A hybrid time-domain approach to the LISA response: performance and applications

Jorge Valencia, Sascha Husa, Maria Rosselló-Sastre, Lluc Planas & Antoni Ramos-Buades

24 June 2025

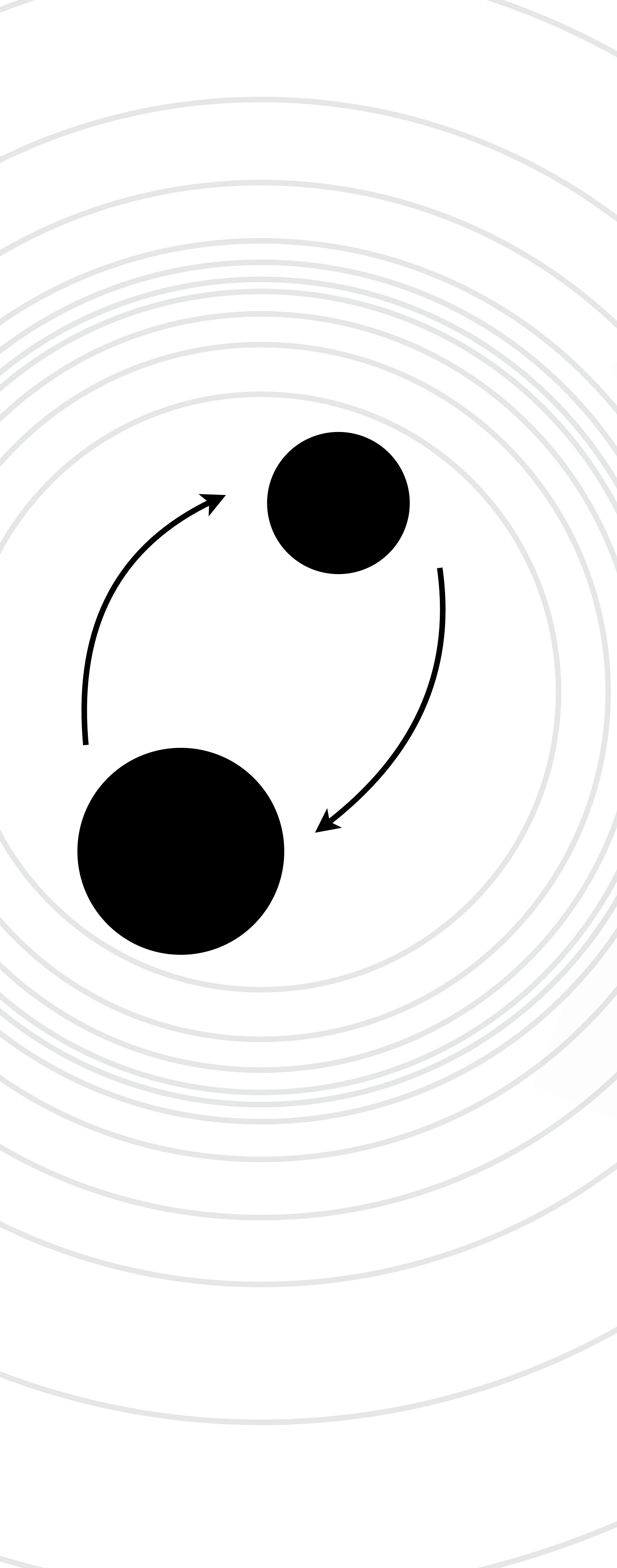
Universitat de les Illes Balears

IAC3 Institut d'Aplicacions Computacionals de Codi Comunitari

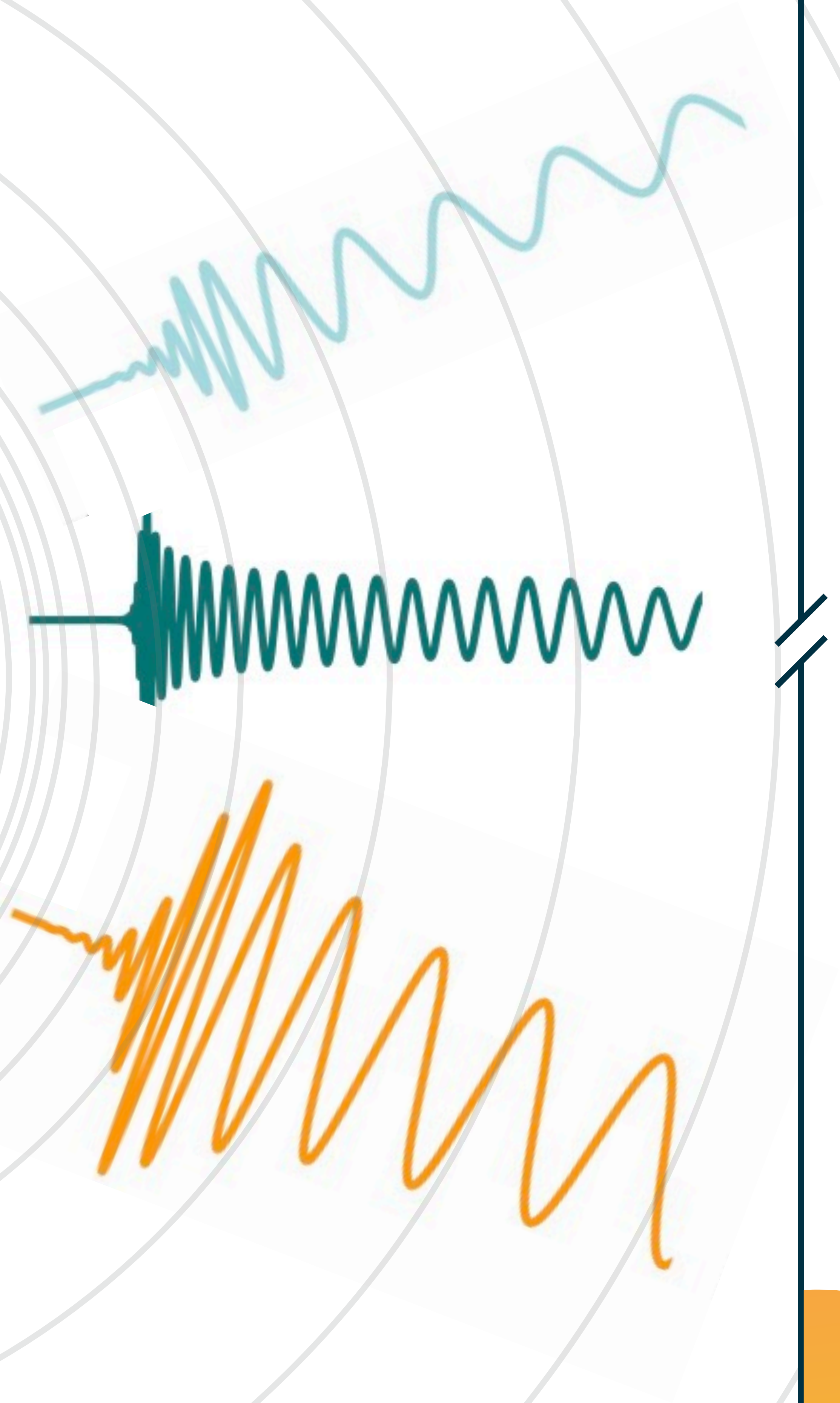
Image adapted from IGWM 2025 webpage: https://agenda.ciemat.es/event/4951/

