Instituto de Física Teórica UAM-CSIC

UNIVERSITY OF BIRMINGHAM | GRAVITATIONAL WAVE ASTRONOMY

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pyEFPE: An improved post-Newtonian waveform model for inspiralling precessing-eccentric compact binaries

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Based on [arXiv:2502.03929]

Motivation - why study eccentricity and precession?

- Measuring eccentric binaries is a very significant milestone for GW science.
- Eccentric binaries could be created by dynamical interactions in dense environments, having also misaligned spins.
- Need to understand possible degeneracy between precession and eccentricity.
- Eccentric-precessing models allow the study of the whole BBH parameter space.
- Models describing eccentricity and precession still have a long way to go

Phenomenology of precessing-eccentric signals

On the largest timescale

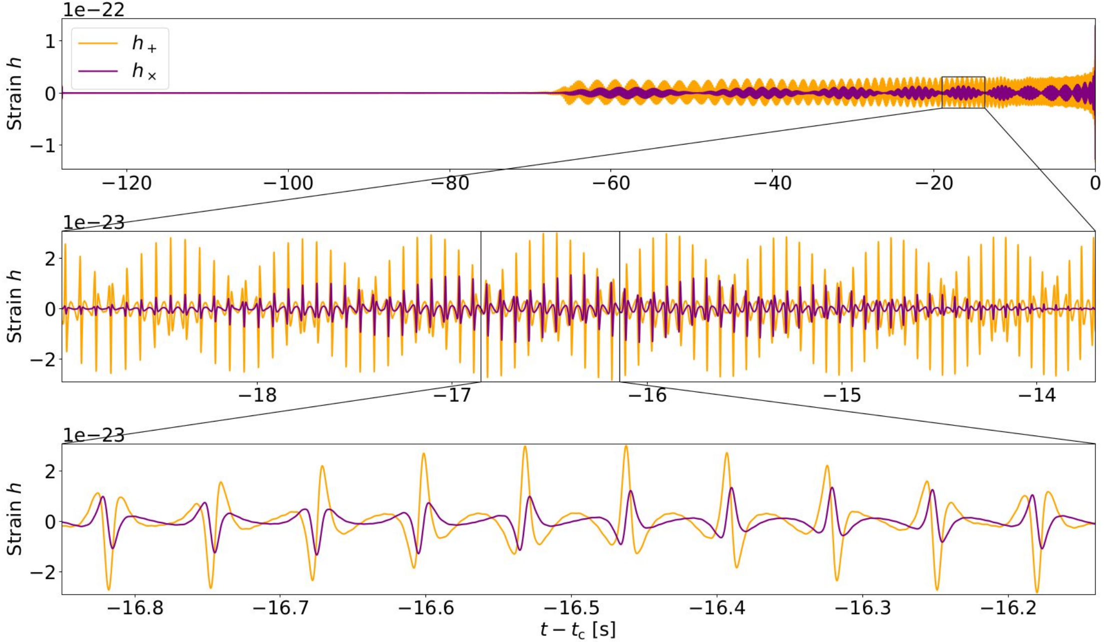
- Waveform slowly chirps due to radiation reaction
- Amplitudes modulated by precession

Zooming into a precession cycle

- Narrow spikes correspond to periastron passages
- Phase at each periastron passage differs due to periastron advance

Zooming into a periastron-advance cycle

- Waveforms no longer quasi-monochromatic
- Require many Fourier modes at multiples of orbital frequency



Time-domain GW polarizations emitted by an edge-on (ι_{10Hz}=π/2) highly eccentric (e_{10Hz}= 0.7) precessing BNS like system (m_1 =2.4$M_\odot$, m_2 =1.2$M_\odot$). We show the iFFT of the pyEFPE frequency-domain waveform, with f_{min}= 20Hz.

