Uncertainty and parameter optimisation in high-energy models

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SANDA Final workshop. General Meeting
Madrid - 3–5 Jul 2024

Plan

- Introduction
- Method
- Results
- Conclusions

Introduction

- The main project ➡ Error assessment and model improvement + associated uncertainty
  - Tool ➡ bayesian statistics(inference)
  - Information ➡ experimental data

- Done within CHANDA
  - Building a method to assess error

- Done within SANDA
  - Building a method to improve the model (INCL) via parameter optimisation

- Next steps (project funded by ANR for 4 years, 2024 → 2027)
  - Combining both
  - Applying to specific cases

Introduction

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  - Building a method to improve the model (INCL) via parameter optimisation
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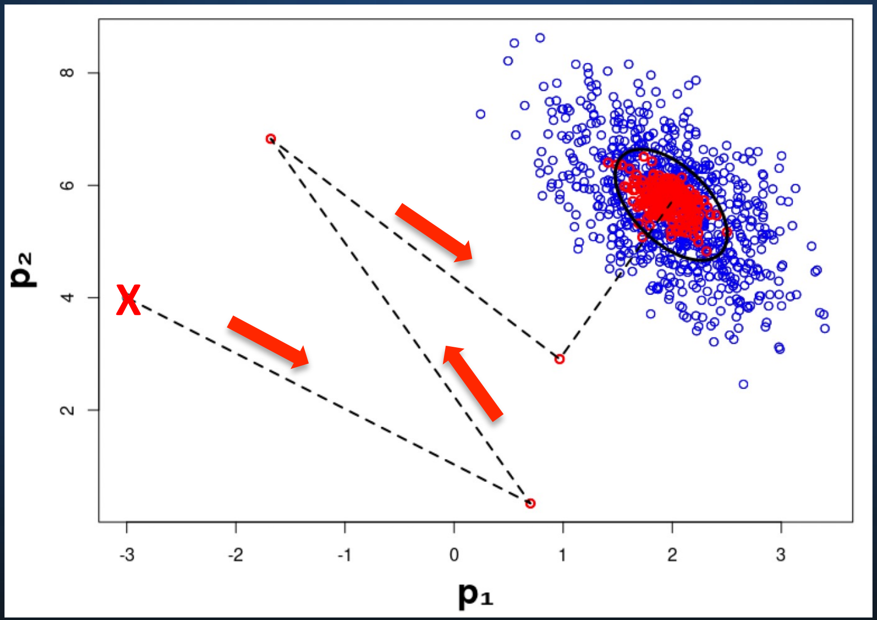
Method

Parameter optimisation

Two steps

- GLS (Generalized Least Square method) → Optimized parameters
- Gibbs-sampling-like method → Uncertainties of the values



More details…

Method

Parameter optimisation
GLS

GLS gives the new values of the parameters and related covariance matrices

$$\vec{p}_{op} = \vec{p}_{ref} + \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} (\vec{\sigma}_{exp} - \mathcal{M}(\vec{p}_{ref}))$$

$$\Sigma_{op} = \Sigma_p - \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} J_p \Sigma_p$$

Method

Parameter optimisation
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reference

link between parameter and model

weight of the information

information

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But

1- Right for linear model → Approximation: $\mathcal{M}(\vec{p}_{ref})$ → $T_i(\vec{p}_{ref}) = \mathcal{M}(\vec{p}_i) + J_{p_i}(\vec{p}_{ref} - \vec{p}_i)$

Jacobian

2- So, Covariance matrix not the right one to get uncertainties

$$\Sigma_{op} = \Sigma_p - \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} J_p \Sigma_p$$

Method

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Steps

(INCL is not linear, so p_{op} from iterations)

$$\vec{p}_{i+1} = \vec{p}_{ref} + \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} (\vec{\sigma}_{exp} - T_i(\vec{p}_{ref}))$$

with

$$T_i(\vec{p}_{ref}) = \mathcal{M}(\vec{p}_i) + J_{p_i}(\vec{p}_{ref} - \vec{p}_i)$$

Method

Parameter optimisation
Gibbs-sampling-like

Similar process as GLS, But…

$$T_i(\vec{p}_{ref}) = \mathcal{M}(\vec{p}_i) + J_{p_i}(\vec{p}_{ref} - \vec{p}_i)$$

