
Testing the cosmological model with the large-scale structure

Benjamin L'Huillier (Sejong University)

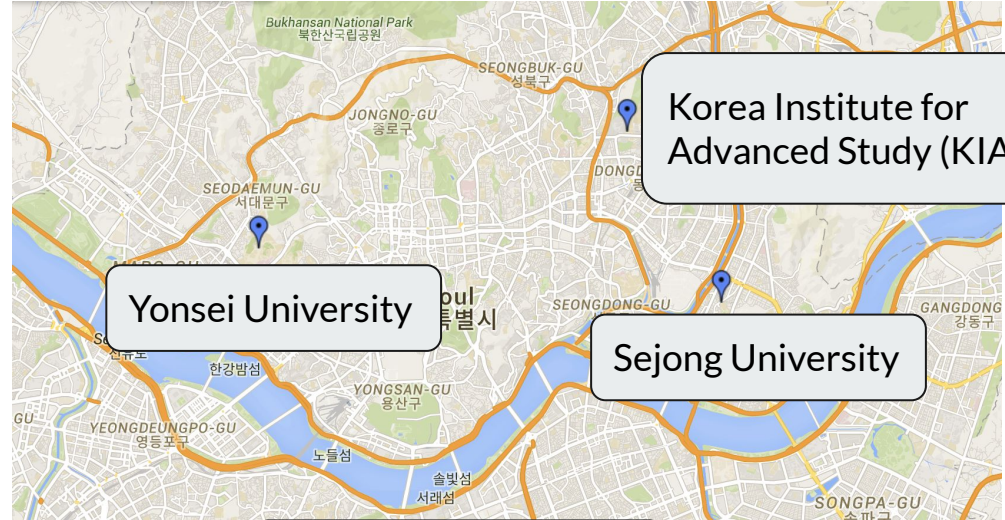
루일리에, 벤자민

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2022-01-13



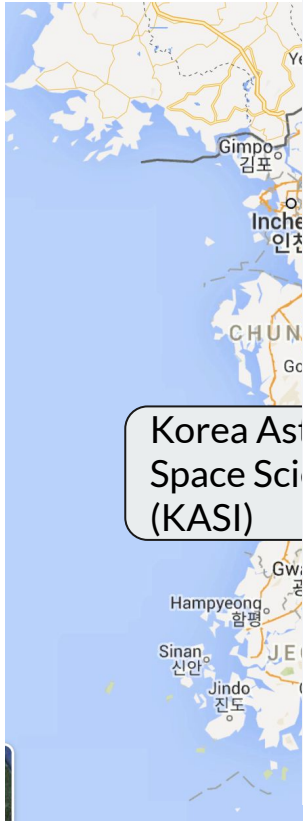
Astronomy & Cosmology in Korea (a totally biased picture!)



Seoul Nat. University

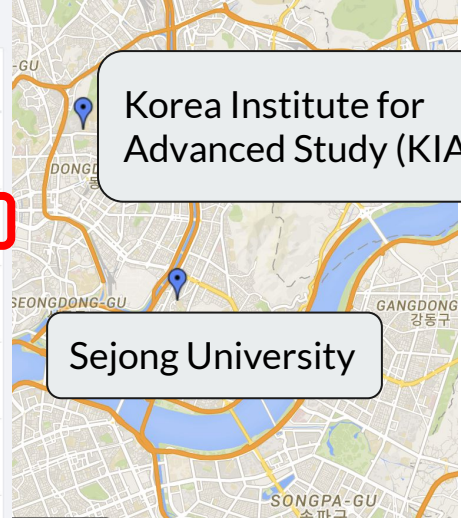
KASI & KIAS cosmology groups:
Members of DESI

Astronomy & Cosmology in Korea (a totally biased picture!)



All Countries and Economies

Country	Most Recent Year	Most Recent Value
Israel	2018	4.95
Korea, Rep.	2018	4.81
Switzerland	2017	3.37
Sweden	2018	3.34
Japan	2018	3.26
Austria	2018	3.17
Germany	2018	3.09
Denmark	2018	3.06
United States	2018	2.84



Source: World Bank (2018)

Astronomy & Cosmology in Korea (a totally biased picture!)



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The Concordance Model

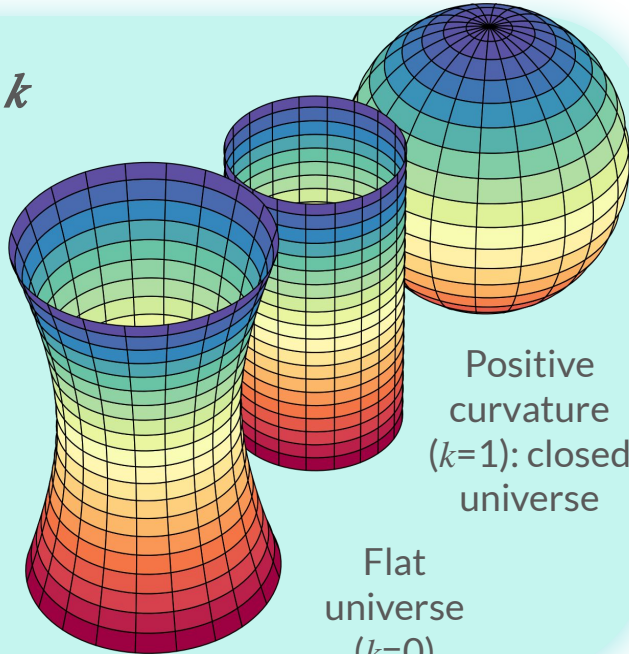
The concordance model of cosmology: the FLRW metric

- **Isotropy & Homogeneity** → FLRW metric

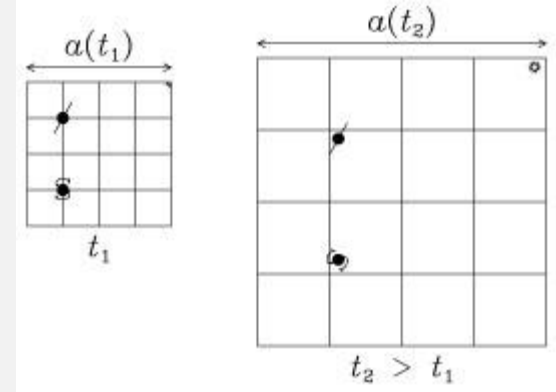
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

Curvature k

Negative curvature
($k=-1$): open universe



Scale Factor a



Cosmology = study of $a(t)$

The concordance model of cosmology: Gravity

- **Gravity: General Relativity**

- **Einstein Equations:** $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$