Introduction to EFPE models

- pyEFPE is based on the Efficient Fully Precessing Eccentric (EFPE) family of models [Klein et al. (2018), Klein (2021), Arredondo et al. (2024)]

- EFPE waveforms model the inspiral of eccentric and precessing systems by
  1. Using the equations of motion derived from the post-Newtonian (PN) approximation
  2. Approximate the orbital dynamics using the quasi-Keplerian (qK) parametrization
  3. Approximate the spin-precession dynamics with the Multiple Scale Analysis (MSA) solution
  4. Evolve the system through radiation reaction similarly to how it is done in TaylorT4
  5. Compute waveform in the time-domain by “twisting-up” a spin-aligned PN waveform
  6. Compute waveform in the frequency domain by applying the Shifted Uniform Asymptotics (SUA) approximation to each Fourier mode

pyEFPE as the next generation of EFPE models

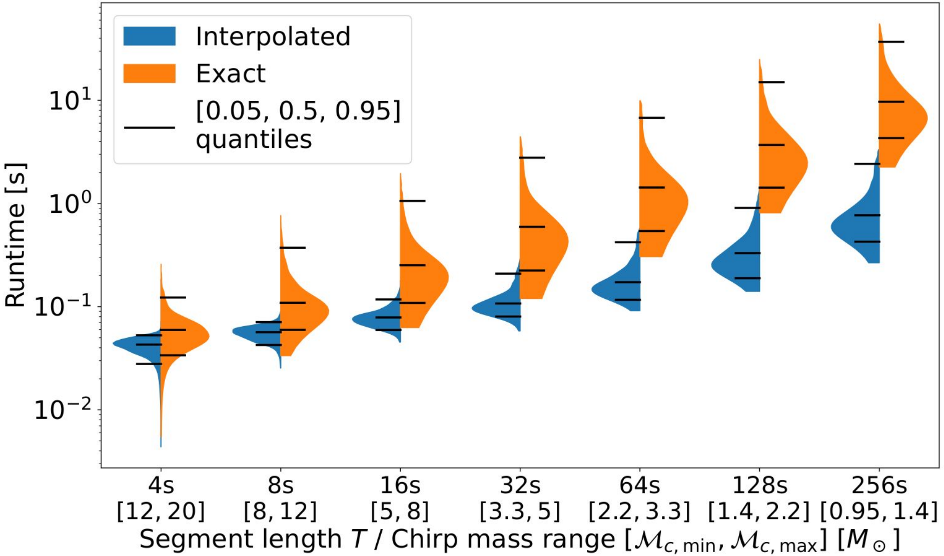
- In pyEFPE we introduce key improvements:
  - Derive and implement *closed analytical* expressions for the Newtonian Fourier mode amplitudes, making the model fast and accurate for large eccentricities ($e \lesssim 0.8$).
  - Improved stability of the MSA description of the spin-precession dynamics.
  - Extended PN information in the evolution equations, adding 2.5PN and 3PN aligned-spin contributions.
  - Simplified numerical implementation and interpolated the amplitudes, leading to an ~O(20) speedup of the waveform.

- The waveform is publicly available at https://github.com/gmorras/pyEFPE

- Python implementation is easy to review, use, and modify.