No more from $\vec{p}_{i+1} = \vec{p}_{ref} + \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} (\vec{\sigma}_{exp} - T_i(\vec{p}_{ref}))$

But from $\vec{p}_{i+1} = \mathcal{N}(\vec{p}_i, \Sigma_i)$ (*break* the linear approximation)

and with $\Sigma_{i+1} = \Sigma_p - \Sigma_p J_p^T (\Sigma_e + J_p \Sigma_p J_p^T)^{-1} J_p \Sigma_p$

Results

Two examples - two cases

- Model needs improvement ⟶ Subthreshold production of K^+
- Model already not bad ⟶ Double differential neutron cross section

Results

Subthreshold production of K+

Description

- K^+ production from p+A (E~1GeV)
- A few parameters involved 🙂
- Very rare event (σ ~nb) + experimental data for each set (target, E) ☹️
  → long calculations
- Parameters:

$\sigma(NN \rightarrow K^+ + X)$	→	**a_{NN}** x $\sigma(NN \rightarrow K^+ + X)$
$\sigma(\pi N \rightarrow K^+ + X)$	→	**$a_{\pi N}$** x $\sigma(\pi N \rightarrow K^+ + X)$
$\sigma(\Delta N \rightarrow K^+ + X)$	→	**$a_{\Delta N}$** x $\sigma(\Delta N \rightarrow K^+ + X)$
Fermi momentum (270 MeV/c)	→	**Fm**

- Time: 7 days with 20 cores

Results

Subthreshold production of K+

Results

Name	old	new
a_{NN}	1.0 +/- 0.1	1.5
$a_{\pi N}$	1.0 +/- 0.1	0.26
$a_{\Delta N}$	1.0 +/- 0.1	0.43
Fm	270. +/- 5.	232.

⚠ Exercice! ⚠

New values, acceptable?

consequences on other quantities?

A complete study must go beyond!

χ^2 was bad

. delicate mechanisms;

. only 3% for exp. uncertainties…

And now…

. not « wonderful »,

. but falls rapidly!

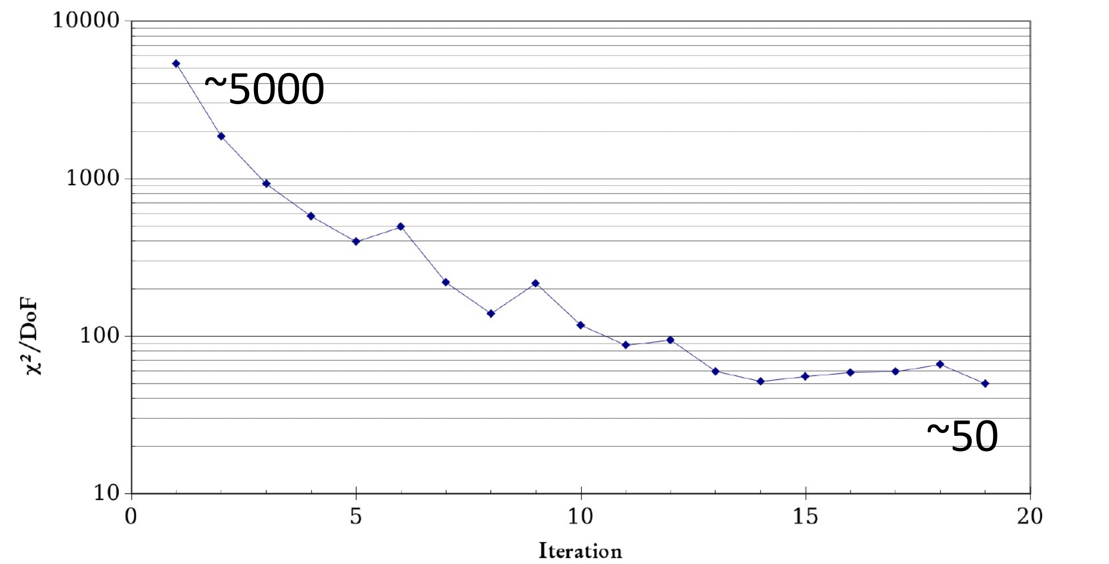

Fig. 2 Figure of merit showing the evolution of the χ^2/DoF after each iteration. Iteration 1 corresponds to the initial version of INCL.