Geometry

Cosmological
constant

Energy

- **Solve for the FLRW metric: obtain the Friedmann Equations**

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

Matter: density ρ , pressure p

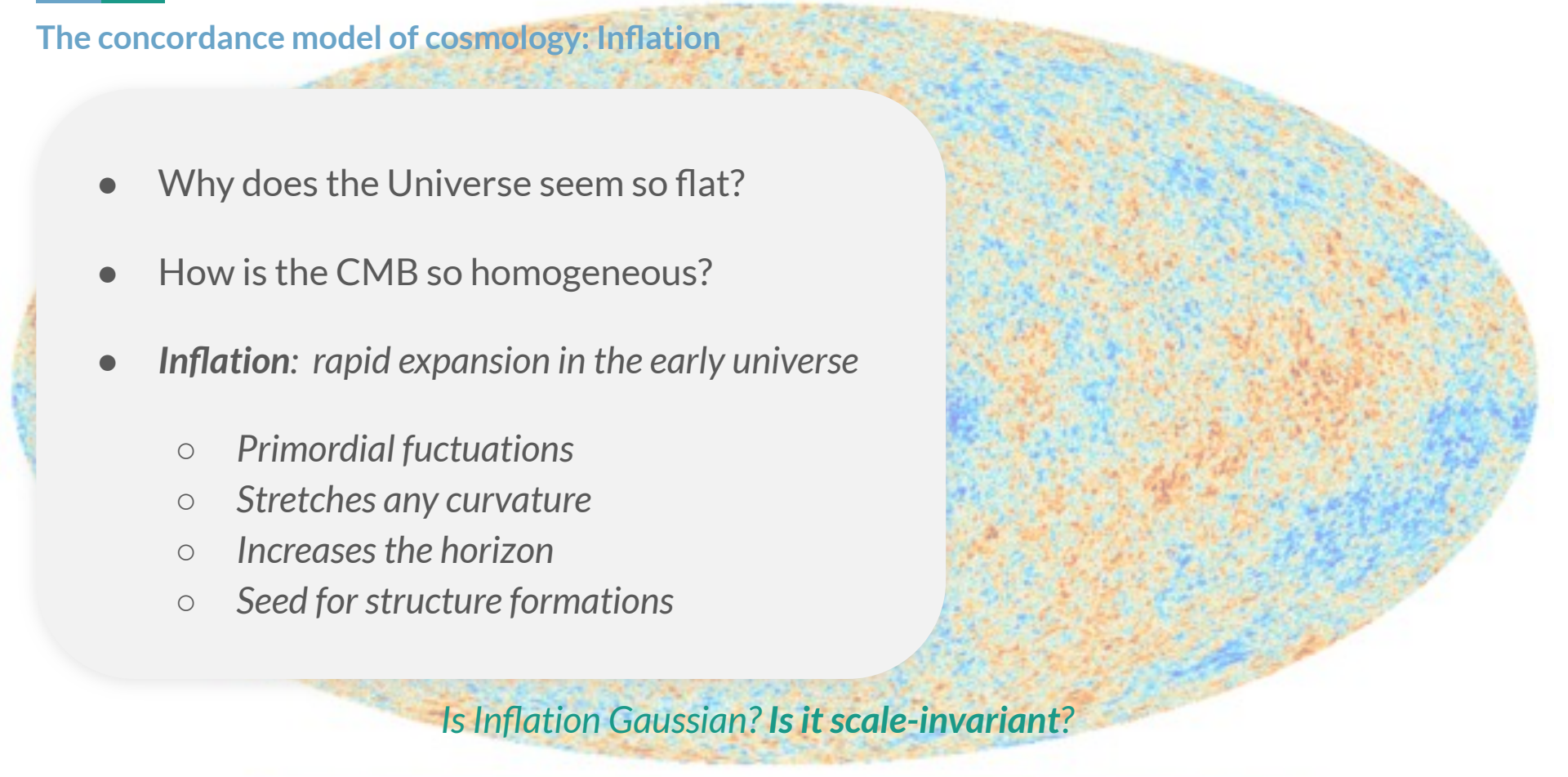
Curvature k

Cosmological constant Λ

The concordance model of cosmology: Inflation

- Why does the Universe seem so flat?
- How is the CMB so homogeneous?
- **Inflation:** *rapid expansion in the early universe*
 - *Primordial fuctuations*
 - *Stretches any curvature*
 - *Increases the horizon*
 - *Seed for structure formations*

Is Inflation Gaussian? Is it scale-invariant?



The concordance model of cosmology

Isotropy & Homogeneity
FLRW metric

A theory of gravity
General relativity

Initial conditions
Inflation

- *Flat Universe dominated by dark energy & dark matter*
- *Concordance: CMB, BAO, SNIa*
- *But... tensions*

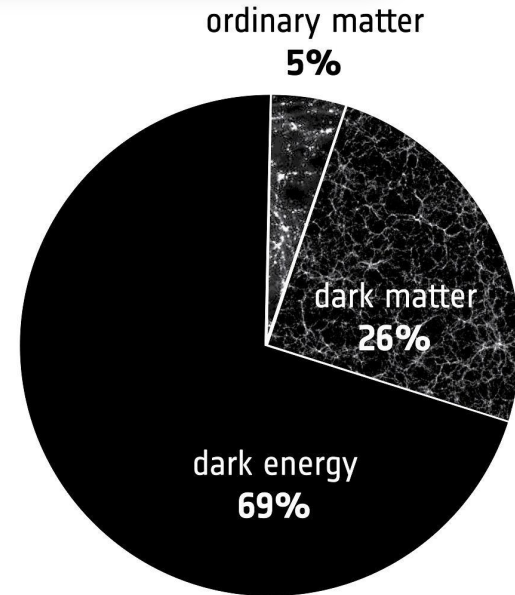


Image Credit:
Science/AAAS

»» The large scale structure hold valuable information

Advanced Statistical Methods

MODEL-DEPENDENT METHODS

- + Widely used; more straightforward
(model-fitting), “easier”
- + More constraining power
- **Bias towards the (assumed) model**
MCMC, Nested sampling

vs.

MODEL-INDEPENDENT METHODS

- Less straightforward; overfitting problem
- Less constraining power
- + More flexibility
- + **No bias towards any model**
- + **Can detect unexpected features in the data**
Gaussian process, iterative smoothing,
Crossing statistics, ...

Both approaches *can* and *should* be used together



The Linear Regime

The concordance model of cosmology

Isotropy & Homogeneity
FLRW metric

A theory of gravity
General relativity

Initial conditions
Inflation

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- **Concordance: CMB, BAO, SNIa**
- **But... tensions**

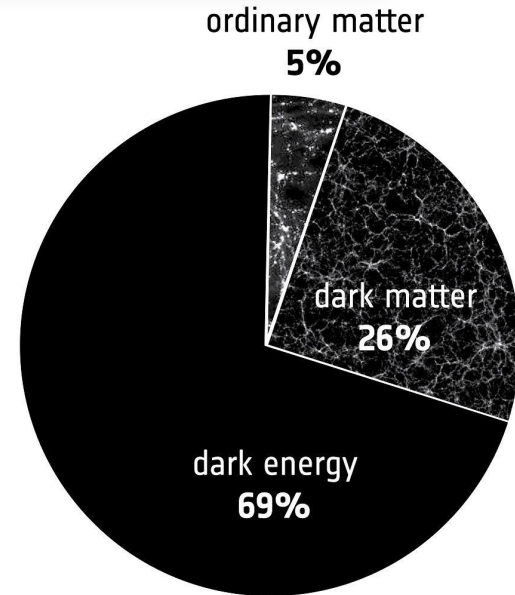
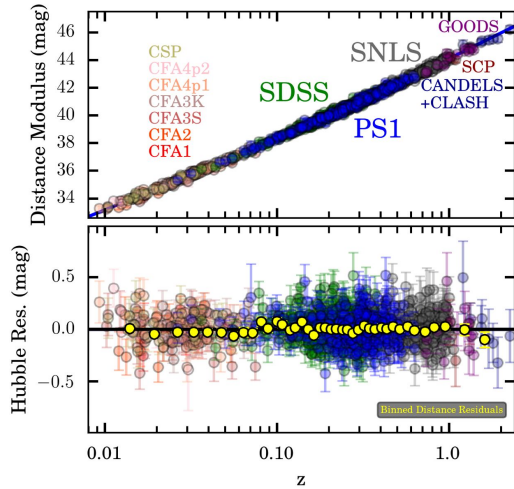


Image Credit:
Science/AAAS

➤➤ **The large scale structure hold valuable information**

Background Expansion: Observables

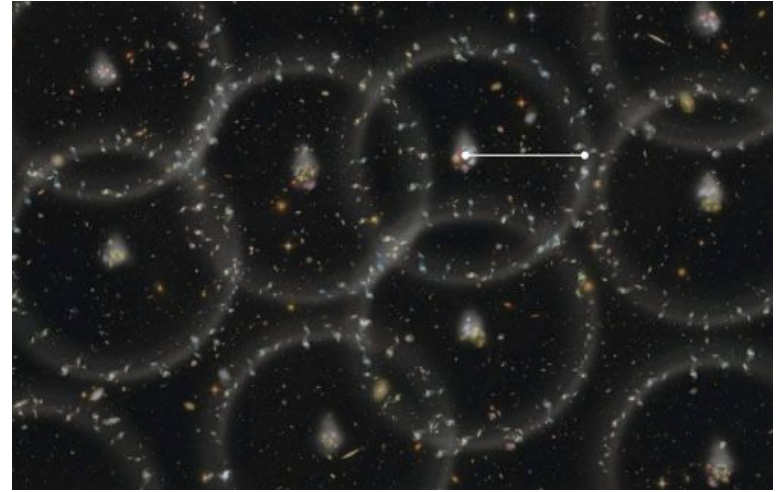
Type Ia Supernovae (SNIa): *standard candles*



Known luminosity \rightarrow Luminosity Distance

$$\mu(z) = 5 \log_{10} d_L(z) + cst$$

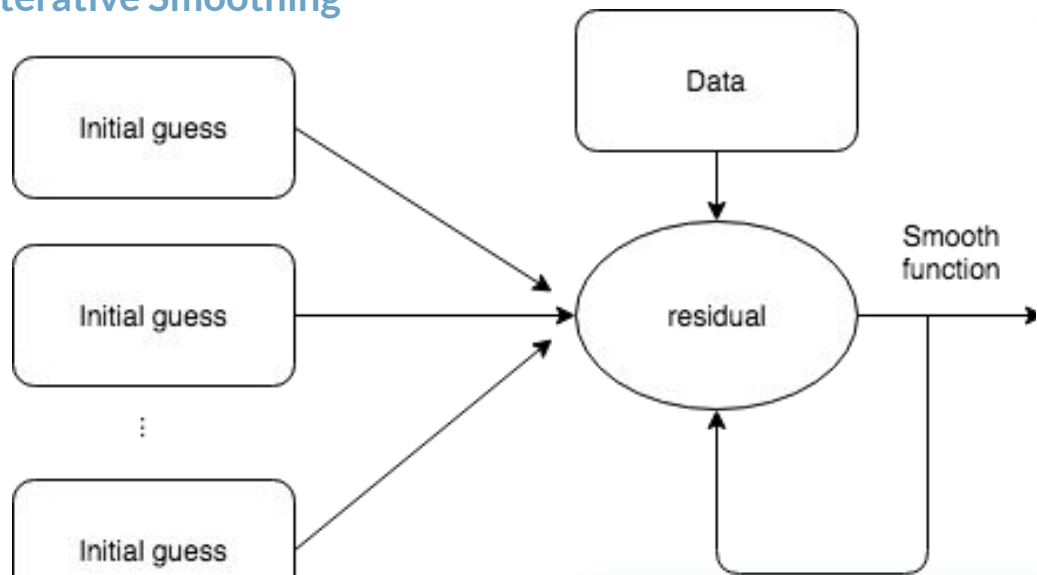
Baryon acoustic oscillations (BAO): *standard ruler*



