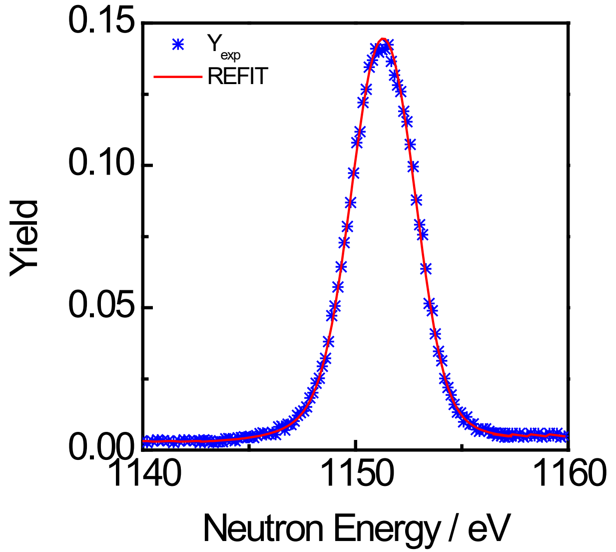
Resonance parameters from capture experiments

- Perform dedicated experiments applying different conditions: $^{56}Fe(n,\gamma)$
  (systematic effect: β)
  - experiments using normalisation samples with different characteristics (sample, γ-ray emission)
  - capture measurements with different iron samples to determine Γ_n for the 1.15 keV resoance
  - experiments with similar contribution of random error component (counting statistics) : ε

- Analysis of the data: determine normalisation N (previous slides) and Γ_n

Sample	N	g/cm^2 Fe	g/cm^2 X	∅ mm		Γ_n / meV
Fe1	Ag	0.105		60		62.6 (7)
Fe2	Ag	0.394		60		62.5 (7)
Fe3	Ag	0.905		60		60.2 (7)
^{206}PbFe*	Ag	0.394	1.213	60		63.1 (7)
PbFe*	Ag	0.422	1.103	60		62.6 (7)
PbFe*	Ag	0.422	2.725	60		62.6 (7)
Fe4	Au	0.202		80		61.2 (7)
Fe5	Au	0.795		80		60.3 (7)
Fe6	Au	0.998		80		61.2 (7)
AuFe	Au	1.708	0.118	80		61.3 (7)
Fe_2O_3	Au	1.404	0.603	80		59.1 (7)
					Mean	61.5
					Std	1.3
					Std (%)	2.1



Resonance parameters from capture experiments

- Perform dedicated experiments applying different conditions
  (systematic effect: β)
  - experiments using normalisation samples with different characteristics (sample, γ-ray emission)
  - capture measurements with iron samples to determine Γ_n for the 1.15 keV resoance
  - experiments with similar contribution of random error component (counting statistics) : ε

- Statistical analysis of results

Reference value (transmission): Γ_n = 61.7 (9) meV

Sample	N	g/cm² Fe	g/cm² X	∅ mm	Γ_n / meV
Fe1	Ag	0.105		60	62.6 (7)
Fe2	Ag	0.394		60	62.5 (7)
Fe3	Ag	0.905		60	60.2 (7)
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				Mean	61.5
				Std	1.3
				Std (%)	2.1

- Difference with reference value is smaller than uncertainty of reference value
- Uncertainty due to systematic effects (normalisation, sample characteristics)

$$u_{\Gamma_n}^2 = u_\varepsilon^2 + u_\beta^2$$

u_ε = 0.7 meV

$$\Rightarrow u_\beta = 1.1 \text{ meV} \Rightarrow \frac{u_\beta}{\Gamma_n} \approx 0.018$$

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Evaluation of uncertainty: Type A and Type B

Purpose of **Type A** and **Type B classification** is to indicate two **different ways of evaluating uncertainty components** and is for convenience only; the classification is **not meant to indicate** that there is any **difference in the nature of the components** resulting from the two types of evaluation.

- Type A

  "**Statistical analysis** of **measured quantity values** obtained under defined conditions"

  "A Type A standard uncertainty is obtained from a probability density function derived from an **observed frequency distribution**"

- Type B

  "Evaluation of a component of measurement uncertainty by means **other than Type A evaluation**"

  "A type B standard uncertainty is obtained from an **assumed probability density function** based on the degree of belief that an event will occur "
  The uncertainty is evaluated by **scientific judgement** based on available information, e.g.:

  - previous measurement data
  - data provided in calibration and other certificates
  - uncertainties assigned to reference data taken from handbooks
  - …

Evaluation and propagation of measurement uncertainty

- Define **measurement model:** $z = f(x_1,\ldots,x_n)$
  - **Identify** basic **metrological parameters** of the measurement **process**
  - Define **a model f** that starts from **independent input quantities**

$V_z = G\, V_x\, G^T$

- Example: $Z = K\,(Y-B)$, e.g. $A_\alpha = K\,(C - B)$ $\quad A_\alpha = \frac{1}{\varepsilon_\alpha\,\Omega\,P_\alpha}(C - B)$

1) <u>define independent input quantities</u>

- Experimental observable (sample measurement) (y, u_y)
- Background (independent background measurement) (b, u_b)
- Normalisation (independent measurement with reference sample) (k, u_k)

$$V_{(y,b,k)} = \begin{bmatrix} u_y^2 & 0 & 0 \\ 0 & u_b^2 & 0 \\ 0 & 0 & u_k^2 \end{bmatrix}$$

2) <u>estimate uncertainties of independent input quantities</u>

Summary

- Statistical analysis:
  - is more than just analysing the result of repeated measurements
  - is a powerful tool to reliably estimate and propagate measurement uncertainties
- All uncertainties are "statistical"
  - uncertainties reflect the width of a statistical distribution
    (uncertainties can be due to counting statistics)
  - uncertainties should be evaluated (estimated) by a statistical analysis
  - such an evaluation requires dedicated experiments
- We differentiate between
  - Random and systematic error and their uncertainties

    $\Rightarrow$ Uncertainties due to systematic and random effects
  - Uncorrelated $\leftrightarrow$ correlated uncertainty components

    Note: - systematic effects introduce a correlated uncertainty component
    - uncertainties due to counting statistics can contribute to a correlated uncertainty component

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Thank you

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