Results

Subthreshold production of K+

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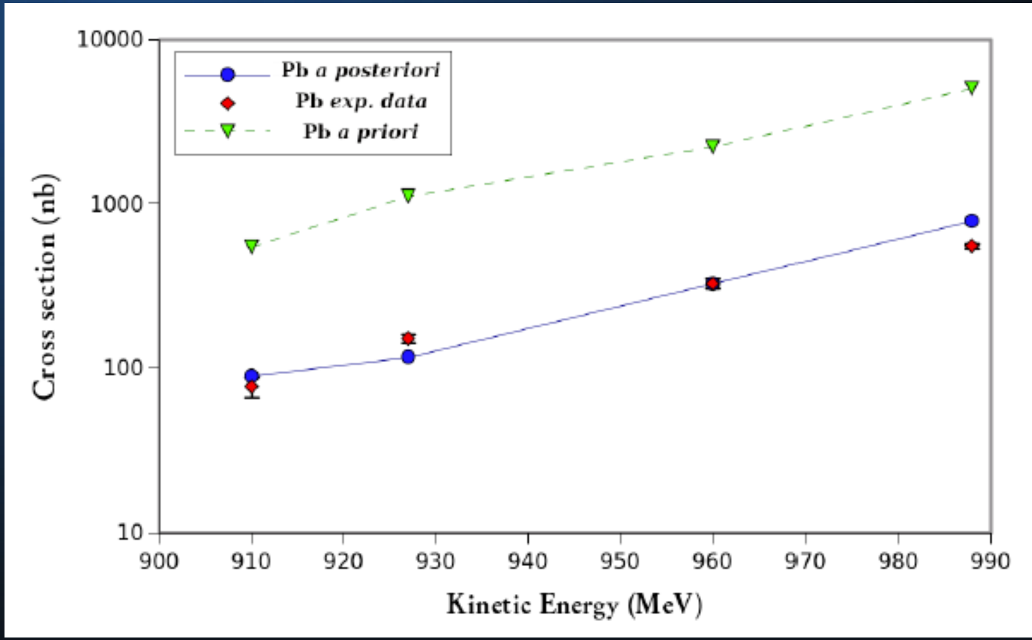
And great improvement for Pb target



Results

Double differential neutron cross section

Description

- Neutron DDXS from p+A   A < Aluminum; $E_p < 200$ MeV
- *Toxic* data (error bar: 0.45%!!!) → exp. data must be studied before ☹️
- Are we able to do better, for an already not too bad DDXS? 🤔
- Parameters:

  $\sigma(N\Delta \rightarrow NN) \propto 3\ \sigma(NN \rightarrow N\Delta)$ → $\sigma(N\Delta \rightarrow NN) \propto$ **DB** x $\sigma(NN \rightarrow N\Delta)$

  $t_{stop} = 29.8 \times A^{0.16}$ fm/c → $t_{stop} =$ **a** x A^{b} fm/c

  Fermi momentum (270 MeV/c) → **Fm**

- Time: 60 hours with 20 cores

Results

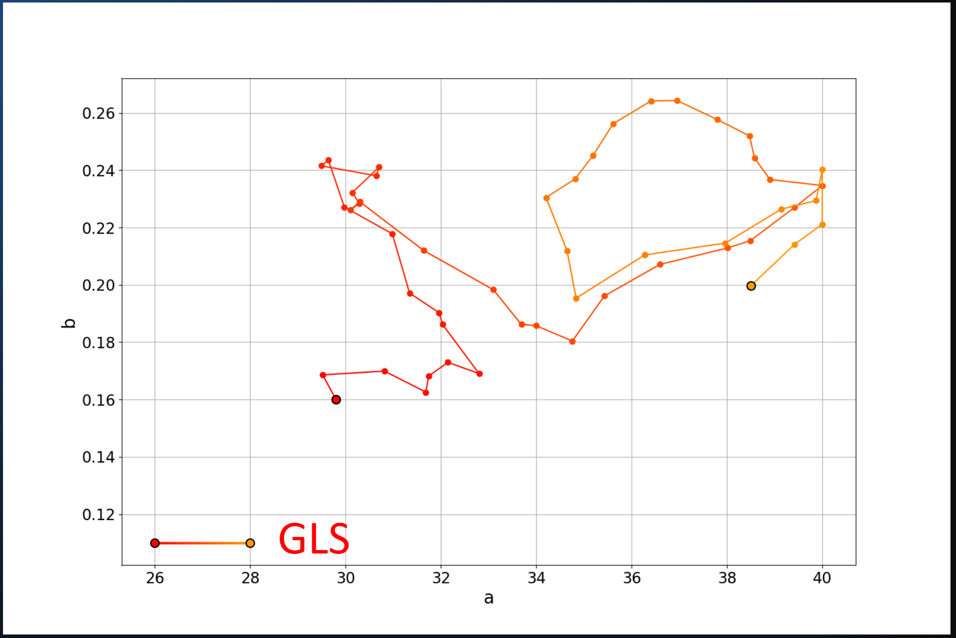
Double differential neutron cross section