Motivation: Data analysis pipeline

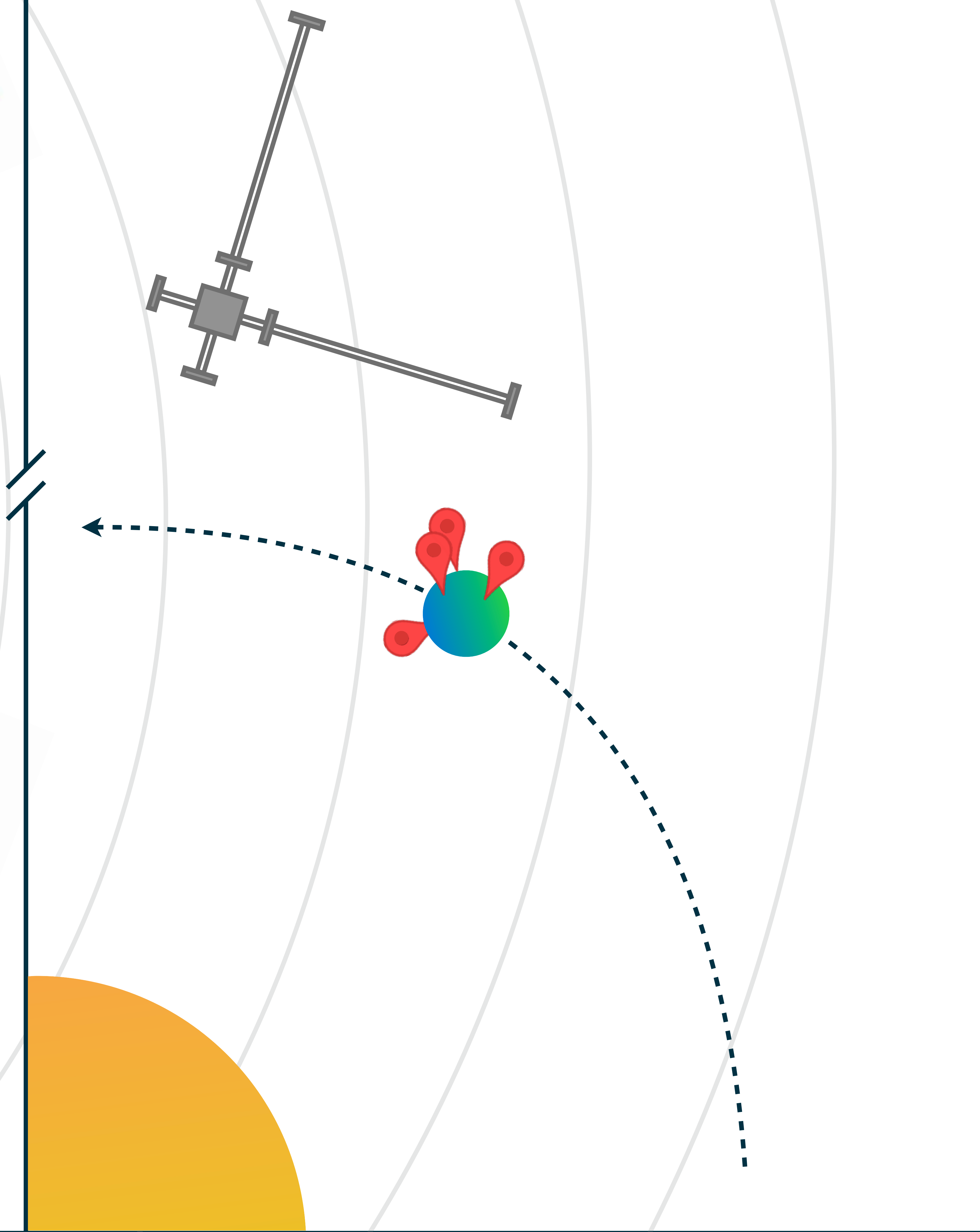
Emission



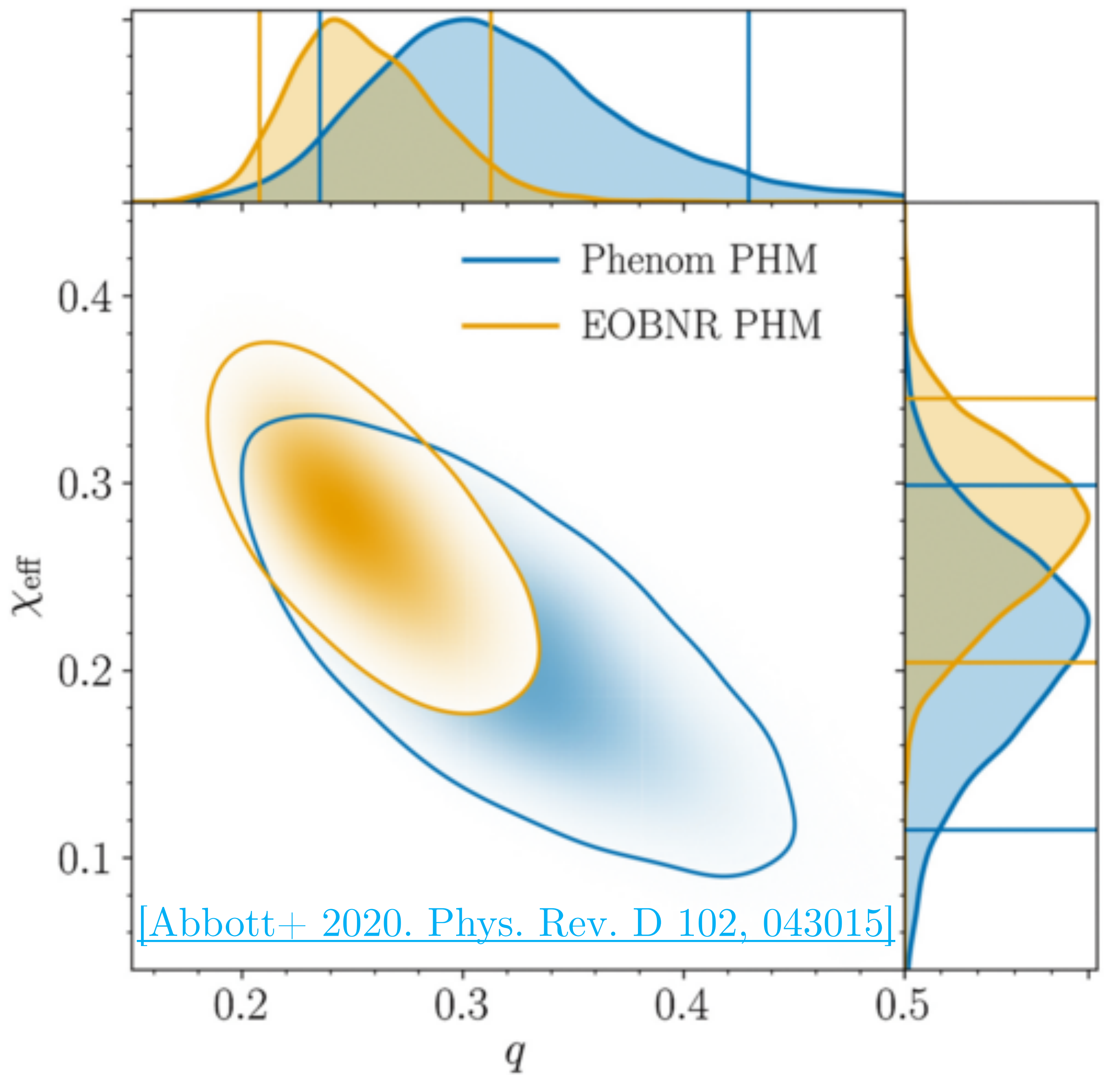
Model



Detector

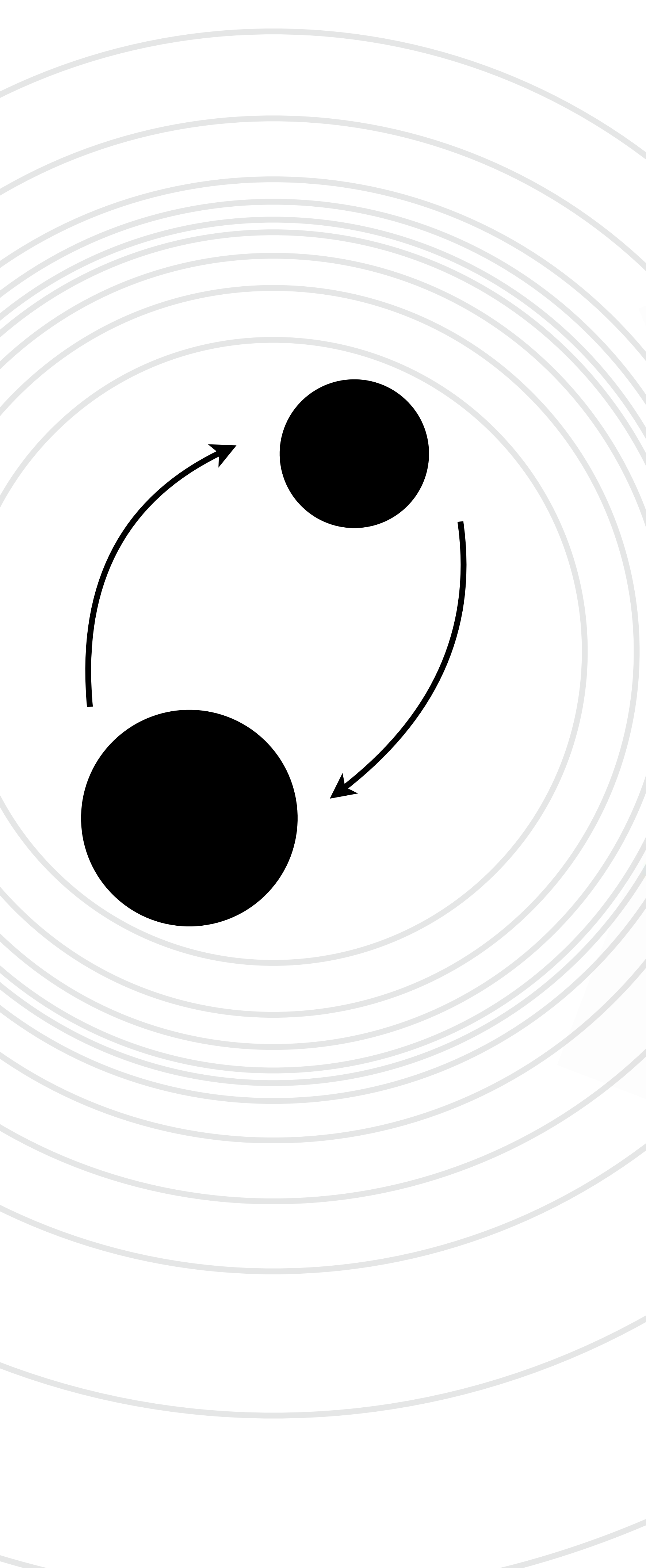