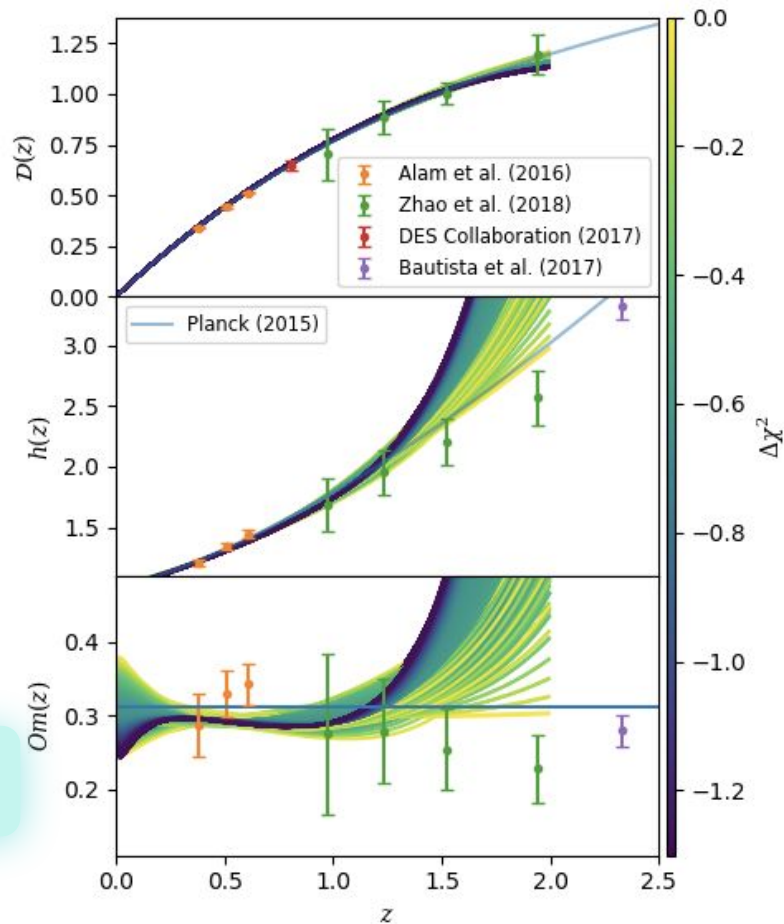
Known size \rightarrow Angular Diameter Distance, Hubble Parameter
 $d_A(z)/r_d, H(z) r_d$

$$r_d = \frac{c}{H_0 \sqrt{3}} \int_0^{1/(1+z_d)} \frac{da}{a^2 h(a) \sqrt{1 + \frac{3\Omega_b}{4\Omega_r} a}}$$

Direct reconstruction of the Expansion History: Iterative Smoothing



Independent of the initial
guess!



Shafieloo et al. (2006), Shafieloo (2007)
BL & Shafieloo (2017)
Shafieloo, BL, Starobinsky (2018)

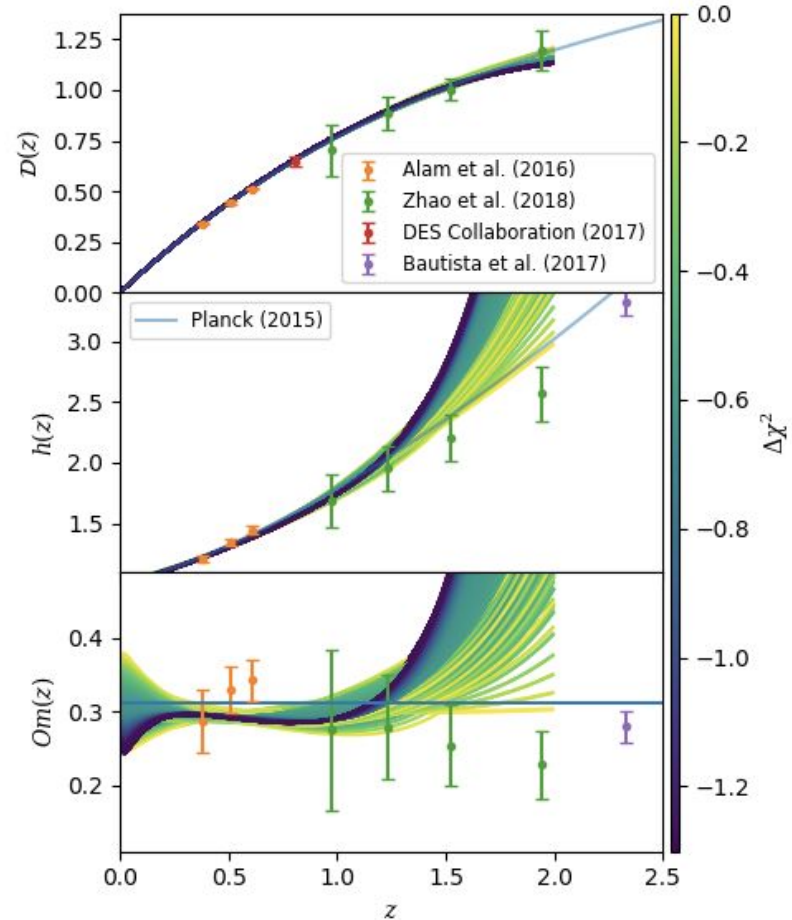
Testing Flat- Λ CDM

Consistent with Flat- Λ CDM at $z < 1.5$

Hints of tension at $z > 1.5$ (cf. Sahni et al. 2014, Zhao et al. 2017)

Om diagnostics (Sahni et al. 2008):

$$Om(z) = \frac{h^2(z)-1}{(1+z)^3-1} \stackrel{\text{flat-}\Lambda\text{CDM}}{\equiv} \Omega_{m,0}$$

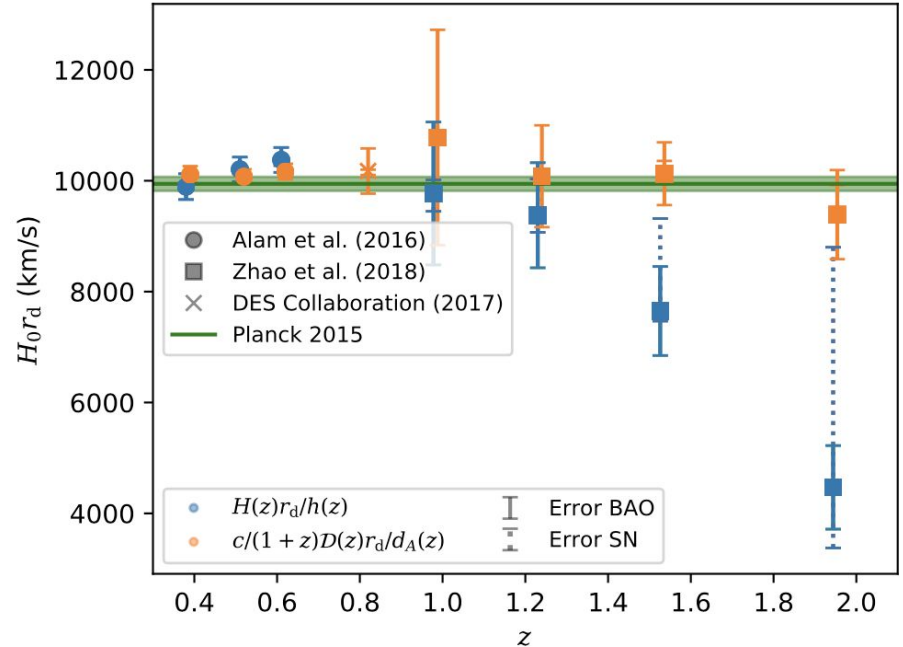


Model-independent Measurement of $H_0 r_d$

SNIa: Pantheon $\rightarrow H_0 d_L(z)/c$

BAO: (e)BOSS $\rightarrow H(z)r_d, d_A(z)/r_d$

$$\begin{aligned} H_0 r_d &= \frac{H(z)r_d}{h(z)} \\ &= \frac{c}{1+z} \frac{D(z)r_d}{d_A(z)} \end{aligned}$$

