Light Element Evaluation
SANDA, WP 4

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Goal of the project work at TU Wien

Task:

Development of a Bayesian evaluation technique of light nuclear systems for R-matrix based descriptions of reaction data in light nuclear systems

Motivation:

Knowledge of reaction data of several light nuclear systems are of great technological interest.

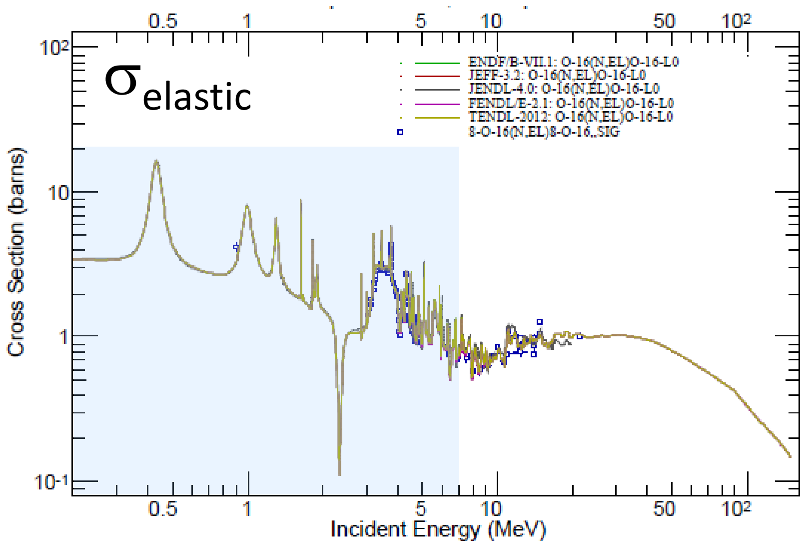
$^{6}Li(n,t)\,^{4}He$, $^{9}Be(n,2n\alpha)\,^{4}He$ nuclear fusion, neutron source

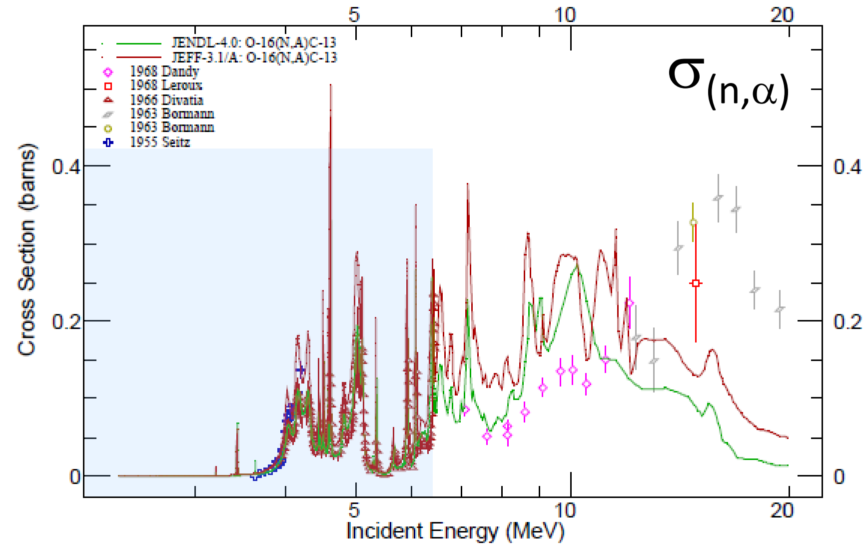
$^{16}O(n,\alpha)\,^{13}C$, $^{12}C(n,\alpha)\,^{9}Be$ embrittlement of structure materials