Parameter estimation

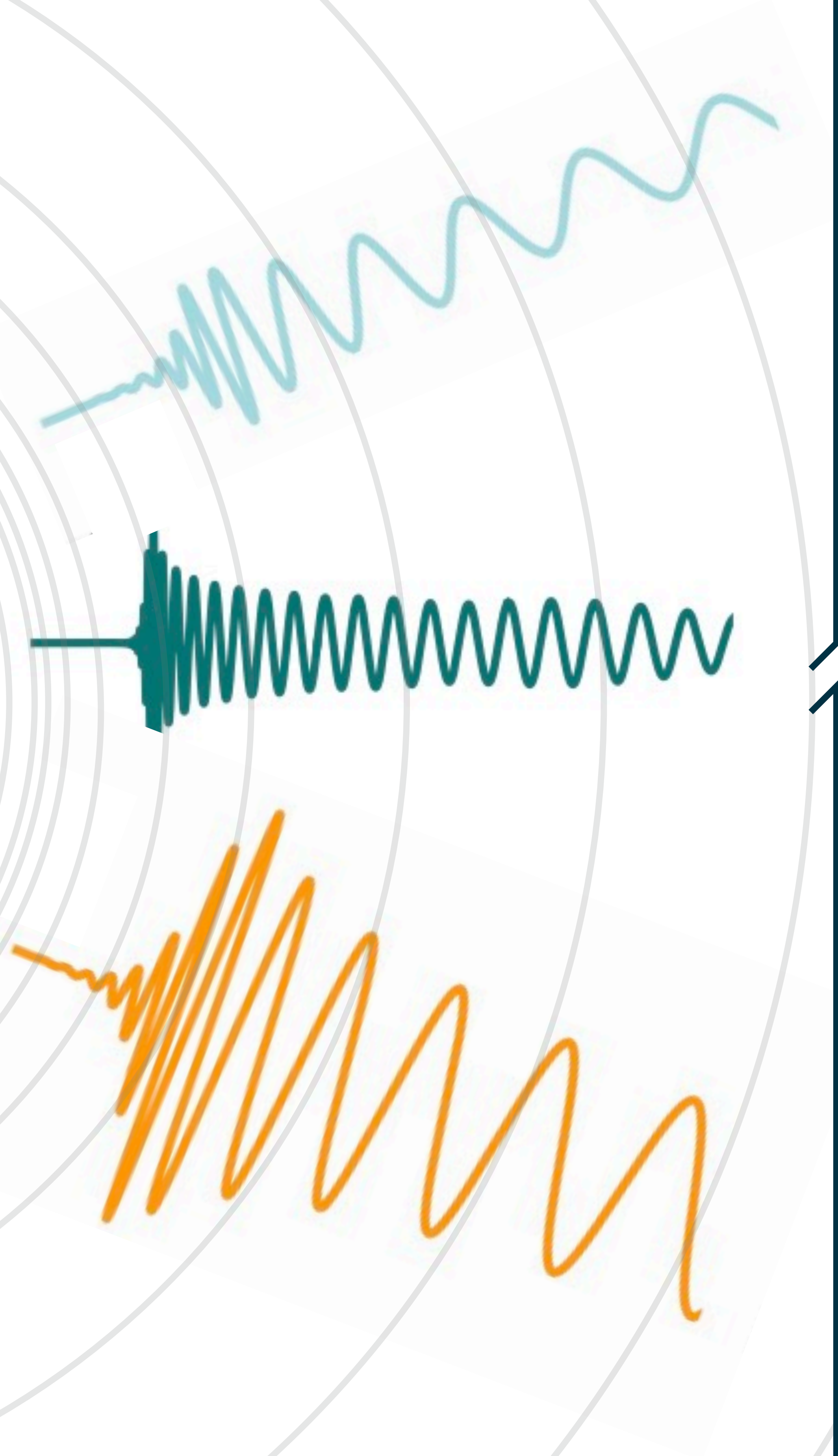


Motivation: Data analysis pipeline

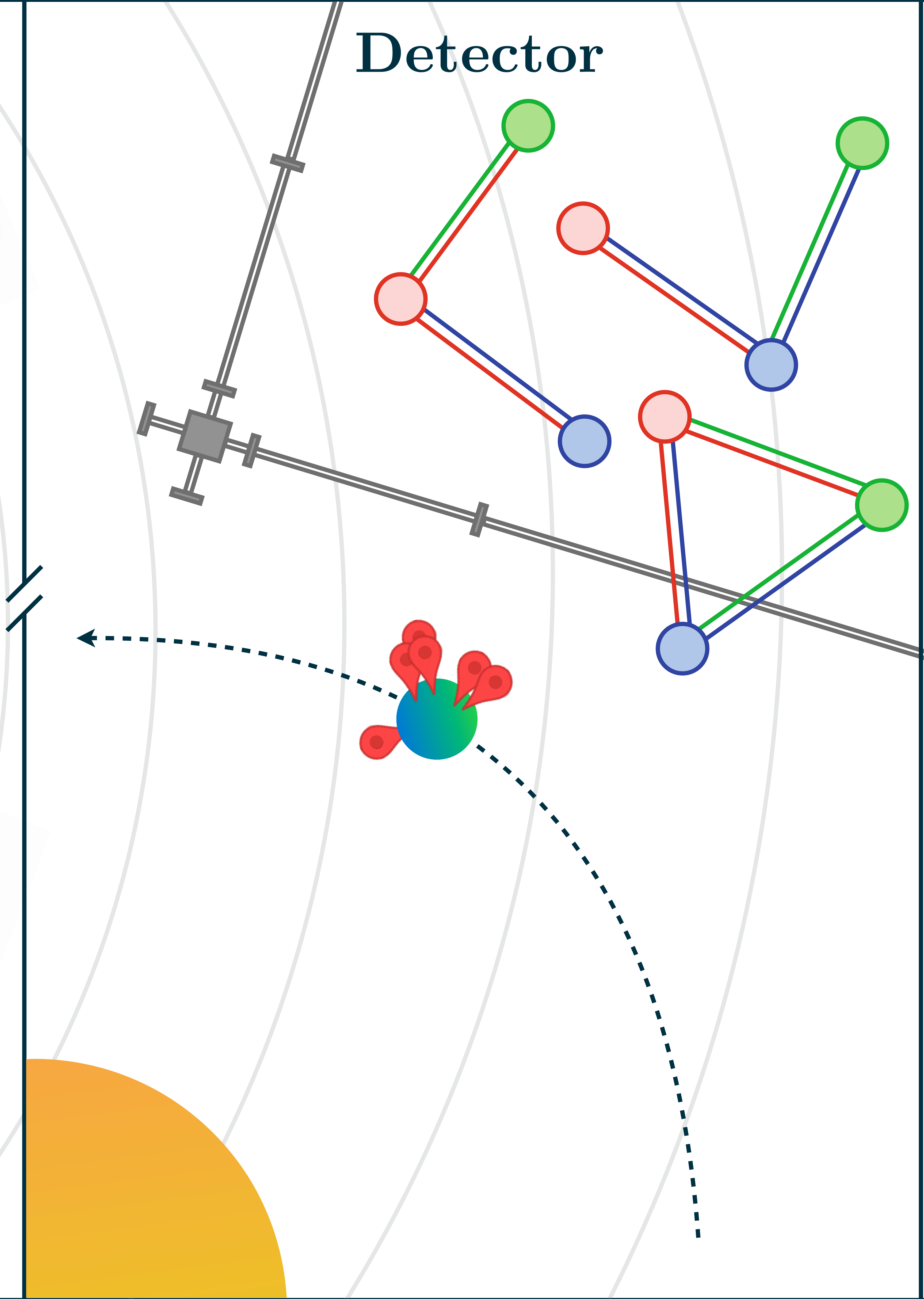
Emission



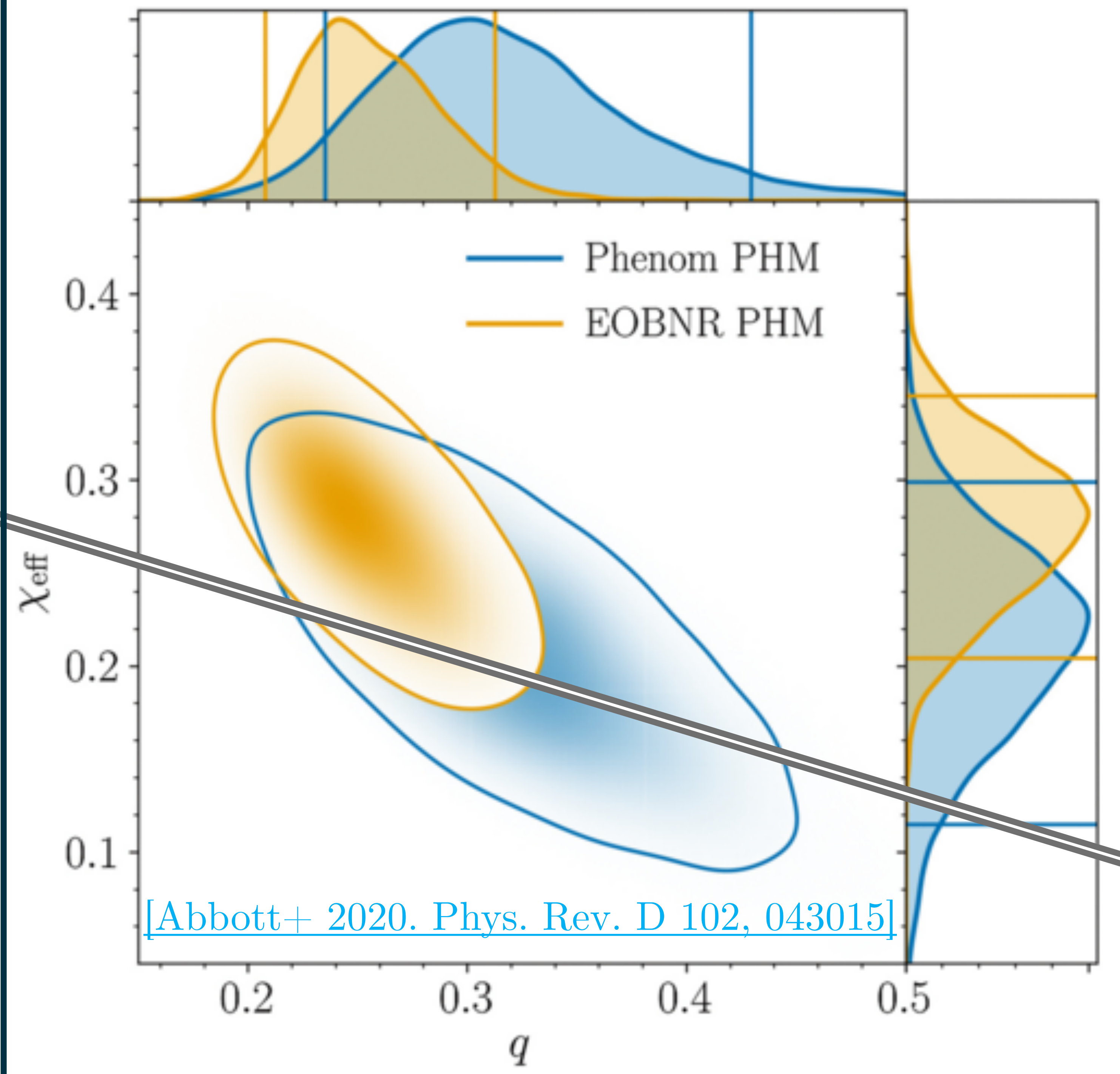