Waveform timings

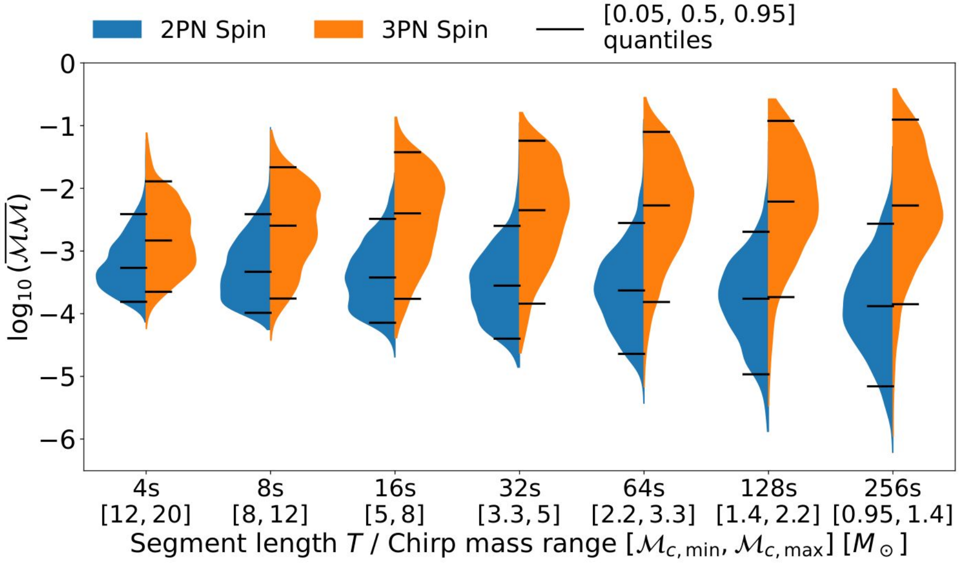
- Without interpolation: runtime of long signals (T≳16s) dominated by amplitude computation ⟶ not feasible to analyze
- With interpolation: takes between 0.05s and 1s to evaluate waveforms, even at very high eccentricities (e_0≲0.7)
⟶ pyEFPE can be used in PE !
- Interpolating amplitudes speeds up their computation by a factor of ~20, at a negligible cost in accuracy (mismatches between both versions ≲10^{-6})



Violin plots of the pyEFPE runtime with interpolated amplitudes (blue) and exact amplitudes (orange). We evaluate pyEFPE on [20Hz, f_{ISCO}] with Δf=20Hz. The code is run on a single core of a 2020 laptop processor.

Comparison with SpinTaylorT4 in the quasi-circular limit

- At 2PN the mismatches are small and below 10^{-2}
  - Using the SUA vs an FFT to compute the frequency domain waveform
  - Using the MSA approximation vs numerical integration of PN equations to describe spin-precession
  - Residual eccentricity induced by precession
- At 3PN, mismatches become larger due to the MSA being limited to 2PN in pyEFPE

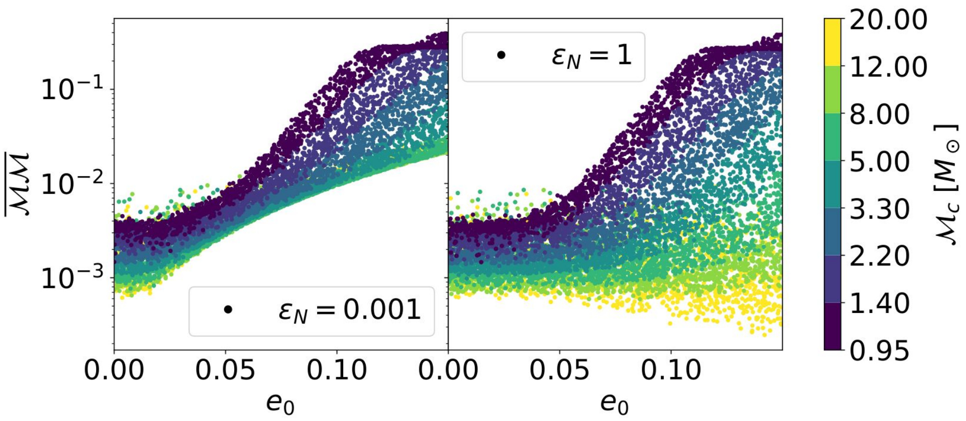


Violin plots showing the distribution of the mismatch between two configurations of pyEFPE and SpinTaylorT4. Taking up to 2PN and 3PN spinning terms in the equations of motion. In the latter case, the precession dynamics for pyEFPE remain at 2PN, while for SpinTaylorT4 they go to 3PN.

Comparison with TaylorF2Ecc in the spin-aligned limit

- Comparing the leading Fourier mode in both cases (ε_N=1 case)
  - For $e_0 \lesssim 0.05$ the mismatches are small and do not depend on eccentricity
  - For $e_0 \gtrsim 0.05$ and small masses (long signals), the mismatches become large due to TaylorF2Ecc neglecting large $O((e_0)^4)$ terms in the phase, starting at 0PN.