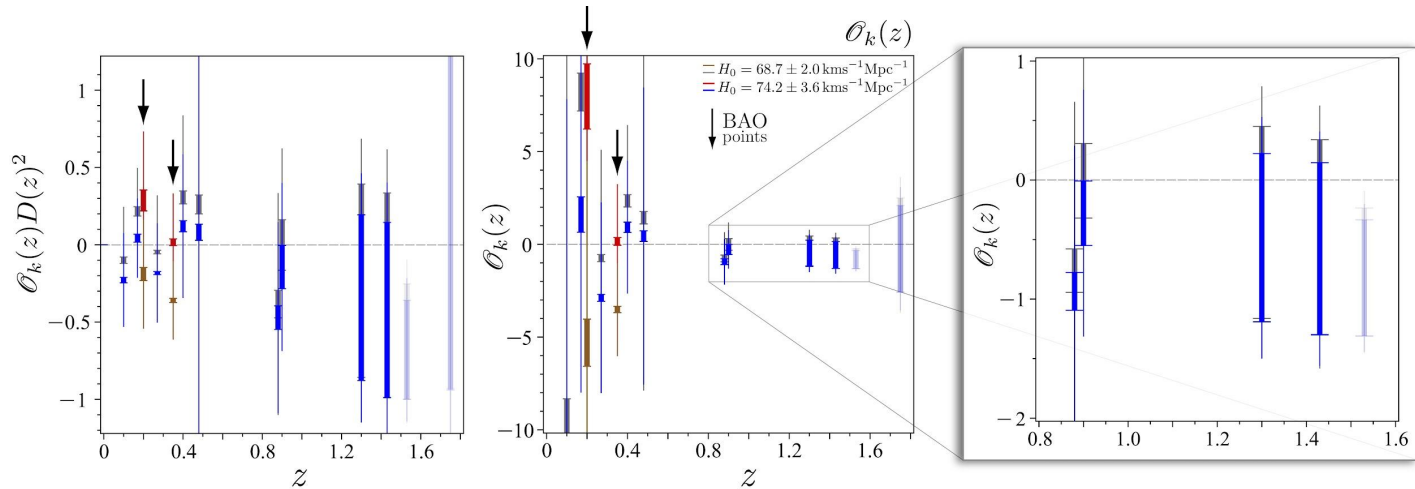


Consistent with Planck 2015: $H_0 r_d = (9944.0 \pm 127.4) \text{ km s}^{-1}$

Model-independent test of FLRW metric & curvature

Clarkson et al. (2008):

$$\mathcal{D}(z) = \frac{1}{\sqrt{-\Omega_k}} \sin \left(\sqrt{-\Omega_k} \int_0^z \frac{dx}{h(x)} \right)$$
$$\mathcal{O}_k(z) = \frac{(h(z)\mathcal{D}'(z)^2 - 1)}{\mathcal{D}^2(z)} = \frac{\left(\frac{H(z)\mathcal{D}'(z)}{H_0} \right)^2 - 1}{\mathcal{D}^2(z)} \stackrel{\text{FLRW}}{\equiv} \Omega_k$$



Shafieloo & Clarkson (2010)

Model-independent test of FLRW metric & curvature

New formulation:

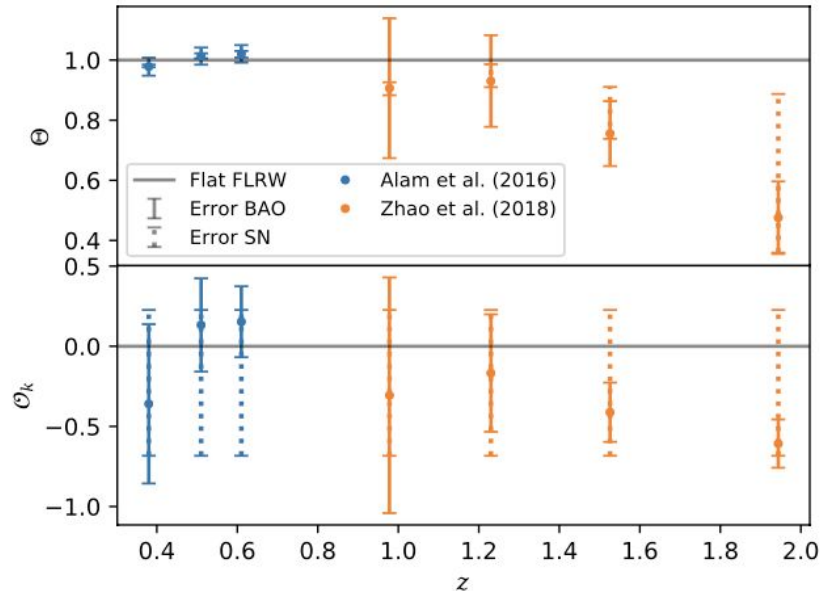
$$\Theta(z) = h(z) \mathcal{D}'(z) = \frac{(1+z)}{c} H(z) r_d \frac{d_A(z)}{r_d} \frac{\mathcal{D}'(z)}{\mathcal{D}(z)} = F_{AP}(z) \frac{\mathcal{D}'(z)}{\mathcal{D}(z)} \stackrel{\text{flat-FLRW}}{\equiv} 1$$

$$\mathcal{O}_k(z) = \frac{\Theta^2(z) - 1}{\mathcal{D}^2(z)} \stackrel{\text{FLRW}}{\equiv} \Omega_k$$

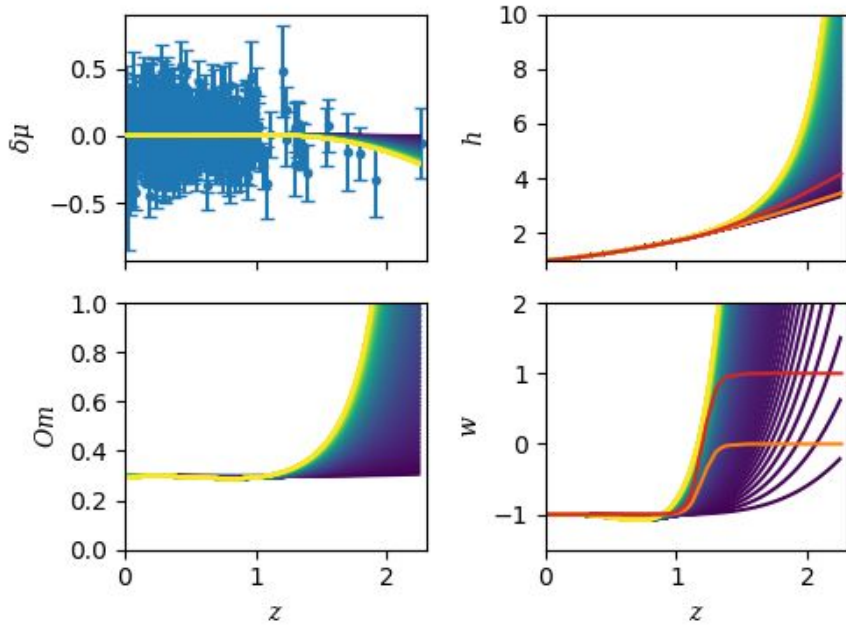
Consistent with flat FLRW

Consistency BAO \leftrightarrow SNIa

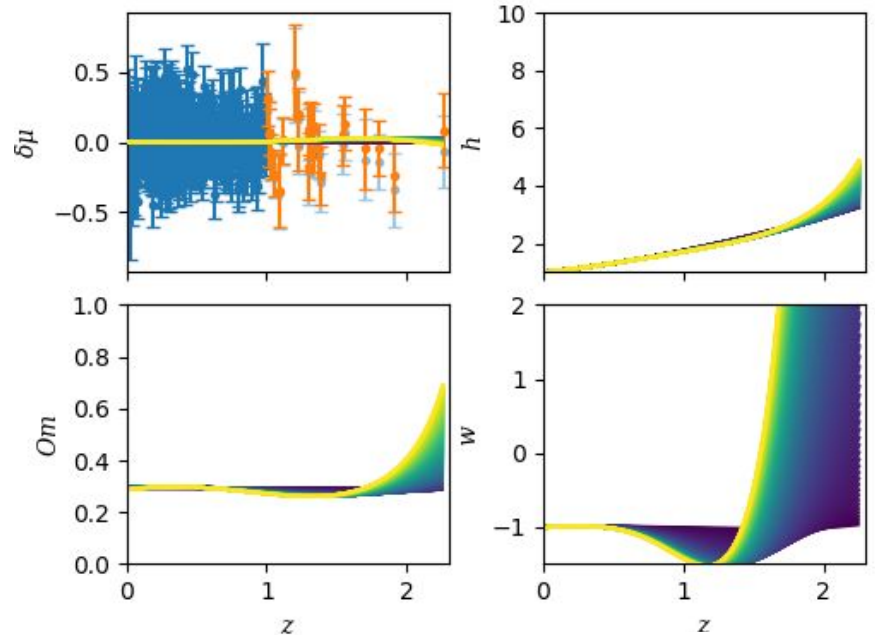
Hints of deviations at high- z ?



High-z SNIa: cosmology or systematics?