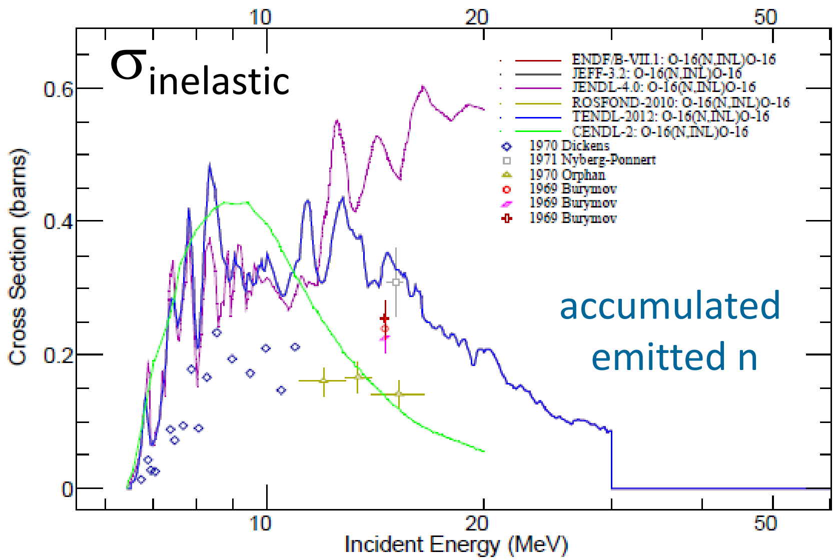
Problem:

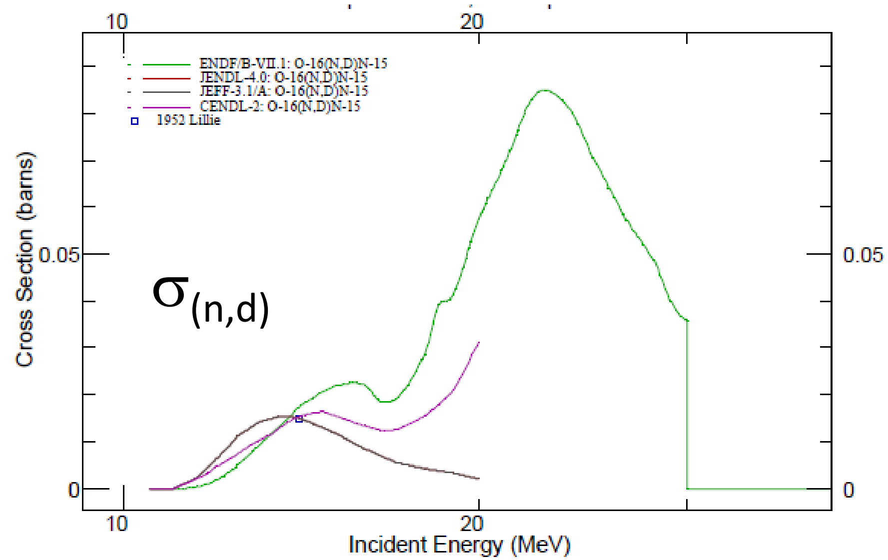
Nuclear data evaluation of light nuclear systems is hampered by the lack of quantitatively reliable microscopic models. Frequently R-matrix based descriptions are used → problem for Bayesian evaluation techniques.

Example: Evaluated Files for n+^{16}O Reactions









Generating evaluated files:
Nuclear reaction data of light nuclear systems

Coarse Overview:
The available evaluated files of nuclear data for light nuclear systems are generated by one of the following methods

1) **Evaluation generated exclusively from available experimental data**
   - limitation to channels with experimental data, consistency not guaranteed, prior of complete ignorance should be used.

2) **Evaluation generated by combining an R-matrix analyses at low energy with a fit of available experimental data at high energies**
   - same problems as in 1) for the fit of experimental data.

Problem: Uncertainties generated from the Hessian of the χ^2-fit either of the experimental data, or the resonance parameters. Frequently too small uncertainties are obtained

In general: No Bayesian evaluation procedure is usually performed.

Bayesian Evaluation of Reaction Data of Light Nuclear Systems

Question:
Is the concept of Bayesian statistics applicable in light nuclear systems?

Bayesian Evaluation Process

$$\pi\left(\vec{p}\mid\vec{\sigma}_{\exp}\right)=\frac{1}{\int d^{d}p\,\ell\left(\vec{\sigma}_{\exp}\mid\vec{p}\right)\pi\left(\vec{p}\right)}\,\ell\left(\vec{\sigma}_{\exp}\mid\vec{p}\right)\pi\left(\vec{p}\right)$$

What is the apriori knowledge ?

Problem:
Parameters of R-matrix are determined from experimental data
What is the a-priori knowledge????