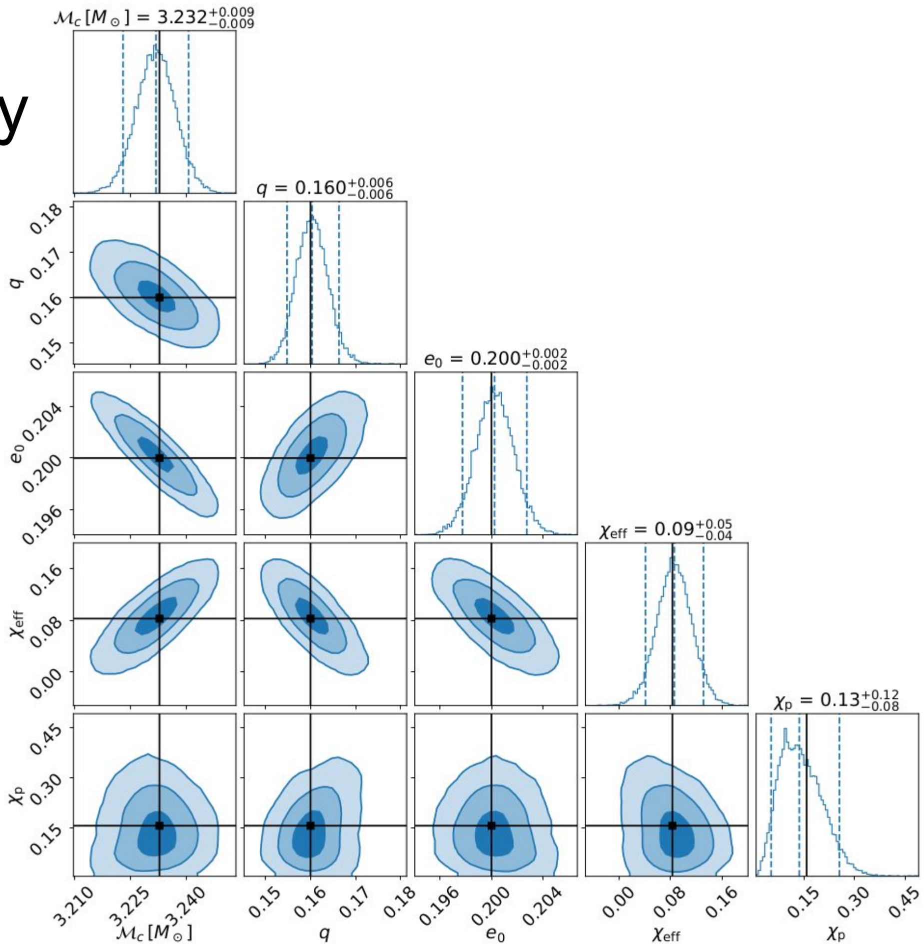
- Taking into account higher order Fourier modes in pyEFPE (with amplitude tolerance $\varepsilon_N = 10^{-3}$)
  - Mismatches grow faster with eccentricity due to TaylorF2Ecc neglecting $O(e_0)$ terms in the amplitude.



Scatter plot of the mismatches between pyEFPE and TaylorF2Ecc as a function of the initial eccentricity e_0 and chirp mass M_c. In the left panel we show when we include as many Fourier modes as necessary in pyEFPE to reach a relative error smaller than 10^{-3} in the amplitude. In the right panel we show when we only include in pyEFPE the same leading Fourier mode modeled by TaylorF2Ecc. We use $f_{min} = f_0 = 20$Hz.