Possible deviation from LCDM at high- z
Difficult to reproduce by dark energy models



Malmquist-like correction: $\mu \rightarrow \mu + A(z - 1)$
 Om better behaved
Not statistically significant

The concordance model of cosmology

Isotropy & Homogeneity
FLRW metric

A theory of gravity
General relativity

Initial conditions
Inflation

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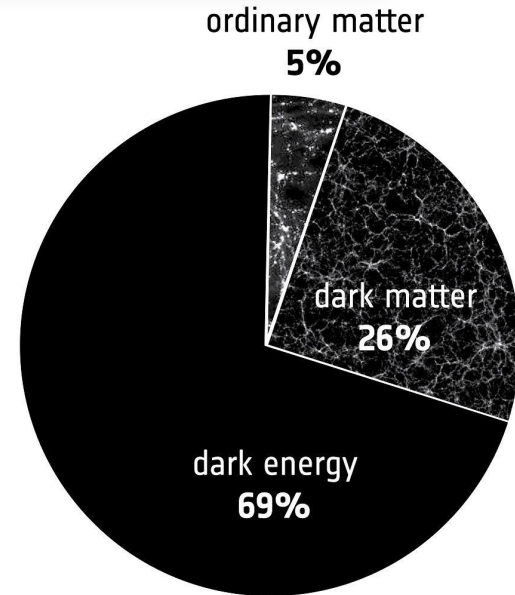


Image Credit:
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The large scale structure hold valuable information

Redshift-space Distortion

Linear growth of perturbations:

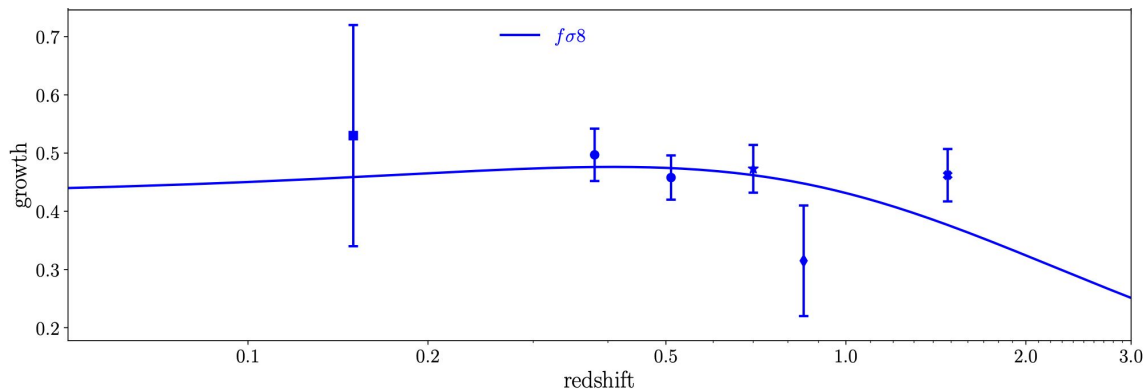
$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2}\Omega_m H^2 \frac{G_{\text{eff}}}{G_N} \delta \longrightarrow D(z)$$

Expansion (background) Modified Gravity

Growth rate:

$$f = \frac{d \ln D}{d \ln a} \simeq \Omega_m^\gamma$$

σ_8 : rms of the fluctuations δ



eBOSS final results (2007.08991)

Reconstructing Expansion from Growth

$$f\sigma_8 = \sigma_{8,0} \Omega_m^\gamma \exp\left(-\int_a^1 \Omega_m^\gamma d \ln a\right)$$

BL et al. (2018).
Shafieloo, BL & Starobinsky (2018)



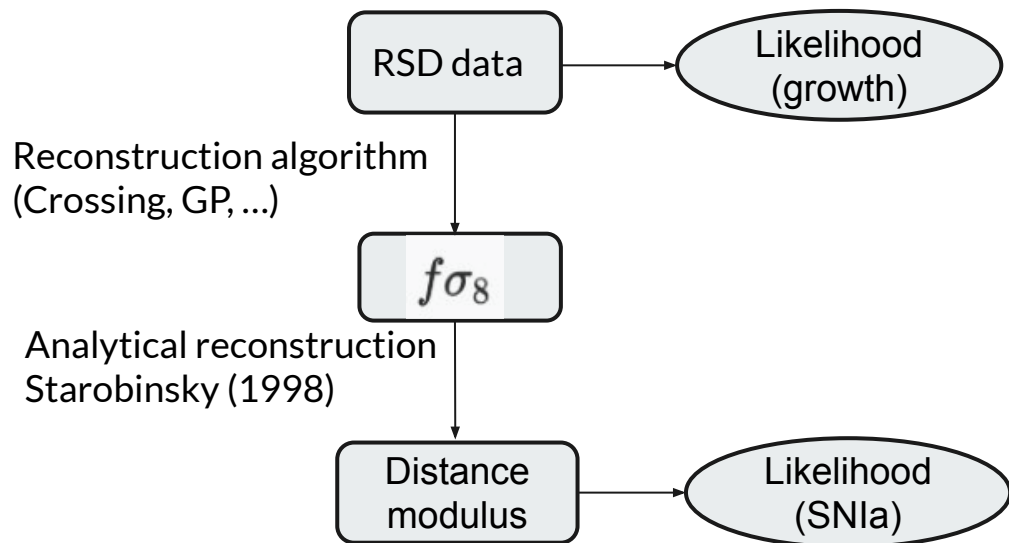
Starobinsky (1998) exact formula (in GR):

$$h^2(z) = \left(\frac{1+z}{\delta'(z)}\right)^2 \left(\delta_0'^2 - 3\Omega_{m0} \int_0^z \delta(u) |\delta'(u)| \frac{du}{1+u}\right)$$

“Only” dependent on
 $(\Omega_{m0}, \sigma_{8,0}, \delta(z))$

*** Independent of Dark Energy!**

Reconstructing Expansion from Growth



BL, Shafieloo, Polarski, Starobinsky (2020)

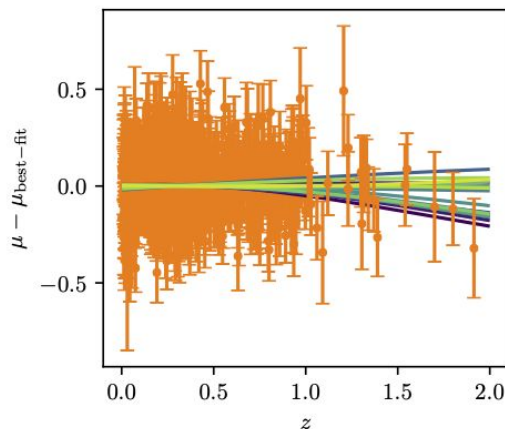
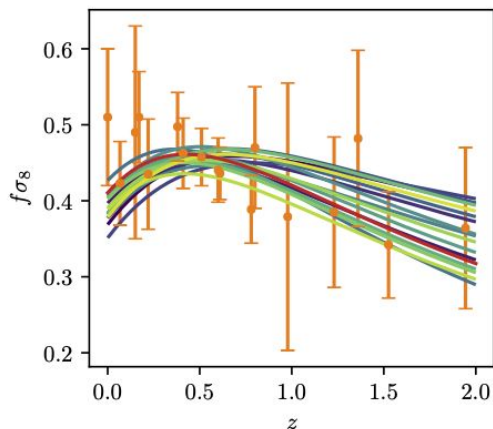
Reconstructing Expansion from Growth: Crossing statistics

Crossing statistics (Shafieloo 2011,2012):
“How consistent is the considered model with the data?”

→ Mean function (“what we want to test”):
best-fit LCDM

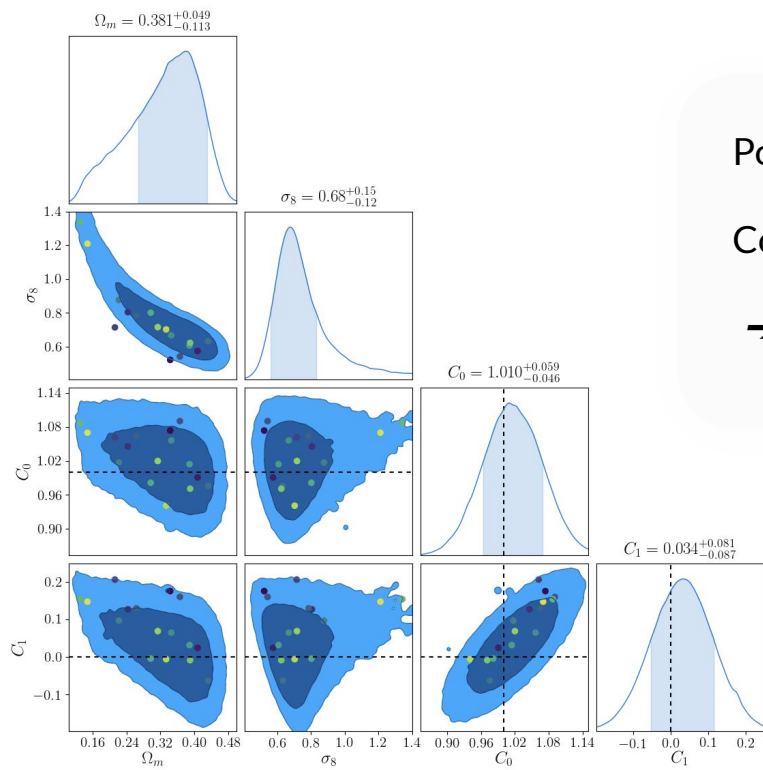
Multiply by hyperfunction (Chebyshev polynomials with coeffs. C_i)

What is the posterior distribution of C_i ?
Deviation from the mean function?