Model



Detector



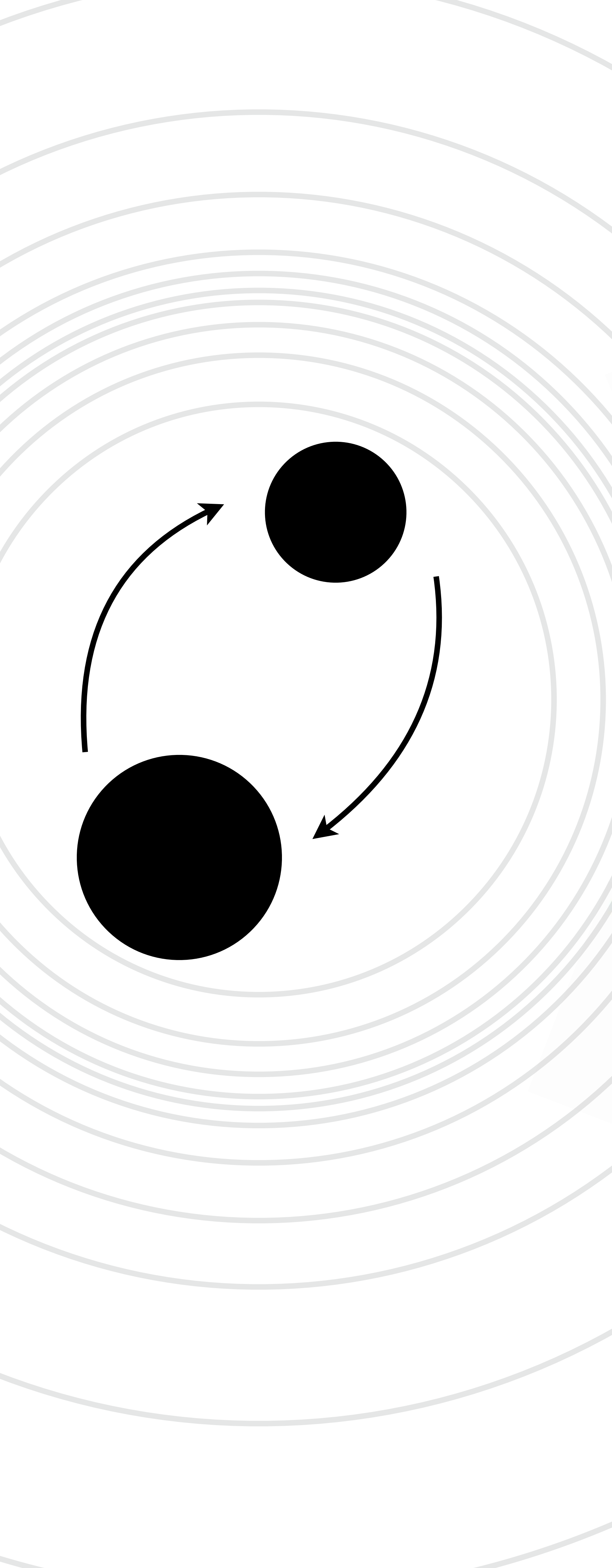
Parameter estimation



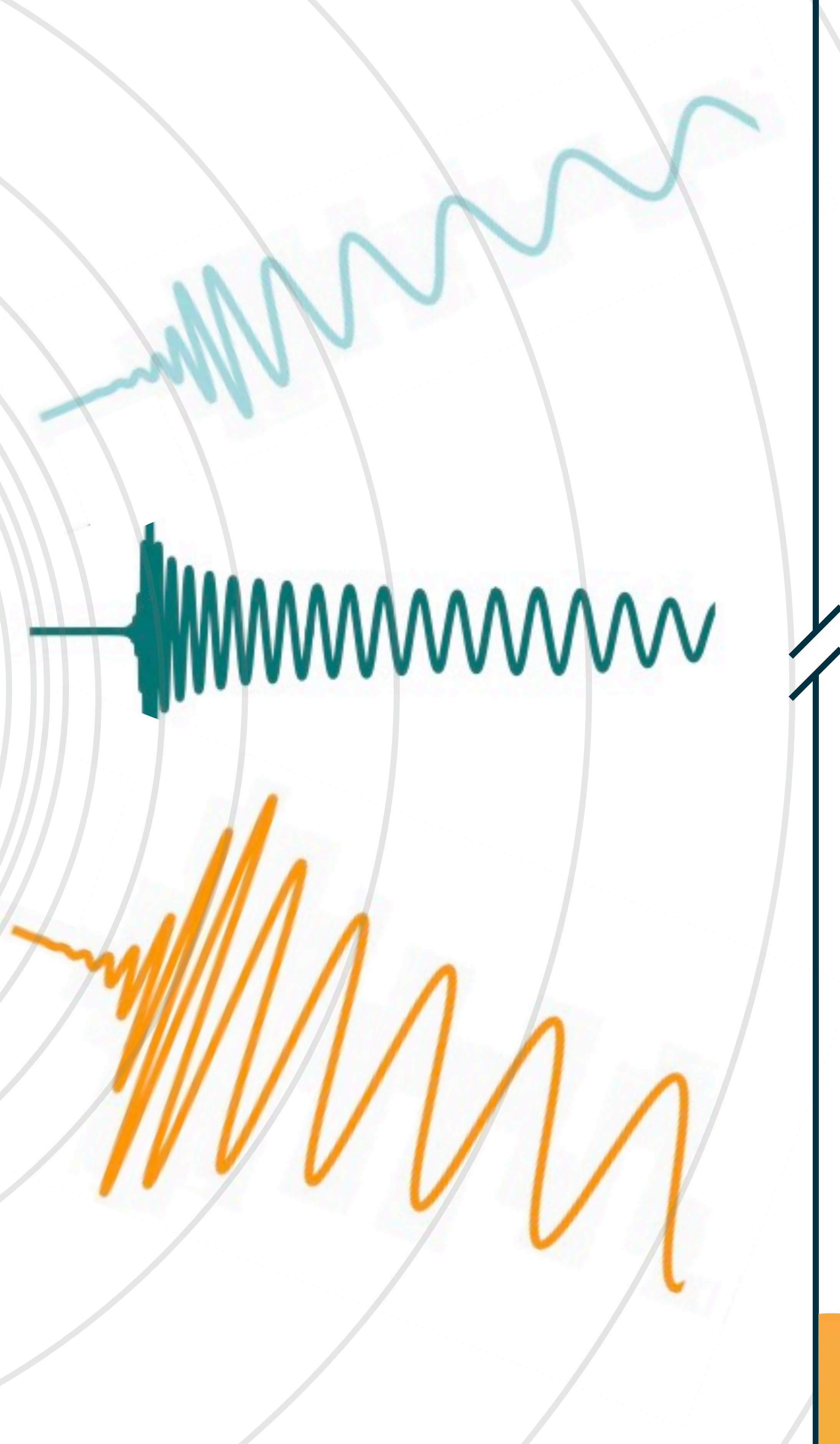
[Abbott+ 2020. Phys. Rev. D 102, 043015]