PE - pyEFPE injection recovery

- We analyze an eccentric (e_0=0.2) NSBH-like (m_1=10$M_\odot$, m_2=1.6$M_\odot$) zero-noise injection from 20Hz (T=32s).
- PE with Bilby for LVK-like eccentric-precessing signals feasible:
  - ~8 days of sampling time with 16 cores
  - ~6.5·10^7 likelihood calls ⟶ ~0.17 core·s/call
- Accurately recover injected parameters
- Mechanisms that may break PE degeneracies
  - Having ~6 Fourier modes significantly contributing
  - Measuring the periastron advance

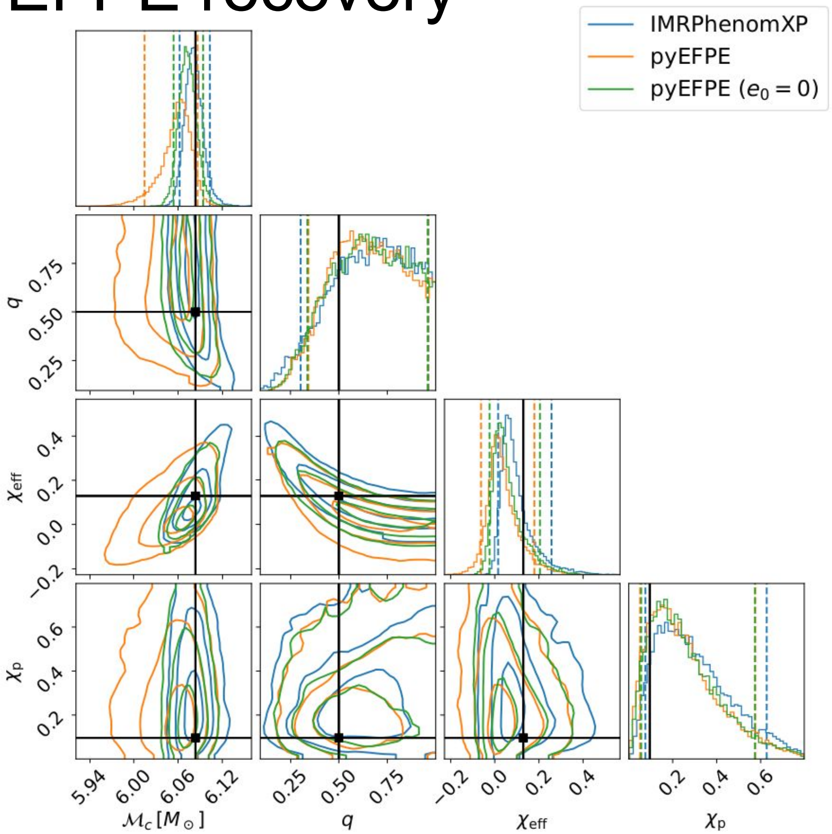


PE - IMRPhenomXP injection - pyEFPE recovery

- We analyze a low mass, asymmetric BBH (m_1=10$M_\odot$, m_2=5$M_\odot$) zero-noise injection from 20Hz (T=16s).

- Use IMRPhenomXP as a baseline, pyEFPE with e_0= 0 and pyEFPE with a flat e_0-prior in [0, 0.4]

- Excellent agreement, with a few differences:
  - pyEFPE with e_0=0 has small deviations from IMRPhenomXP due to the added information from PN, EOB and NR in the phase and amplitude
  - pyEFPE with $e_0 \neq 0$ has bigger deviations, especially in the chirp mass due to e_0-M_c correlations