Deformations of the mean function
(best-fit, red)

Reconstructing Expansion from Growth: Crossing statistics



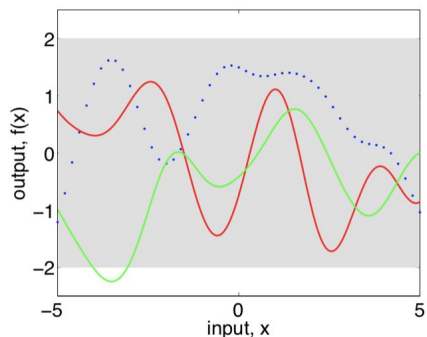
Posterior distribution of C_i :

Consistent with $C_0 = 1, C_i = 0$:

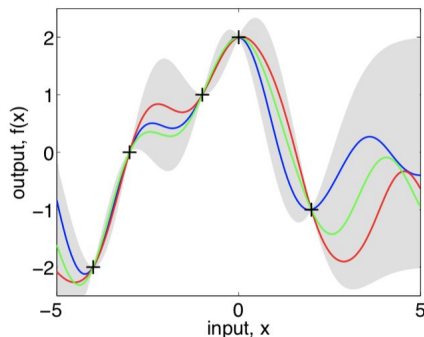
→ *The data does not require deviation from Λ CDM+GR*

BL, Shafieloo, Polarski, Starobinsky (2020)

Reconstructing Expansion from Growth: Gaussian Process



(a), prior

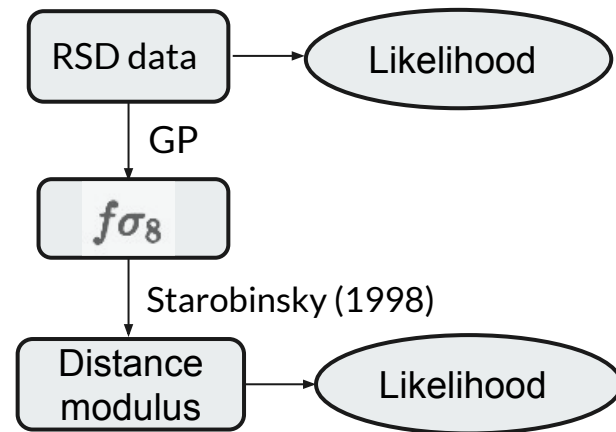


(b), posterior

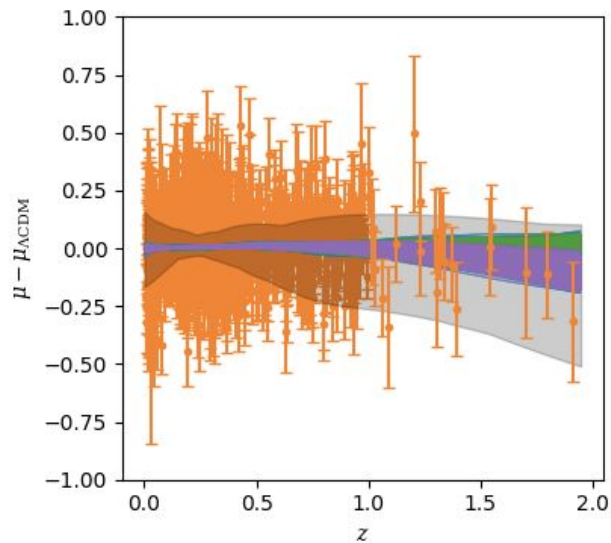
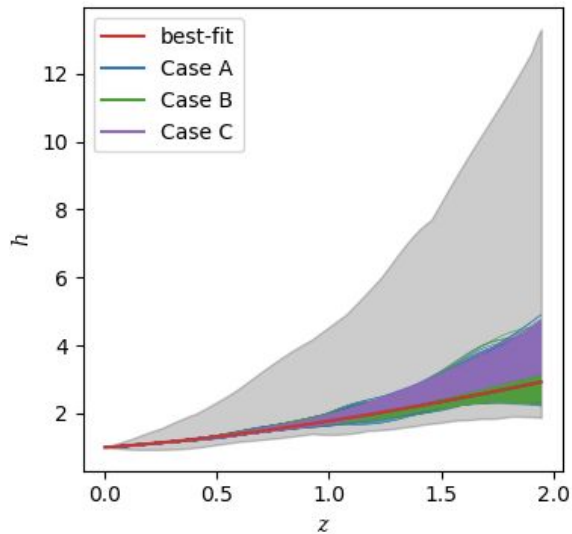
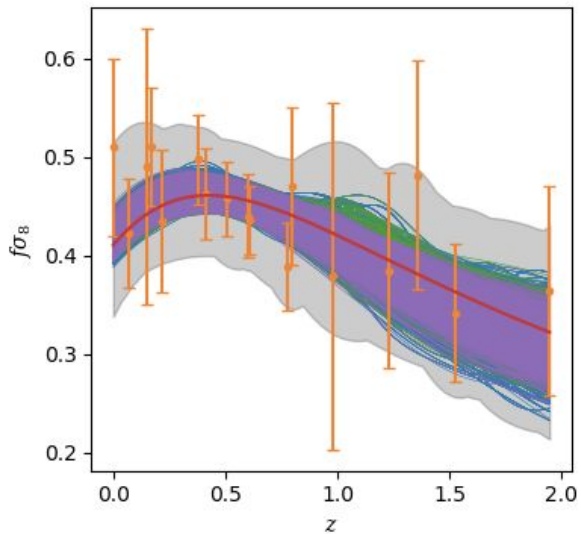
Observations y_i reconstruct $f(z)$?

Ansatz: $(y_i, f(z_i))$ are jointly Gaussian, with an input-covariance

The hyper-parameters of the GP are trained on the data



Reconstructing Expansion from Growth: Gaussian Process



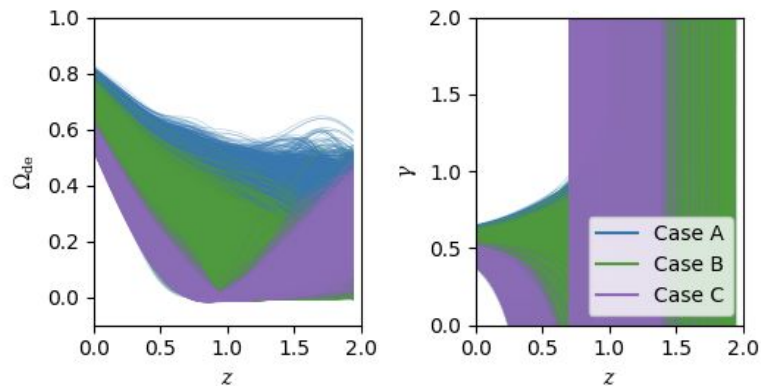
Consistent with GR+ΛCDM

DE energy density is not guaranteed to be positive!

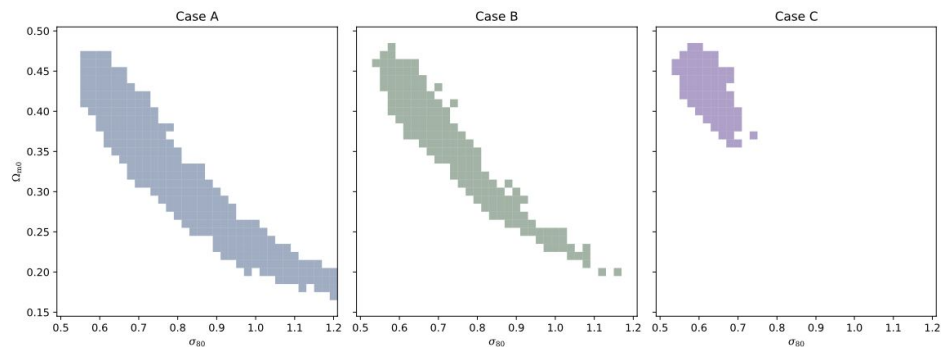
$$\Omega_{\text{DE}}(z) = 1 - \frac{\Omega_{\text{m}0}(1+z)^3}{h^2(z)}$$

- **Case A:** $\Omega_{\text{DE}}(z < 2) > 0$
- **Case B:** $\Omega_{\text{DE}}(z < 1) > 0$
- **Case C:** $\Omega_{\text{DE}}(z < 0.7) > 0$

Negative DE Energy Density?



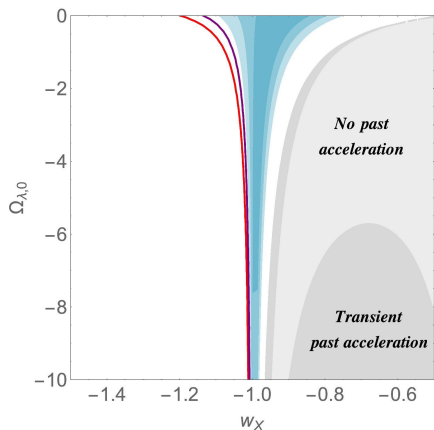
- Case A: $\Omega_{DE}(z < 2) > 0$
- Case B: $\Omega_{DE}(z < 1) > 0$
- Case C: $\Omega_{DE}(z < 0.7) > 0$



High $\Omega_{m,0}$ and low $\sigma_{8,0}$ allowed by the data

BL, Shafieloo, Polarski, Starobinsky (2020)

Negative DE Energy Density?