Motivation: Data analysis pipeline

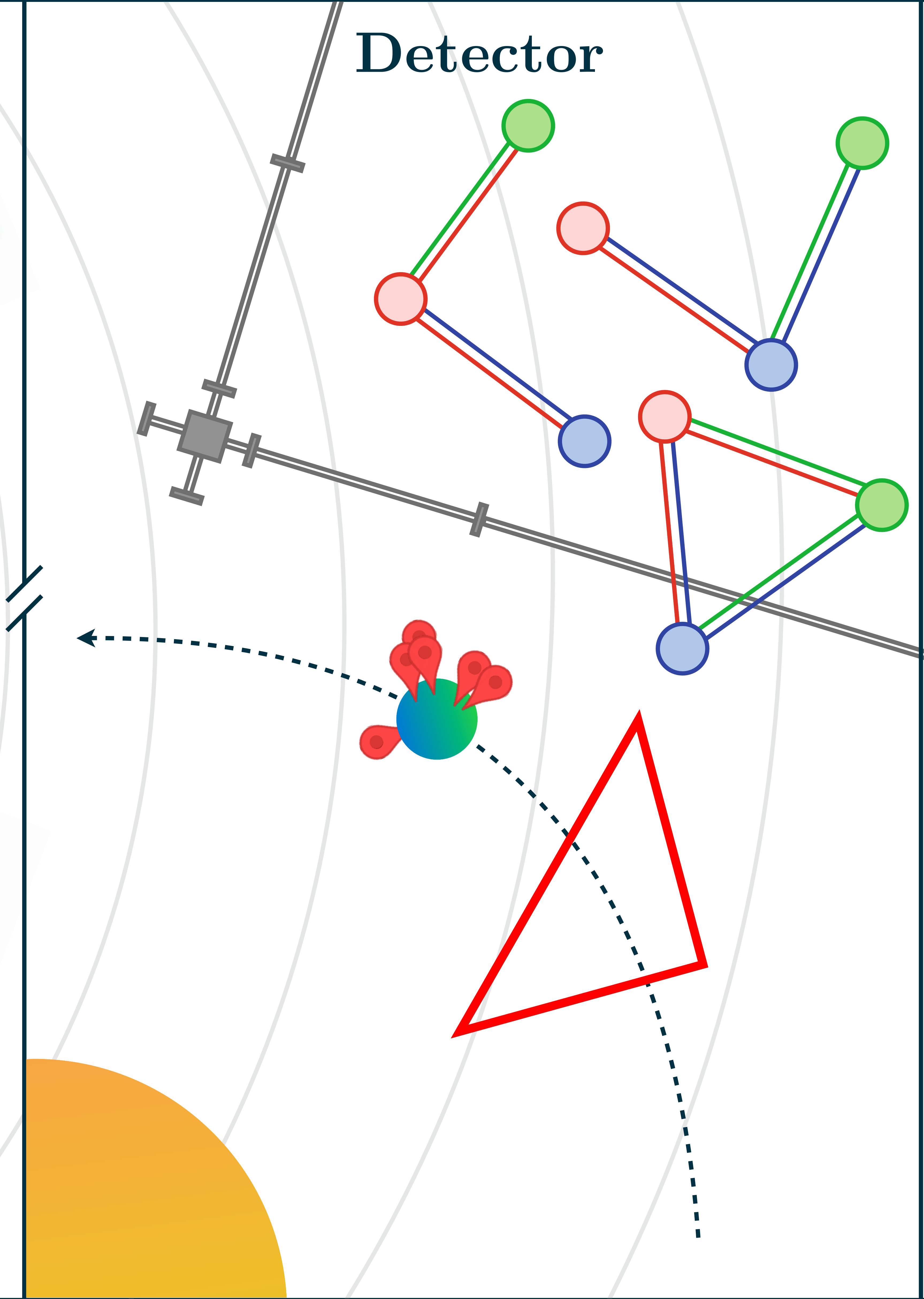
Emission



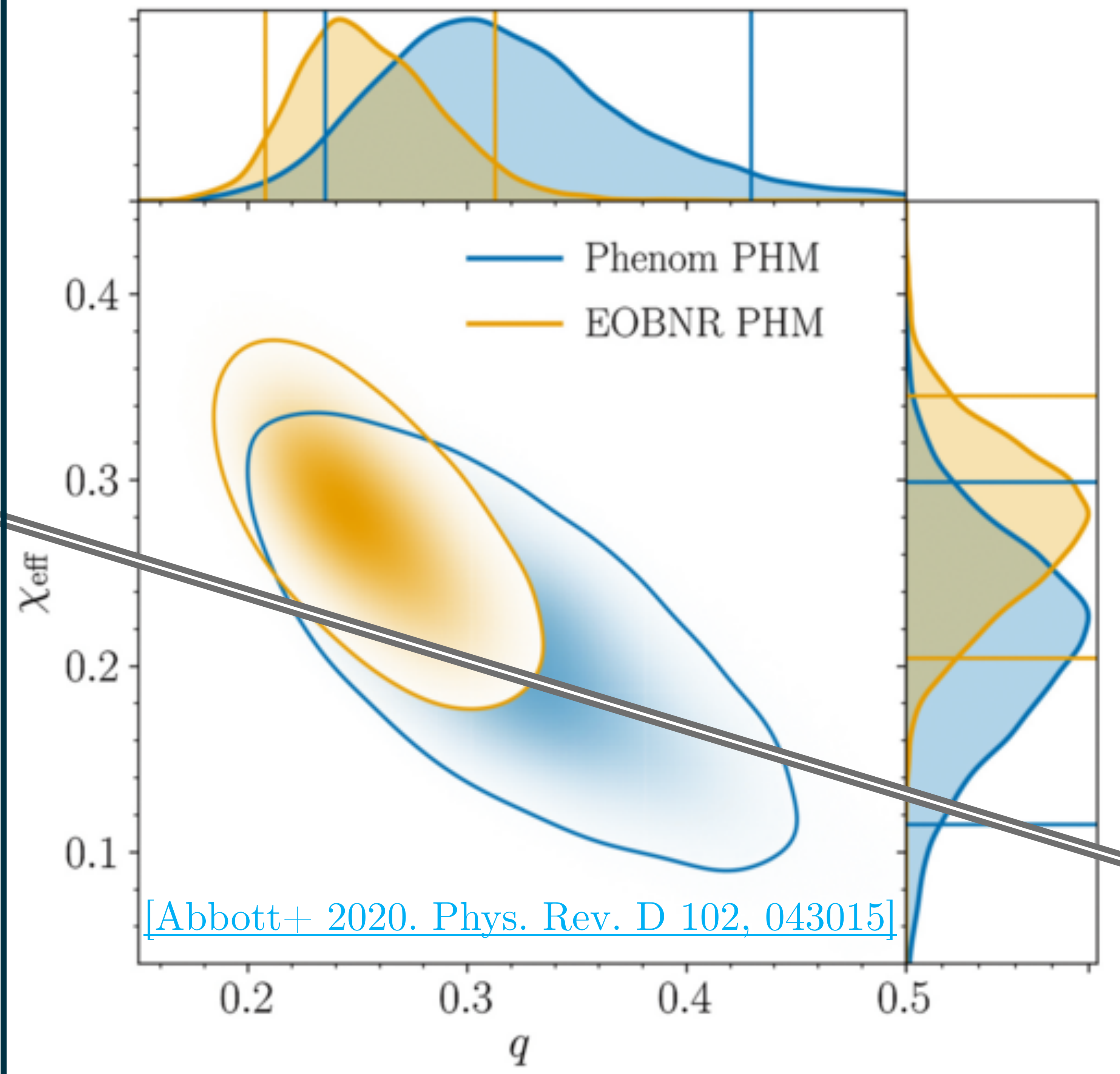
Model



Detector



Parameter estimation



[Abbott+ 2020, Phys. Rev. D 102, 043015]

Motivation: Detector Response

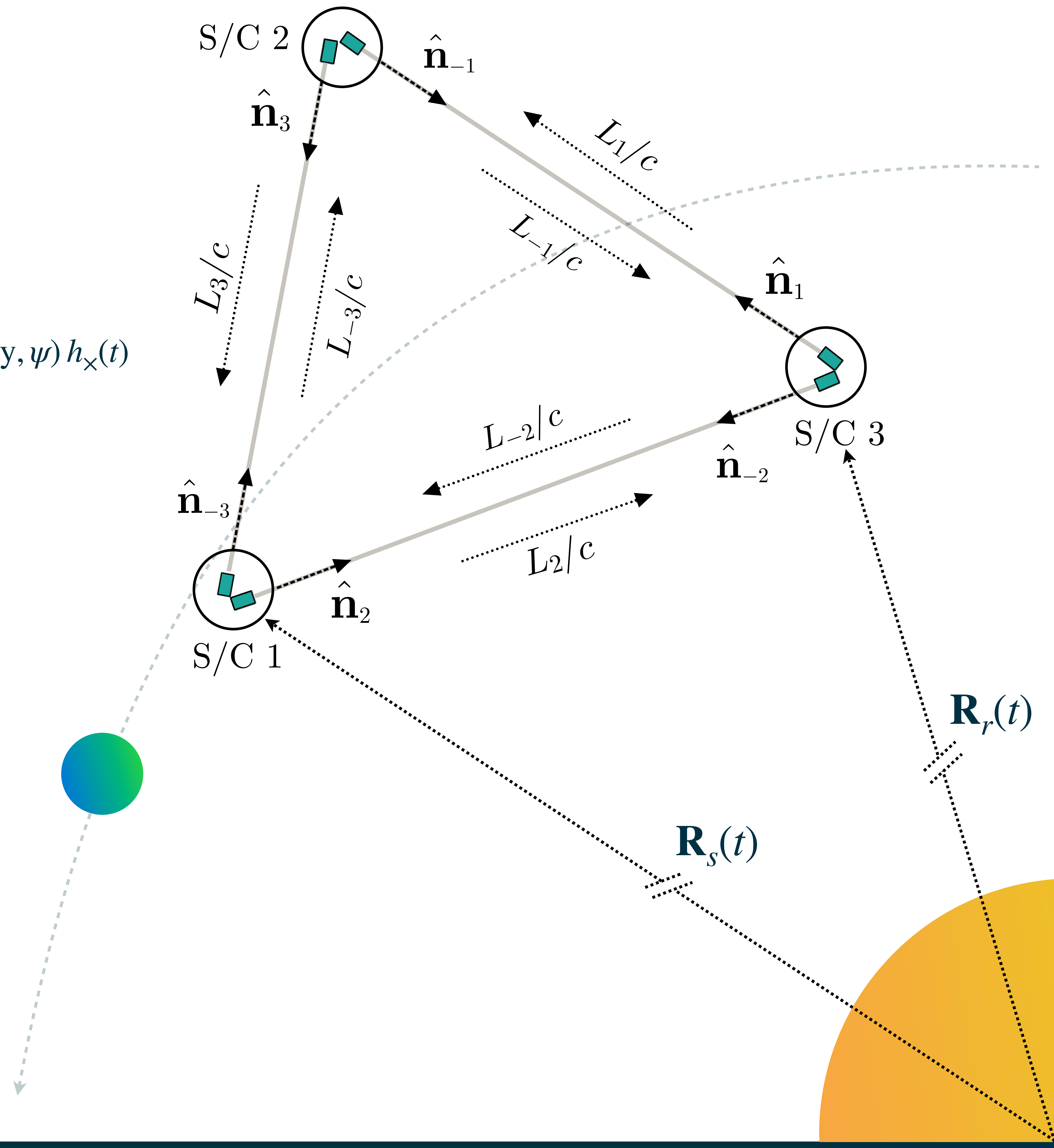
CBCs & Current ground-based detectors: → $\boxed{h(t) = F_{+}(\text{sky}, \psi)\, h_{+}(t) + F_{\times}(\text{sky}, \psi)\, h_{\times}(t) \ \text{Ready for DA}}$

Motivation: Detector Response

CBCs & Current ground-based detectors: $\boxed{h(t) = F_+(\text{sky},\psi)\, h_+(t) + F_\times(\text{sky},\psi)\, h_\times(t)\ \text{Ready for DA}}$

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))} \quad \text{with} \quad \left| \begin{aligned} &\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_s(t)}{c}, \\ &\delta_r(t) = \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_r(t)}{c}, \\ &H_l(t) = F_l^+(t;\text{sky},\psi)\, h_+(t) + F_l^\times(t;\text{sky},\psi)\, h_\times(t) \end{aligned} \right.$$