Results

Name	old	new
DB	3.0 +/- 0.1	4.406 +/- 0.131
a	29.8 +/- 0.5	37.13 +/- 0.59
b	0.16 +/- 0.05	0.226 +/- 0.005
Fm	270. +/- 3.	266.4 +/- 0.97

χ2/DoF 7.805 → 7.34
(6% better)

GLS gives the values
… and uncertainties, but not really usable (linearity)



Results

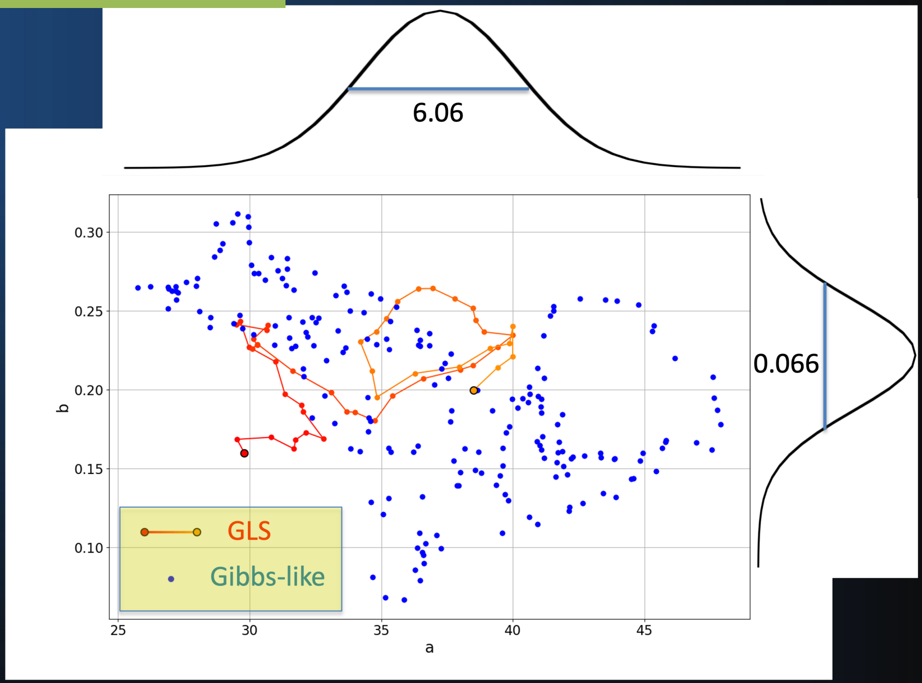
Double differential neutron cross section

Results

Name	old	new
DB	3.0 +/- 0.1	4.406 +/- 0.131 +/- 0.986
a	29.8 +/- 0.5	37.13 +/- 0.59 +/- 6.06
b	0.16 +/- 0.05	0.226 +/- 0.005 +/- 0.066
Fm	270. +/- 3.	266.4 +/- 0.97 +/- 8.82

GLS gives the values
… and uncertainties, but not really usable (linearity)

Gibbs-like, distributions of the possible values
Fit by a gaussian → uncertainties



Conclusions

- The method to optimize parameters of Monte Carlo nuclear reaction models like INCL is built (with associated uncertainty).
- The report has been delivered.
- An article has been accepted in EPJA and will be published soon.

Thanks for your attention!

And thanks to

J. Hirtz

who did the work

in collaboration with

G. Schnabel