Interesting transient acceleration in the past

Toy model: Negative cosmological constant

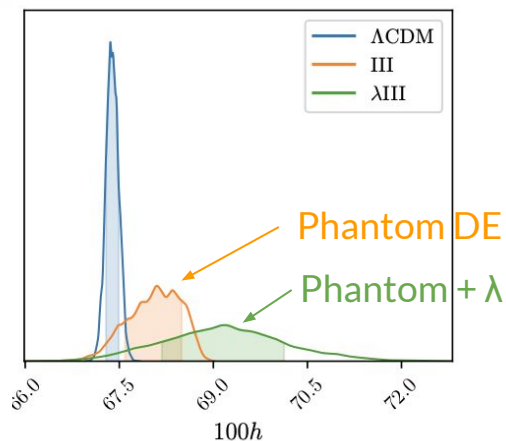
Two DE components X and λ :

Constant $\lambda < 0$

X: Phantom behaviour: $w_X(0.1 < z < 1) < -1$

Not excluded by the data
(similar Bayesian evidence as
LCDM)

Can mitigate the H_0 tension



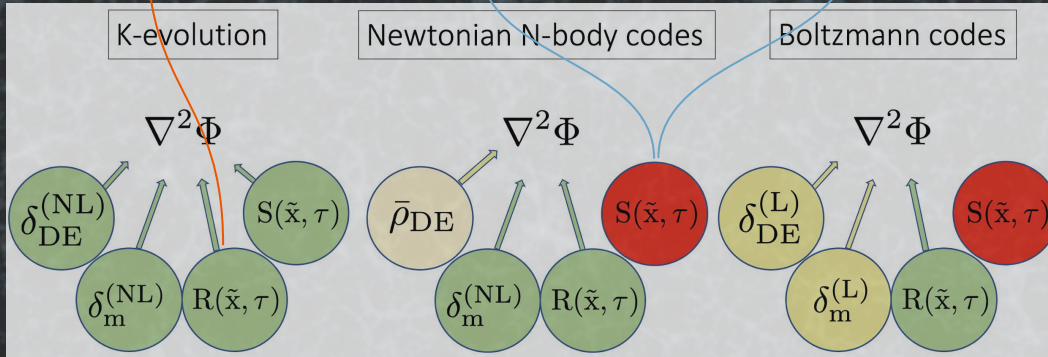
Calderón, Gannouji, BL, Polarski (2021)



The non-linear Regime: N-body Simulations

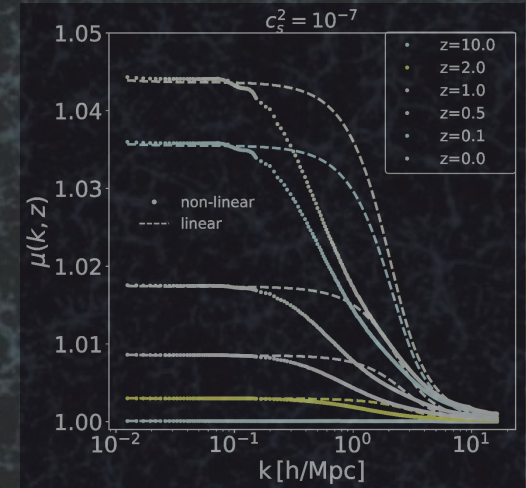
Parametrizing Nonlinear DE Perturbations

$$\nabla^2\Phi = 3\mathcal{H}\Phi' + 3\mathcal{H}^2\Psi + \frac{1}{2}\delta^{ij}\Phi_{,i}\Phi_{,j} + 4\pi G_N a^2(1 - 2\Phi)\sum_X \bar{\rho}_X \delta_X,$$



- Relativistic N-body code k-evolution (Hassani et al)
 - Effects of **non-linearity**
 - **Non- Λ Dark Energy** (k-essence)

Hassani, BL+(2020)



Deviation from GR ($\mu \equiv 1$)

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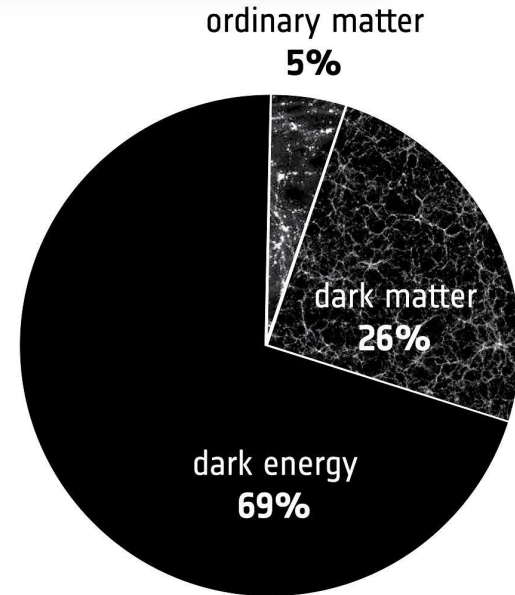


Image Credit:
Science/AAAS

➤➤ The large scale structure hold valuable information

Planck 2018 constraints: power law

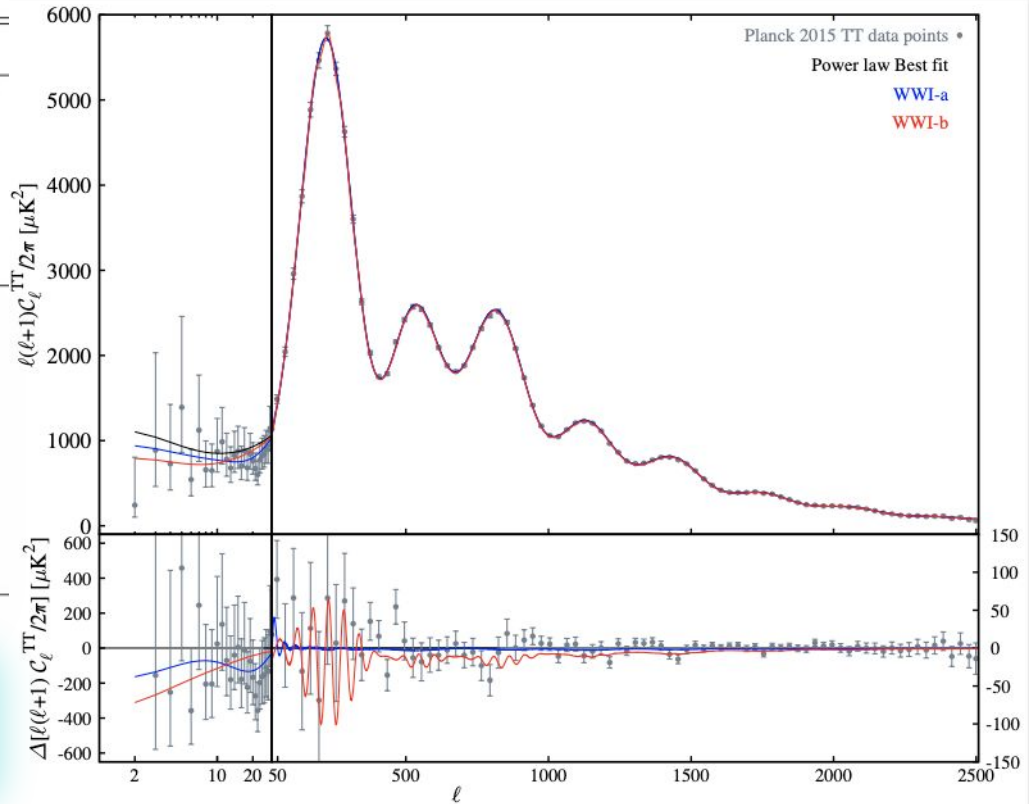
Parameter	Plik best fit	Plik [1]
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031
τ	0.0543	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014
n_s	0.96605	0.9649 ± 0.0042

Power-law fits very well

But features still allowed at $\ell < 500$
Example: wiggly whipped inflation

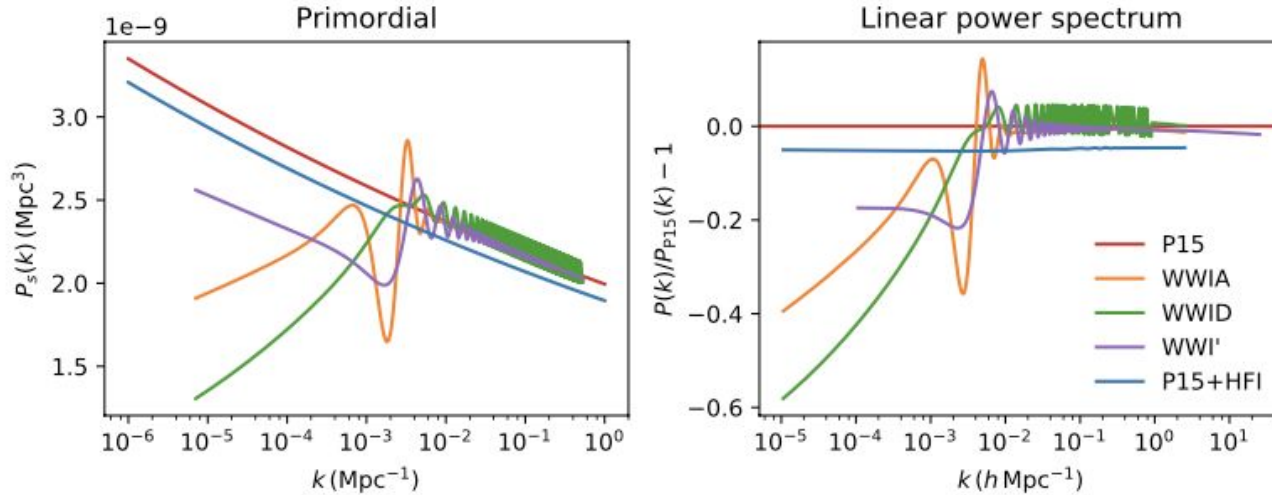
CMB: 2D info only

Can the LSS break the degeneracy?