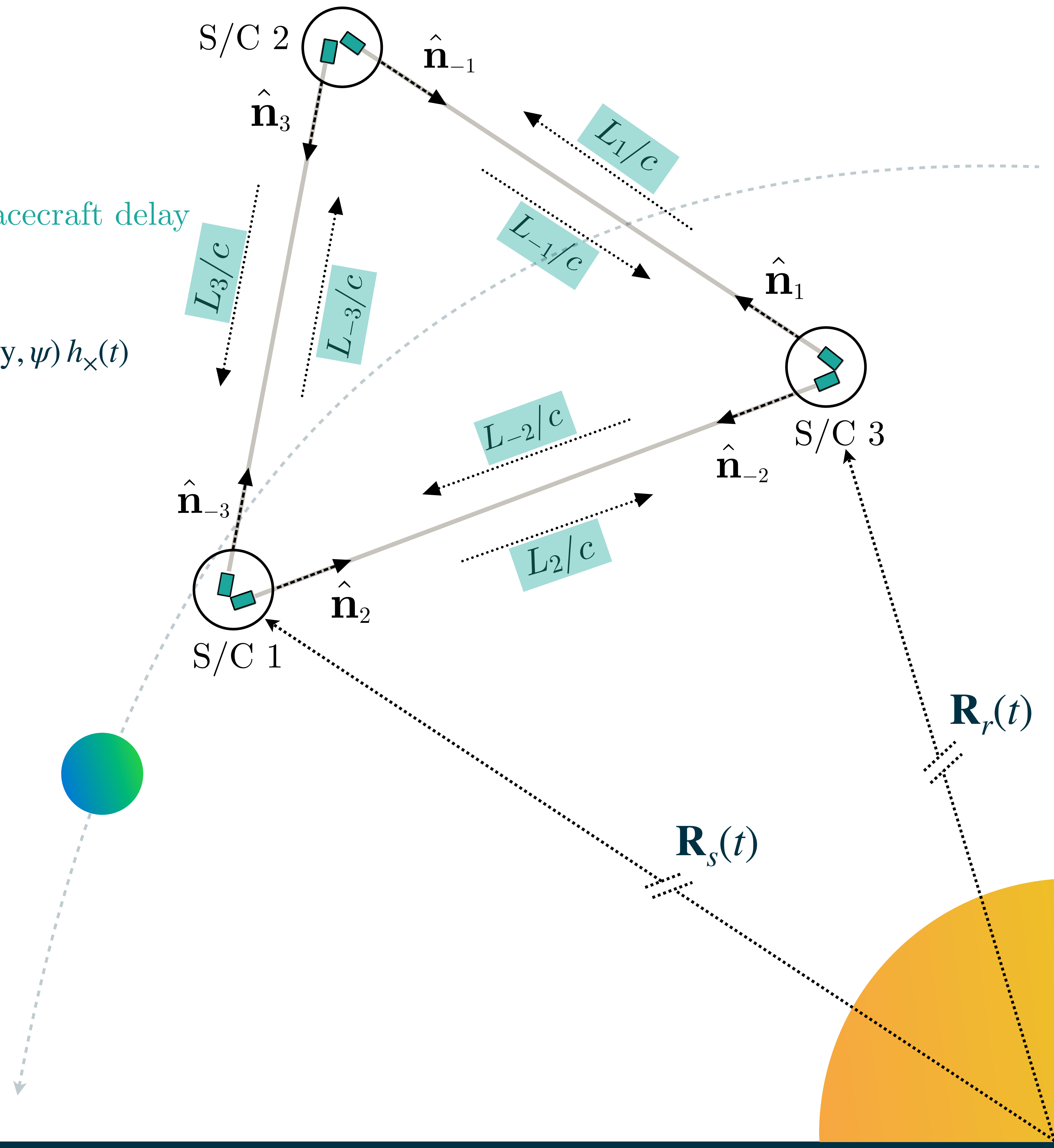


CBCs & Current ground-based detectors: → $h(t) = F_+(\text{sky}, \psi)\, h_+(t) + F_\times(\text{sky}, \psi)\, h_\times(t)$ Ready for DA

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))} \quad \text{with} \quad \begin{cases} \delta_s(t) = \boxed{\dfrac{L_l(t)}{c}} + \dfrac{\hat{\mathbf{k}} \cdot \mathbf{R}_s(t)}{c}, \\ \delta_r(t) = \dfrac{\hat{\mathbf{k}} \cdot \mathbf{R}_r(t)}{c}, \\ H_l(t) = F_l^+(t; \text{sky}, \psi)\, h_+(t) + F_l^\times(t; \text{sky}, \psi)\, h_\times(t) \end{cases}$$

Inter-spacecraft delay



Motivation: Detector Response

CBCs & Current ground-based detectors: $h(t) = F_+(\text{sky}, \psi)\, h_+(t) + F_\times(\text{sky}, \psi)\, h_\times(t)$ Ready for DA

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))}$$

with

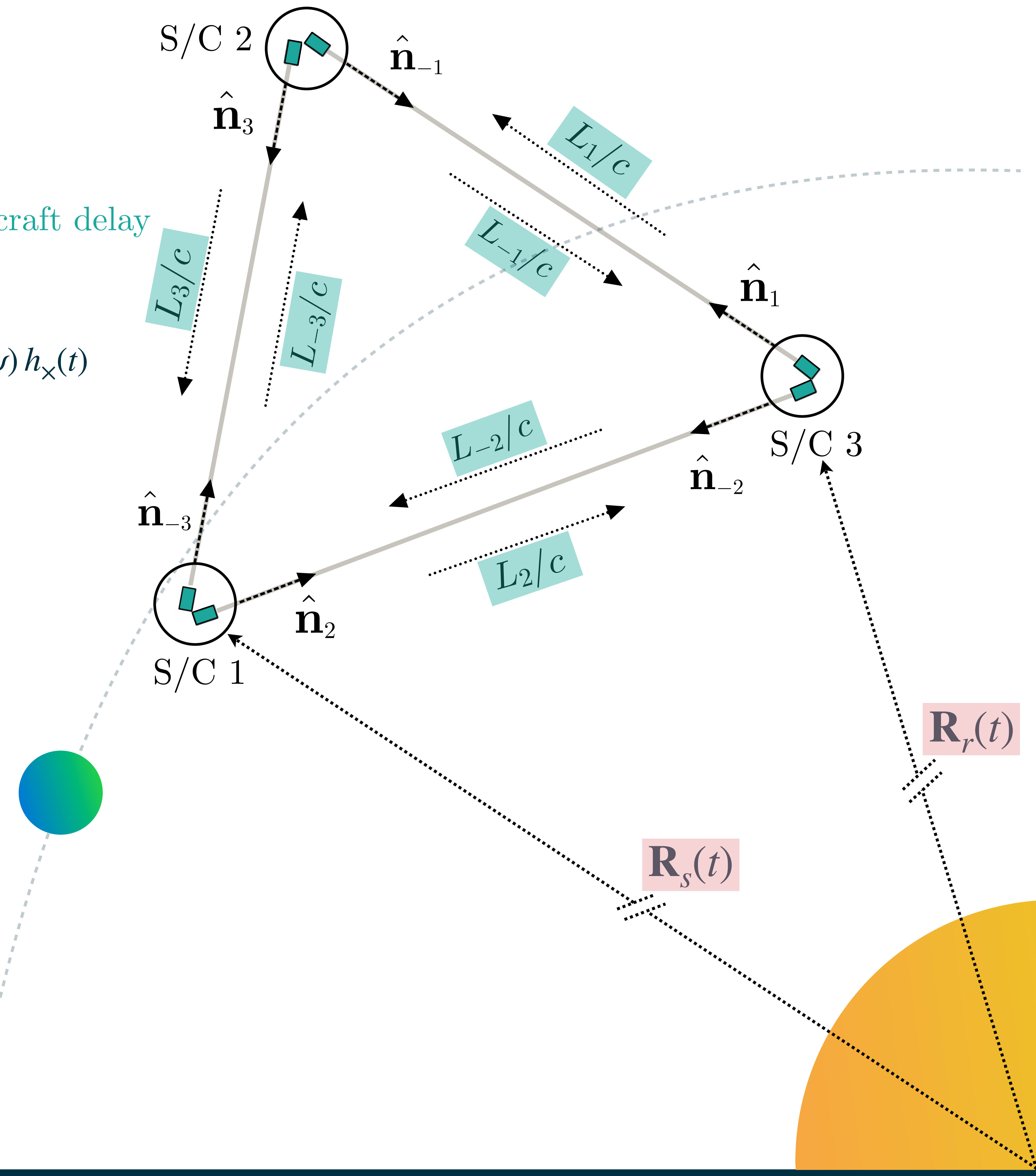
$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_s(t)}{c},$$

$$\delta_r(t) = \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_r(t)}{c},$$

$$H_l(t) = F_l^+(t; \text{sky}, \psi)\, h_+(t) + F_l^\times(t; \text{sky}, \psi)\, h_\times(t)$$