Proposal for generating a prior for the R-matrix analyses

Available a-priori information

level scheme of the compound nucleus (E_{lv} , J^{π})
partitions, thresholds E_{th}

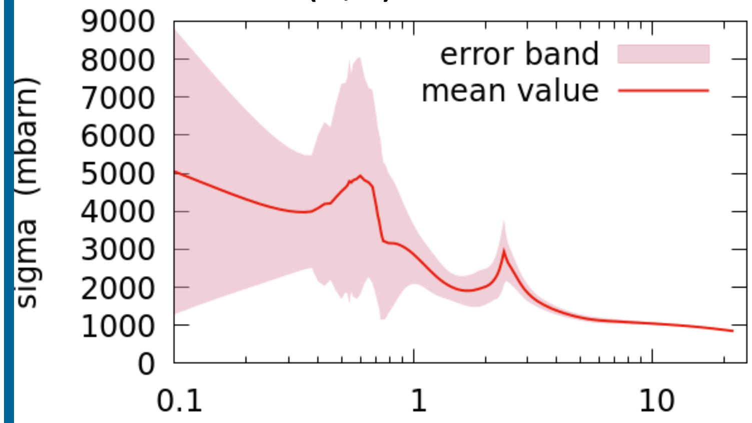
R-matrix parameters: resonance energy E_r form of resonance
reduced widths γ_i

Generate Monte Carlo sweeps of cross sections with R-matrix code
variation of E_r within 0.5 MeV, matching radius 0.2 fm,
γ-widths of previous R-matrix analysis varied within Turchin/0,25

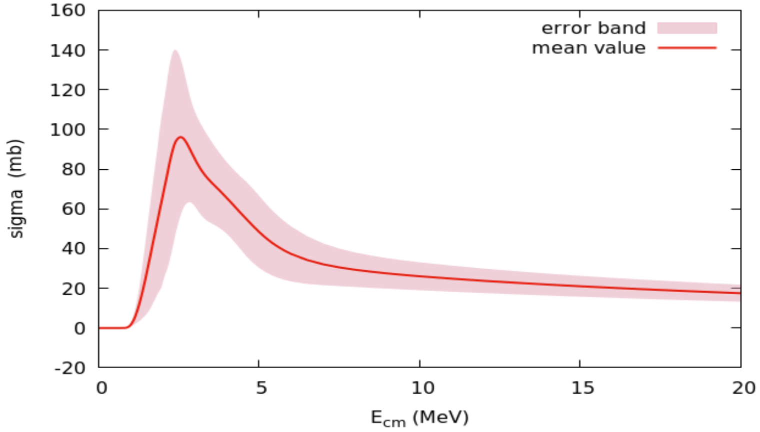
Quasi a-priori covariance matrix extracted:
limited knowledge on the position of resonances
knowledge of J^{π} included and thus features of the resonance
high energy behaviour determined variation of matching radius a