PE - IMRPhenomXP injection - pyEFPE recovery

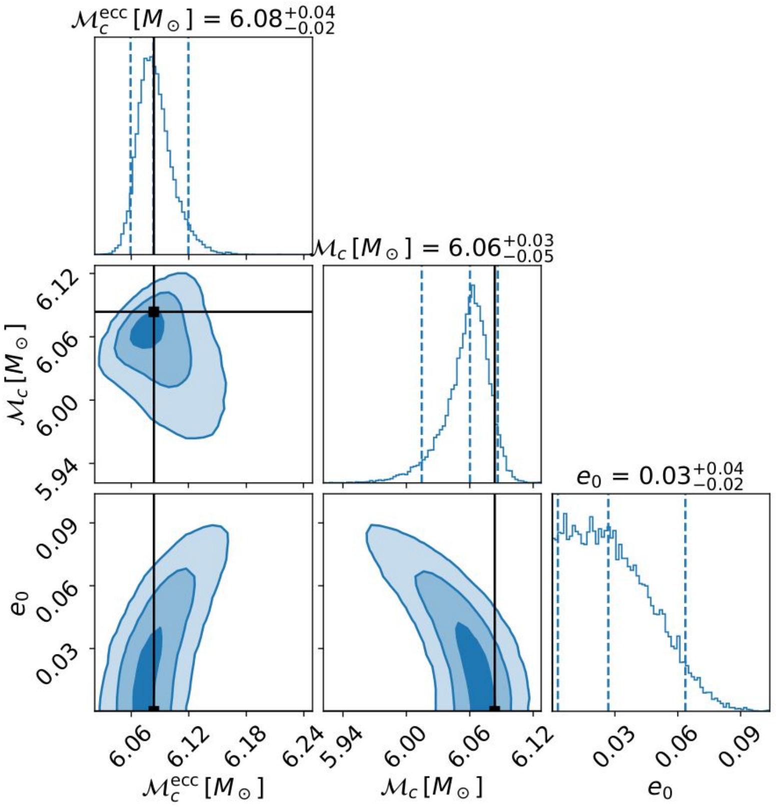
- Eccentricity posterior peaked towards e_0=0, consistent with XP being quasi-circular

- Long tail in eccentricity pushes the chirp mass posterior to be almost incompatible with the injected value

- Using the eccentric chirp mass [Favata et al. (2021)]

$$\mathcal{M}_c^{\mathrm{ecc}} = \left(1 + \frac{157}{40} e_0^2\right) \mathcal{M}_c$$

  - We capture most of the M_c-e_0 degeneracy
  - The posterior is centered nicely at the injected value

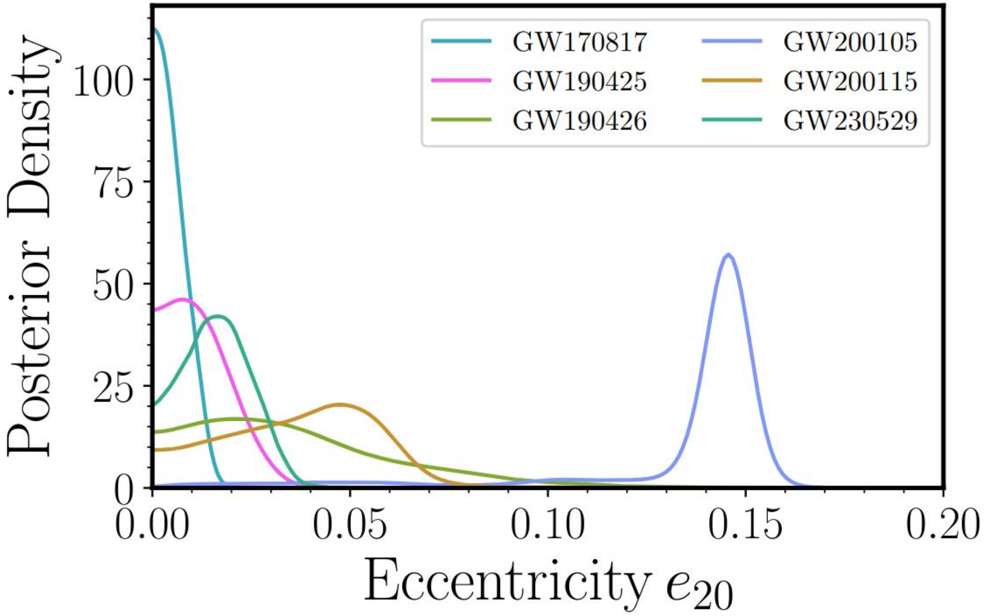


First eccentric-precessing PE of low-mass LVK events

- Events in range of validity of pyEFPE
  - 2 BNS: *GW170817* and *GW190425*
  - 4 NSBH: *GW190426*, *GW200105*, *GW200115* and *GW230529*

- Only *GW200105* shows clear evidence for eccentricity (see arXiv:2503.15393)

- A large fraction of NSBHs might have significant eccentricities in LVK-band !