Inter-spacecraft delay

SSB delay



MOTIVATION: DETECTOR RESPONSE

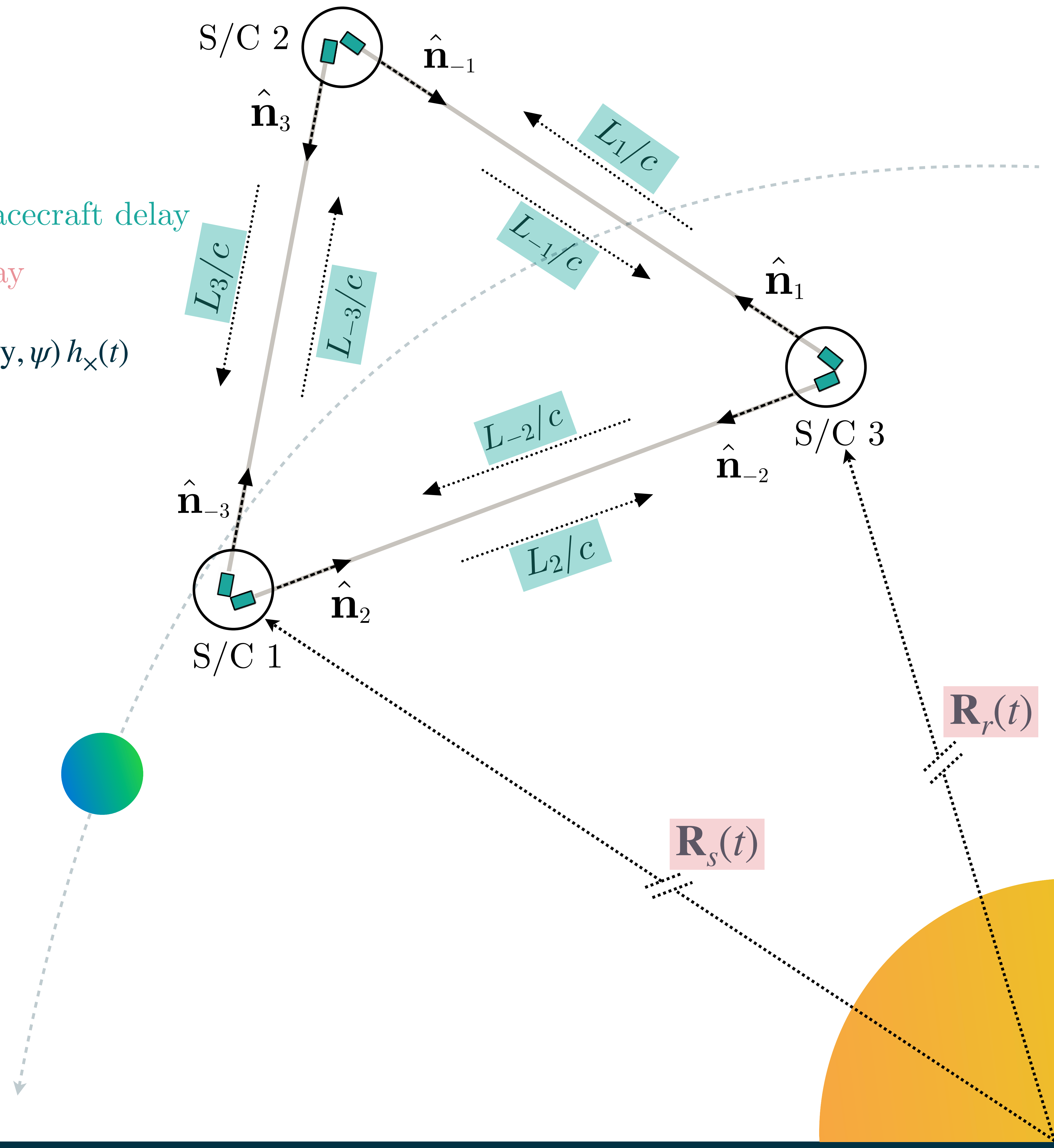
CBCs & Current ground-based detectors: $h(t) = F_+(\text{sky}, \psi)\, h_+(t) + F_\times(\text{sky}, \psi)\, h_\times(t)$ Ready for DA

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))} \quad \text{with} \quad \begin{aligned} \delta_s(t) &= \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_s(t)}{c}, \\ \delta_r(t) &= \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_r(t)}{c}, \\ H_l(t) &= F_l^+(t; \text{sky}, \psi)\, h_+(t) + F_l^\times(t; \text{sky}, \psi)\, h_\times(t) \end{aligned}$$

Inter-spacecraft delay

SSB delay



Setting $c = 1$ and dropping the time dependence on the delays for simplicity, the X Michelson combination are

$$\begin{aligned} X_{1.5}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3) \\ & - y_{1-32}(t - L_3) \\ & + y_{231}(t - L_2 - L_{-2}) \\ & - y_{231}(t) \\ & + y_{123}(t - L_{-2}) \\ & - y_{123}(t - L_{-2} - L_{-3} - L_3) \\ & + y_{3-21}(t) \\ & - y_{3-21}(t - L_{-3} - L_3)\,, \end{aligned}$$

$$\begin{aligned} X_{2.0}(t) = {} & X_{1.5}(t) \\ & + y_{1-32}(t - L_{-2} - L_2 - 2L_3 - L_{-3}) \\ & - y_{1-32}(t - 2L_3 - L_{-3} - 2L_{-2} - 2L_2) \\ & + y_{231}(t - L_{-2} - L_2 - L_3 - L_{-3}) \\ & - y_{231}(t - L_3 - L_{-3} - 2L_{-2} - 2L_2) \\ & + y_{123}(t - 2L_{-2} - L_2 - 2L_3 - 2L_{-3}) \\ & - y_{123}(t - L_3 - L_{-3} - 2L_{-2} - L_2) \\ & + y_{3-21}(t - L_{-2} - L_2 - 2L_3 - 2L_{-3}) \\ & - y_{3-21}(t - L_3 - L_{-3} - L_{-2} - L_2) \end{aligned}$$

Motivation: Detector Response

CBCs & Current ground-based detectors: $h(t) = F_+(\text{sky},\psi)\, h_+(t) + F_\times(\text{sky},\psi)\, h_\times(t)$ Ready for DA

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))} \quad \text{with}$$

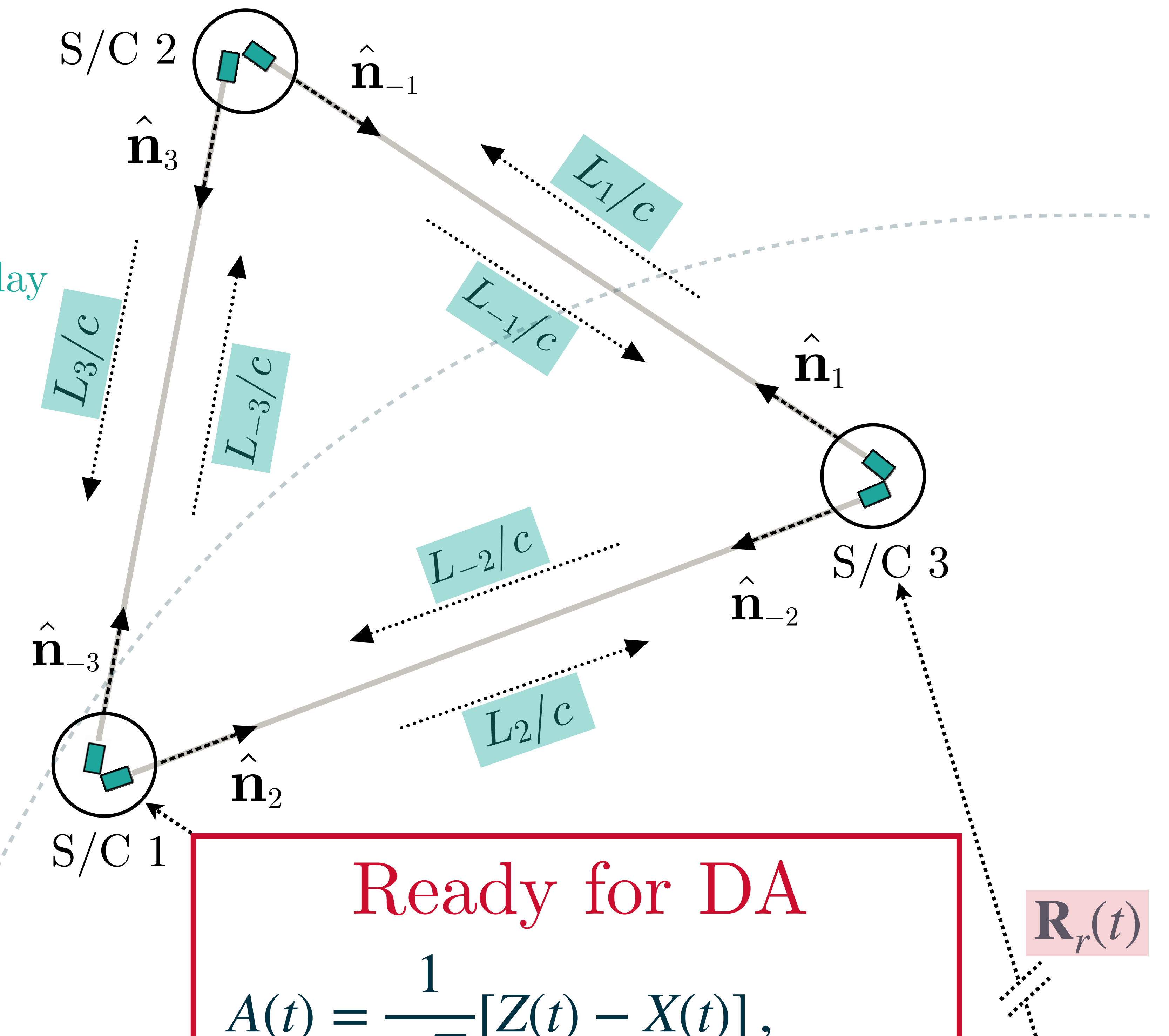
$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_s(t)}{c},$$

$$\delta_r(t) = \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_r(t)}{c},$$

$$H_l(t) = F_l^+(t;\text{sky},\psi)\, h_+(t) + F_l^\times(t;\text{sky},\psi)\, h_\times(t)$$

Inter-spacecraft delay

SSB delay



Setting $c=1$ and dropping the time dependence on the delays for simplicity, the X Michelson combination are

$$\begin{aligned}
X_{1.5}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3) \\
& -y_{1-32}(t-L_3) \\
& +y_{231}(t-L_2-L_{-2}) \\
& -y_{231}(t) \\
& +y_{123}(t-L_{-2}) \\
& -y_{123}(t-L_{-2}-L_{-3}-L_3) \\
& +y_{3-21}(t) \\
& -y_{3-21}(t-L_{-3}-L_3)\,,
\end{aligned}$$

$$\begin{aligned}
X_{2.0}(t) = {} & X_{1.5}(t) \\
& +y_{1-32}(t-L_{-2}-L_2-2L_3-L_{-3}) \\
& -y_{1-32}(t-2L_3-L_{-3}-2L_{-2}-2L_2) \\
& +y_{231}(t-L_{-2}-L_2-L_3-L_{-3}) \\
& -y_{231}(t-L_3-L_{-3}-2L_{-2}-2L_2) \\
& +y_{123}(t-2L_{-2}-L_2-2L_3-2L_{-3}) \\
& -y_{123}(t-L_3-L_{-3}-2L_{-2}-L_2) \\
& +y_{3-21}(t-L_{-2}-L_2-2L_3-2L_{-3}) \\
& -y_{3-21}(t-L_3-L_{-3}-L_{-2}-L_2)
\end{aligned}$$