Generated Prior for n+^{9}Be Evaluation in the R-matrix regime

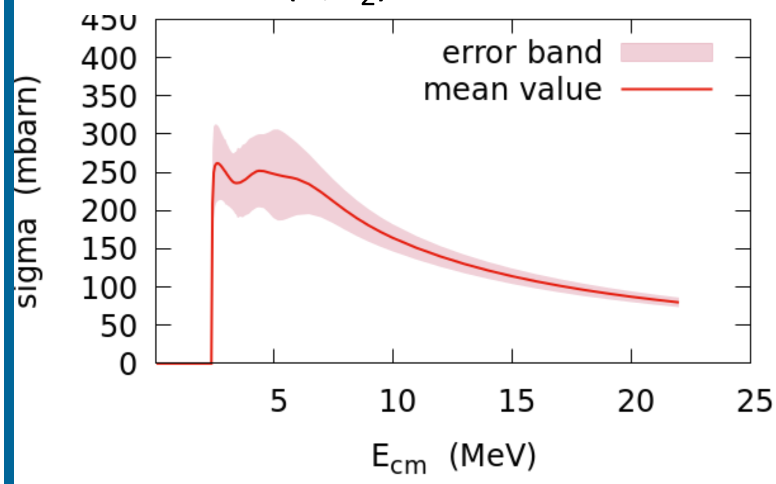
^{9}Be(n,n)^{9}Be elastic



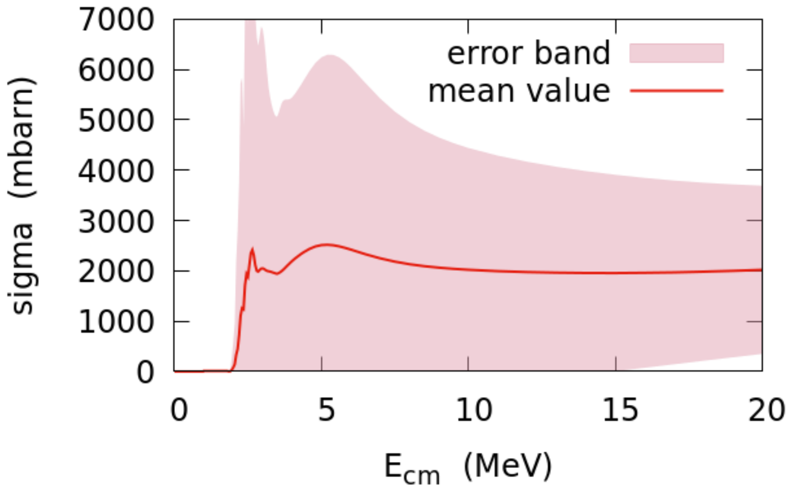
^{9}Be(n,^{4}He)^{6}He



^{9}Be(n,n$_2$)^{9}Be inelastic



^{9}Be(n,α2n)^{4}He breakup



Linearized Bayesian Update

normal distributions assumed for

experimental uncertainties, $\vec{\varepsilon}_{\exp} \sim N(0, \mathbf{B})$

likelihood and $\ell(\vec{\sigma}_{\exp} \mid \vec{p}) \sim N(M(\vec{p}), \mathbf{B})$

model parameters $\pi(\vec{p}) \sim N(\vec{p}_0, \mathbf{A}_0)$,

Generalised Least Square (GLS): Using multi-variate normal distributions allows linearization of Bayesian Theorem for update:

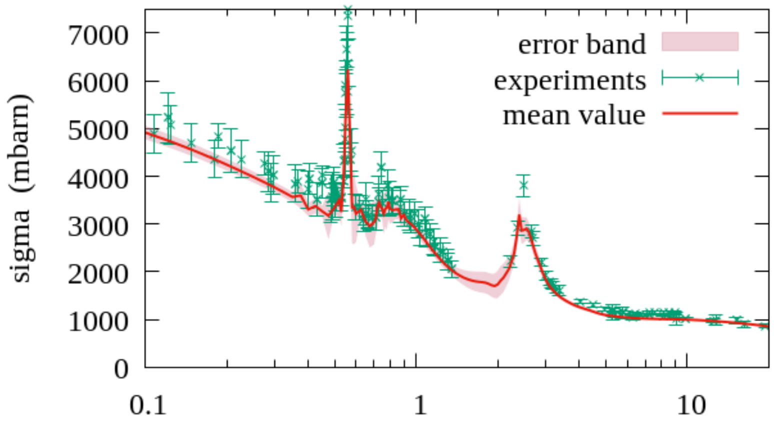
$$\vec{\sigma}_1 = \vec{\sigma}_0 + \mathbf{A}_0\mathbf{S}^T\left(\mathbf{S}\mathbf{A}_0\mathbf{S}^T + \mathbf{B}\right)^{-1}\left(\vec{\sigma}_{\exp} - \mathbf{S}\vec{\sigma}_0\right)$$

$$\mathbf{A}_1 = \mathbf{A}_0 - \mathbf{A}_0\mathbf{S}^T\left(\mathbf{S}\mathbf{A}_0\mathbf{S}^T + \mathbf{B}\right)^{-1}\mathbf{S}\mathbf{A}_0$$