Primordial Power Spectrum

BL, Shafieloo, Hazra, Smoot, Starobinsky (2018)



N-body simulations

15 realizations x 5 models

$L = 1.89 \text{ Gpc}/h$

$N = 1024^3$

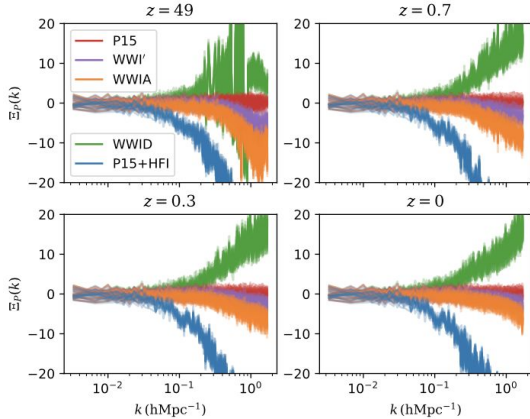
Reference model: Planck 2015 TTTEEE (P15)

Wiggly-whipped inflation (Hazra et al. 2014ab, 2016): WWIA, WWID, WWI'

Planck 2015 TTTEEE+HFI (P15+HFI)

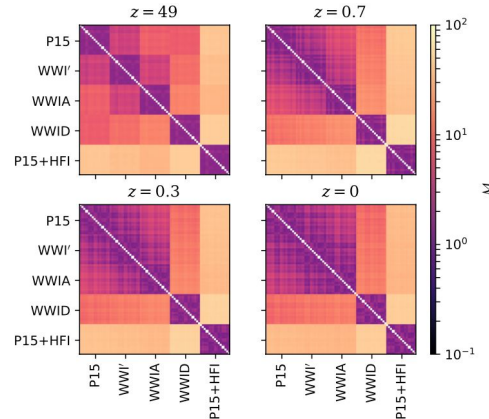
Matter Power Spectrum

BL, Shafieloo, Hazra, Smoot, Starobinsky (2018)



$$\Xi(k) = \frac{P(k) - \langle P_{P15}(k) \rangle}{\sigma_P(k)}$$

$$M_{P,i,j}(k) = \sqrt{\frac{1}{N_k} \sum_k \left(\frac{P_i(k) - P_j(k)}{\sigma_P(k)} \right)^2}$$



Initial conditions: features visible

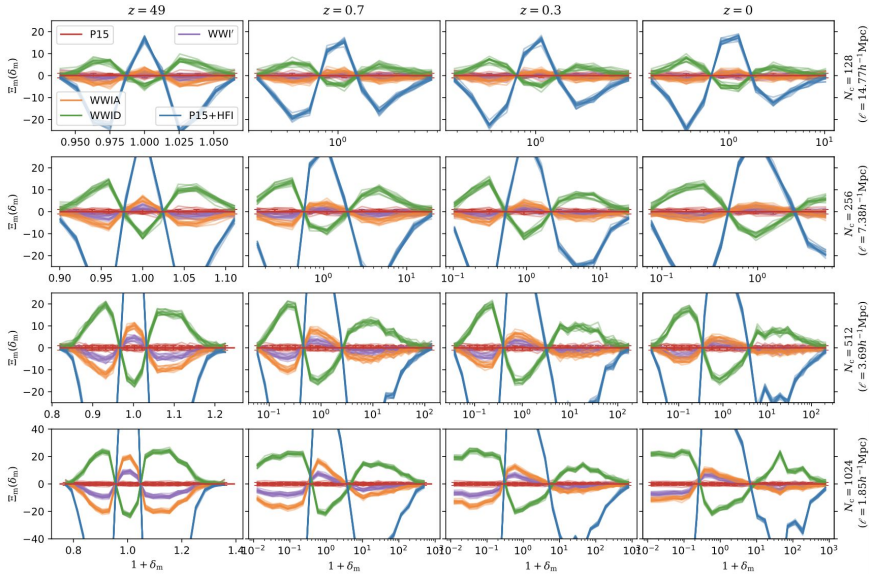
Features disappear in the non-linear regime

WWID, P15+HFI can be distinguished till $z=0$

WWIA, WWI' cannot be distinguished

Power spectrum may not be sufficient

Count-in-cell: Matter density

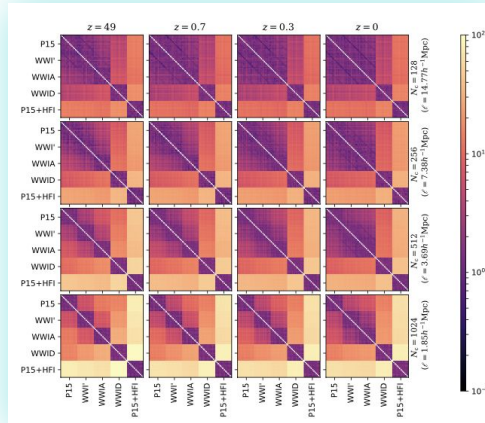


BL, Shafieloo, Hazra, Smoot, Starobinsky (2018)

Can distinguish between models!

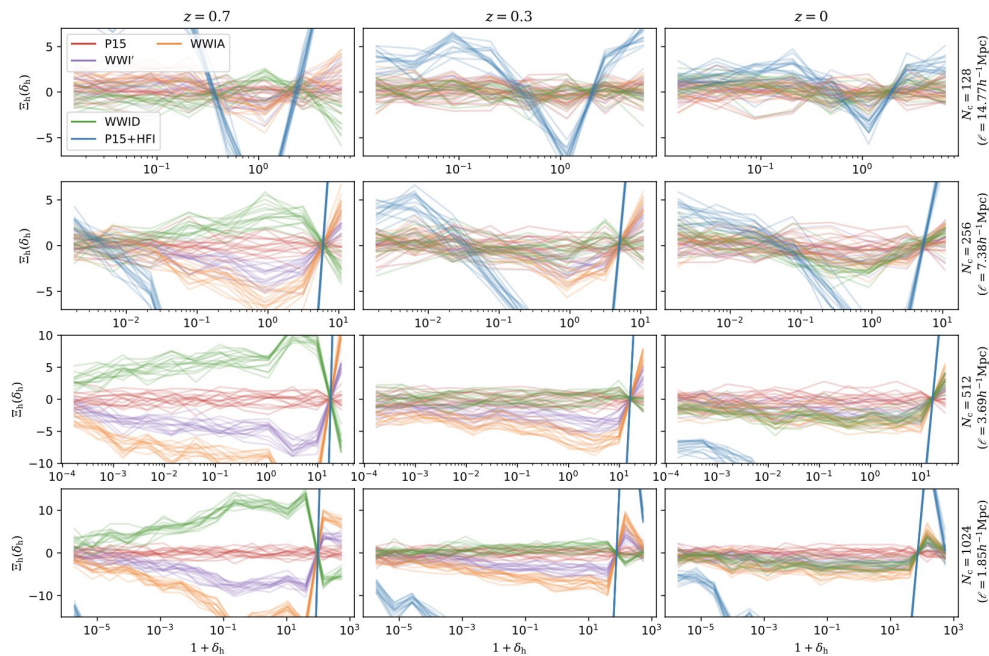
From 1D $P(k)$ to 3D count-in-cell

Non-linear regime: $P(k)$ does not capture all the information



Count-in-cell: Haloes

BL, Shafieloo, Hazra, Smoot, Starobinsky (2018)



Still holds!

But by $z=0$, difficult to distinguish

Summary

- **Current data are consistent with Flat FLRW, Λ as dark energy, GR as gravity, but room for deviation**
- **Modeling the non-linear regime with N-body simulations beyond Λ CDM**
 - **Relativistic simulations of DE: quantifying and parametrizing the deviation from Λ CDM**
 - **LSS can help constraining the early Universe: 3 dimensional info**
- **Future surveys (DESI) will help**
 - ¡Gracias! -
 - 감사합니다 -