Conclusions

- We have developed a fast PN waveform model with eccentricity and spin precession
- Due to improvements in stability, PN information and runtime it can be applied to LVK data
- Comparing with LAL waveforms in the quasi-circular and eccentric-spin-aligned limits, pyEFPE works as expected
- Demonstrated model efficacy for PE, recovering accurate parameters for both IMRPhenomXP and pyEFPE injections
- Applied model to real GW data, constraining eccentricity and precession for low-mass events
- Found that GW200105 has evidence of being eccentric

Backup slides

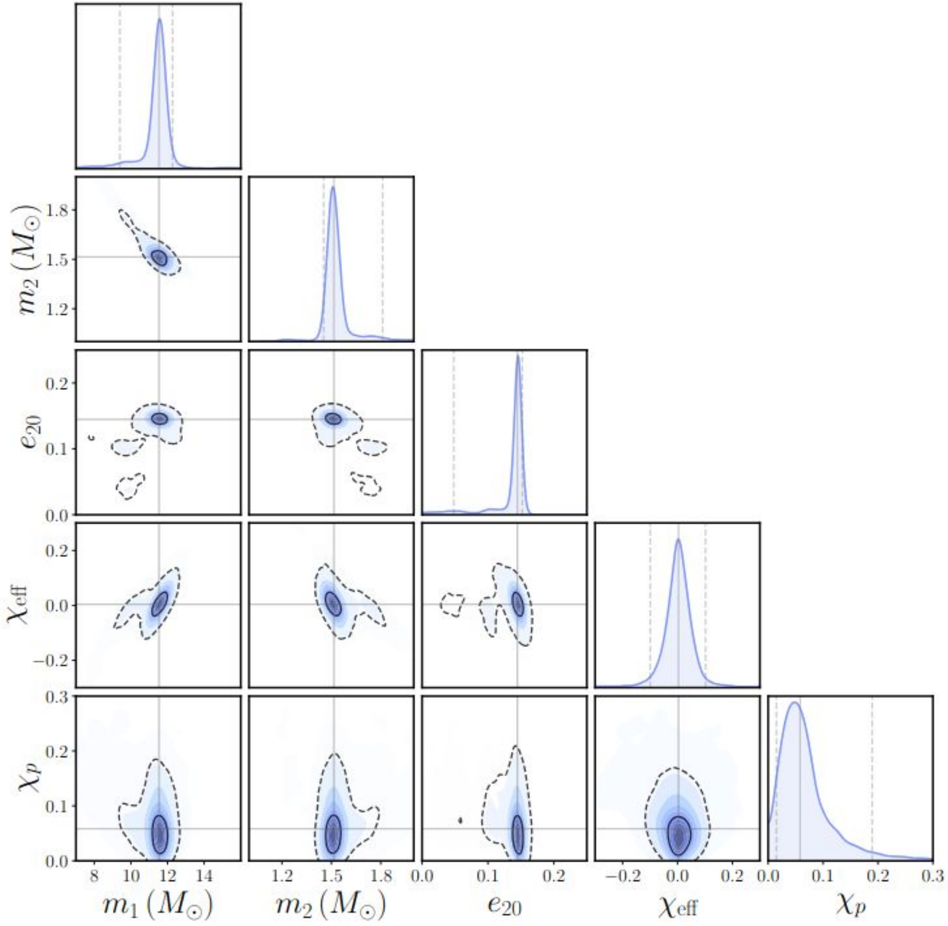


FIG. 3. **Measured properties of GW200105.** One- and two-dimensional posterior probability distributions of the component masses, effective inspiral spin $\chi_{\rm eff}$, effective precession spin χ_p, and the orbital eccentricity at a GW frequency of 20 Hz inferred with the eccentric-precessing PYEFPE waveform model. Solid lines indicate the median value, dashed lines the 90% credible interval. Two-dimensional contours indicate the 50% (solid) and 90% (dashed) credible regions.