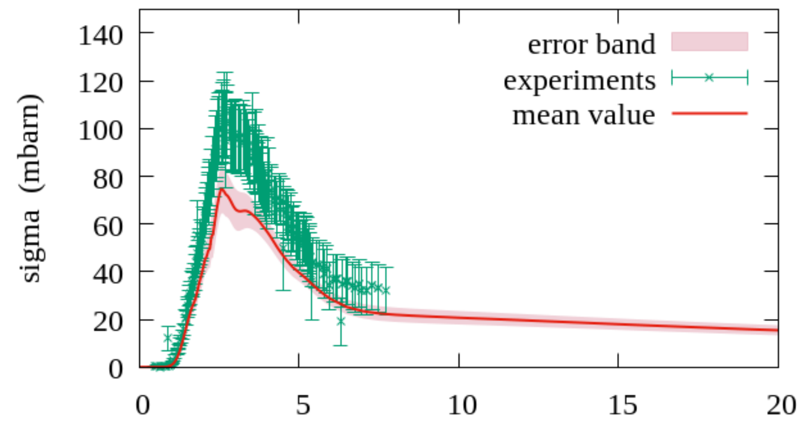
Sensitivity Matrix: $\vec{\sigma}_{\text{int}} = M_{\text{surr}}\left(\vec{\sigma}_{\text{mod}}\right) = \mathbf{S}\vec{\sigma}_{\text{mod}}$

Bayesian Evaluation of n+^{9}Be via GLS

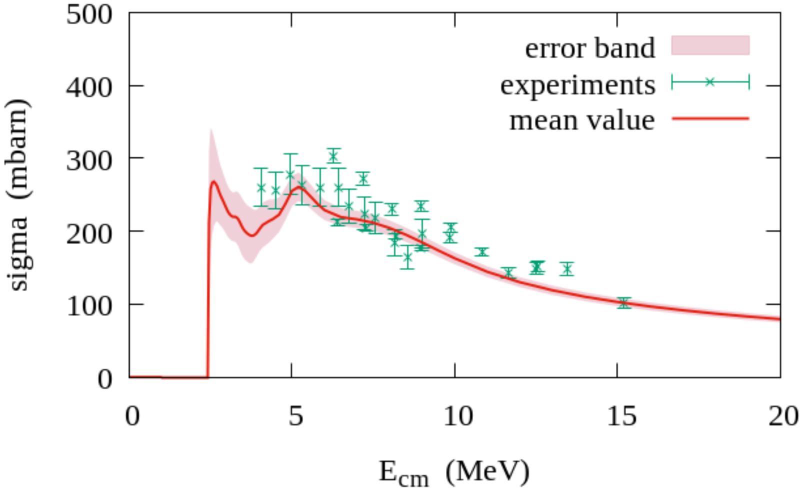
^{9}Be(n,n)^{9}Be elastic



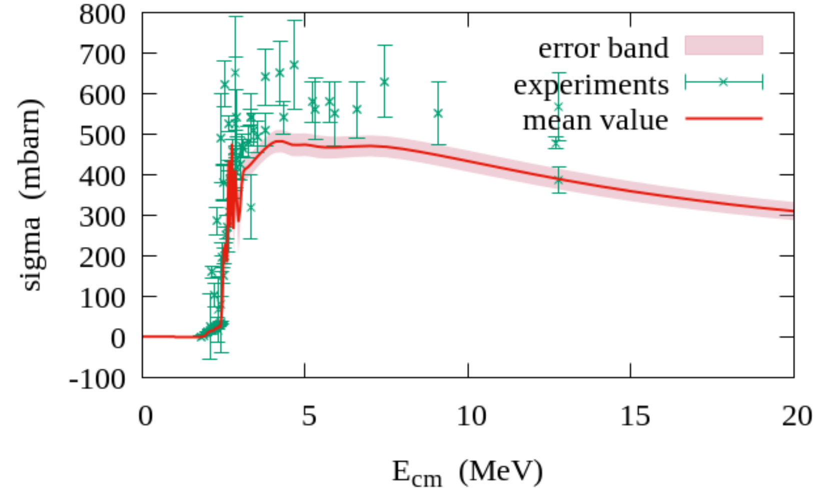
^{9}Be(n,^{4}He)^{6}He



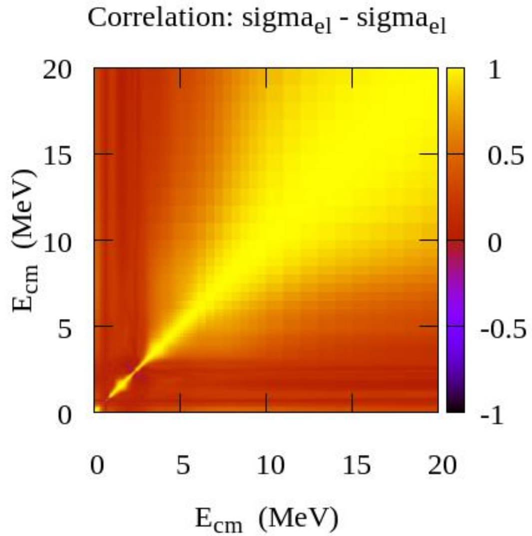
^{9}Be(n,n$_2$)^{9}Be inelastic

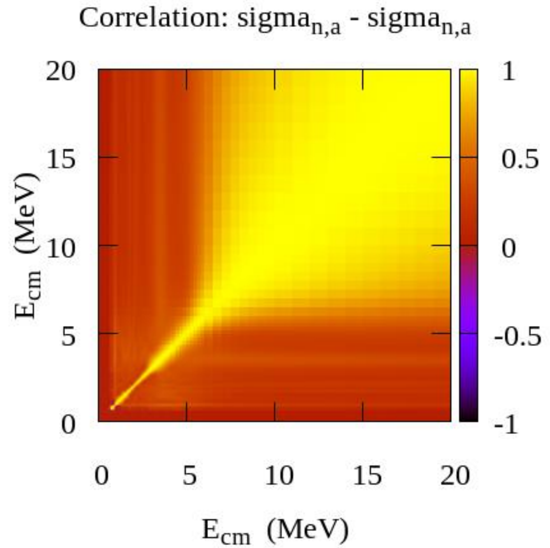


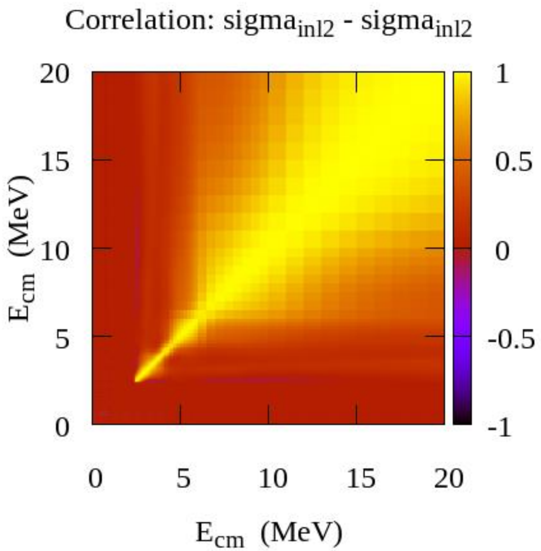
^{9}Be(n,α2n)^{4}He breakup

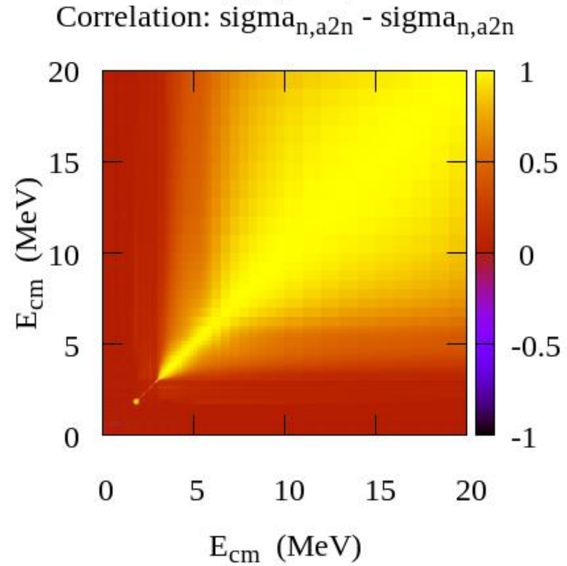


Bayesian Evaluation of n+^{9}Be via GLS Correlations









Summary

Deliverable was submitted by the task leader in spring 2023

Publications:

H. Leeb, Th. Srdinko,
Towards a Bayesian evaluation technique for light nuclear systems,
EPJ Web of Conferences 294, 04006 (2024)
https://doi.org/10.1051/epjconf/202429404006

Outlook:

The developed proposal for an R-matrix based prior is currently the starting point for further developments on the basis of the level matrix in the recently proposed R-matrix parametrisation of Park

Thank you for your attention