Ready for DA

$$A(t) = \frac{1}{\sqrt{2}}[Z(t)-X(t)]\,,$$

$$E(t) = \frac{1}{\sqrt{6}}[X(t)-2Y(t)+Z(t)]\,,$$

$$T(t) = \frac{1}{\sqrt{3}}[X(t)+Y(t)+Z(t)]\,.$$

Motivation: Detector Response

CBCs & Current ground-based detectors: $h(t) = F_+(\mathrm{sky}, \psi)\, h_+(t) + F_\times(\mathrm{sky}, \psi)\, h_\times(t)$ Ready for DA

LISA: The relative laser frequency shifts between spacecraft are related to the GW polarizations as:

$$y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))} \quad \text{with}$$

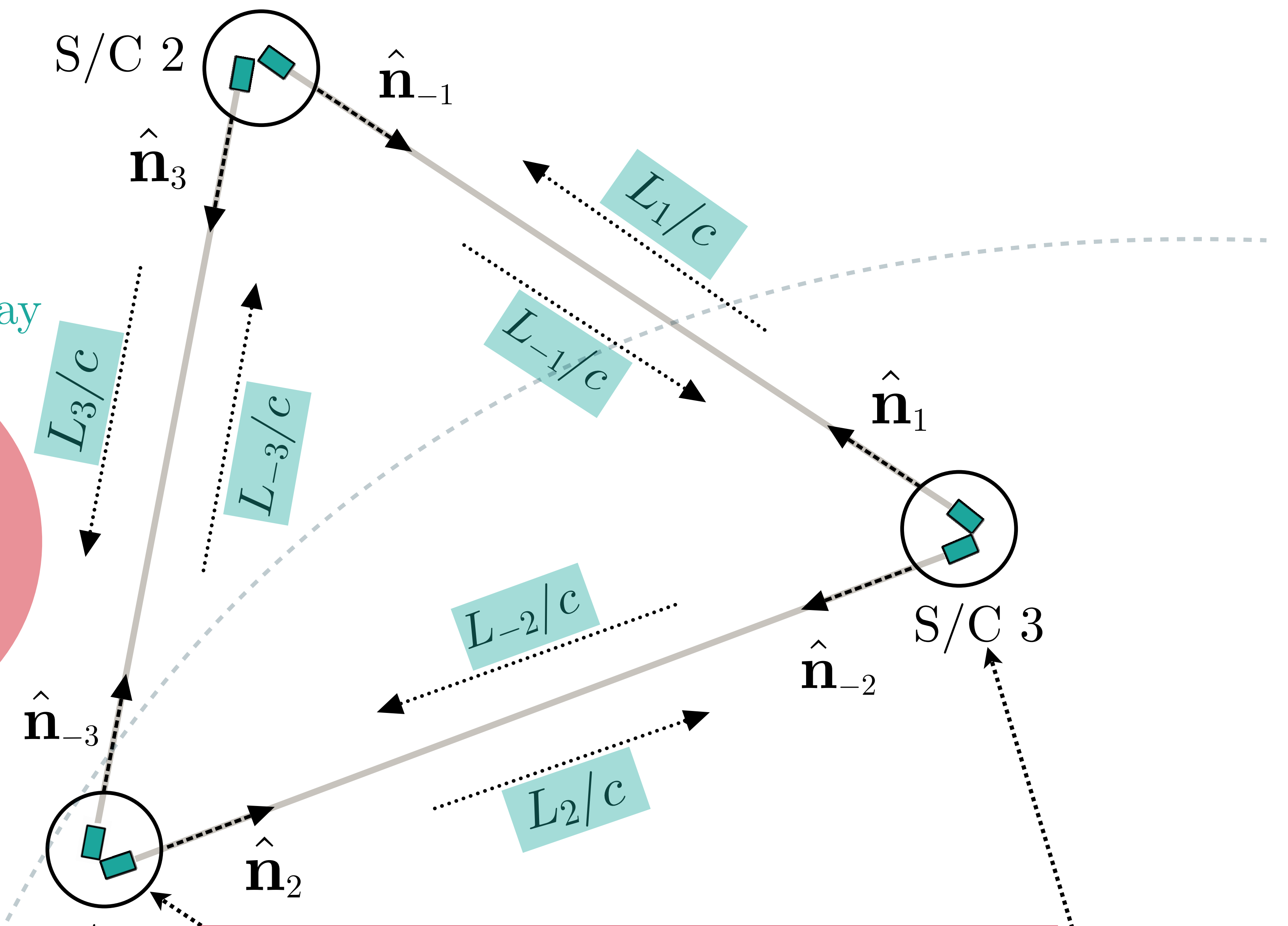
$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_s(t)}{c},$$

$$\delta_r(t) = \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_r(t)}{c},$$

$$H_l(t) = F_l^+(t; \mathrm{sky}, \psi)\, h_+(t) + F_l^\times(t; \mathrm{sky}, \psi)\, h_\times(t)$$

Inter-spacecraft delay

SSB delay



$\mathbf{R}_r(t)$

Setting $c = 1$ and dropping the time dependence on the delays for simplicity, the X Michelson combination are

$$\begin{aligned}
X_{1.5}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3) \\
& - y_{1-32}(t - L_3) \\
& + y_{231}(t - L_2 - L_{-2}) \\
& - y_{231}(t) \\
& + y_{123}(t - L_{-2}) \\
& - y_{123}(t - L_{-2} - L_{-3} - L_3) \\
& + y_{3-21}(t) \\
& - y_{3-21}(t - L_{-3} - L_3)\,,
\end{aligned}$$

$$\begin{aligned}
X_{2.0}(t) = {} & X_{1.5}(t) \\
& + y_{1-32}(t - L_{-2} - L_2 - 2L_3 - L_{-3}) \\
& - y_{1-32}(t - 2L_3 - L_{-3} - 2L_{-2} - 2L_2) \\
& + y_{231}(t - L_{-2} - L_2 - L_3 - L_{-3}) \\
& - y_{231}(t - L_3 - L_{-3} - 2L_{-2} - 2L_2) \\
& + y_{123}(t - 2L_{-2} - L_2 - 2L_3 - 2L_{-3}) \\
& - y_{123}(t - L_3 - L_{-3} - 2L_{-2} - L_2) \\
& + y_{3-21}(t - L_{-2} - L_2 - 2L_3 - 2L_{-3}) \\
& - y_{3-21}(t - L_3 - L_{-3} - L_{-2} - L_2)
\end{aligned}$$

Ready for DA

$$A(t) = \frac{1}{\sqrt{2}}[Z(t) - X(t)]\,,$$

$$E(t) = \frac{1}{\sqrt{6}}[X(t) - 2Y(t) + Z(t)]\,,$$

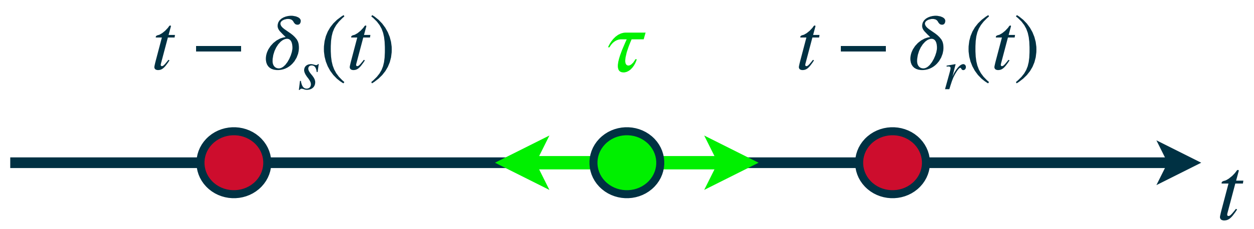
$$T(t) = \frac{1}{\sqrt{3}}[X(t) + Y(t) + Z(t)]\,.$$

LFA: SINGLE-LINK MEASUREMENTS

The laser frequency shifts between spacecraft are related to the time **derivatives** of the strain as:

$$y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))}$$

$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_s(t)}{c}, \qquad \delta_r(t) = \frac{\hat{\mathbf{k}} \cdot \mathbf{R}_r(t)}{c},$$



Finite difference scheme

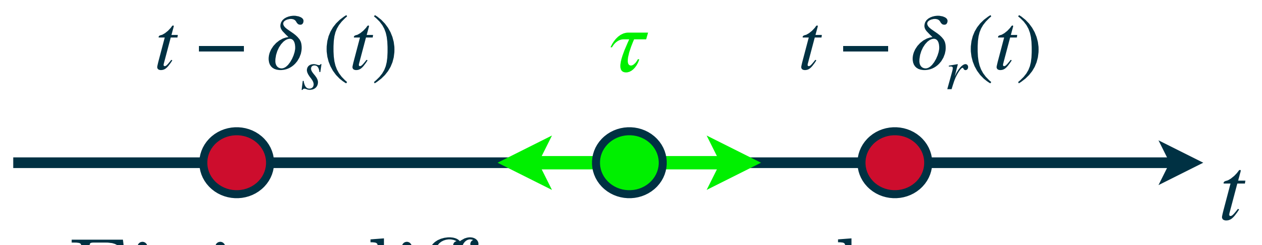
$\dot{H}_l(\tau)$ but... what is τ?

LFA: SINGLE-LINK MEASUREMENTS

The laser frequency shifts between spacecraft are related to the time **derivatives** of the strain as:

$$y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))}$$

$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_s(t)}{c}, \qquad \delta_r(t) = \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_r(t)}{c},$$



Finite difference scheme

$\dot{H}_l(\tau)$ but... what is τ?

τ	Finite difference type	Single-link measurement (leading order)
$\tau = t-\delta_s(t)$	Backward	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\delta_s(t)\right) + \boxed{\mathcal{O}\left[\frac{L_l\dot{L}_l}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^2}{c^2}\ddot{H}_l\right]}$
$\tau = t-\delta_r(t)$	Forward	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\delta_r(t)\right) + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^2}{c^2}\ddot{H}_l\right]}$ Eq.(51) of [Babak+ 2021]
$\tau = t-\frac{\delta_s(t)+\delta_r(t)}{2}$	Central	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\frac{\delta_s(t)+\delta_r(t)}{2}\right) + \boxed{\mathcal{O}\left[\frac{L_l\dot{L}_l}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^3}{c^3}\dddot{H}_l\right]}$

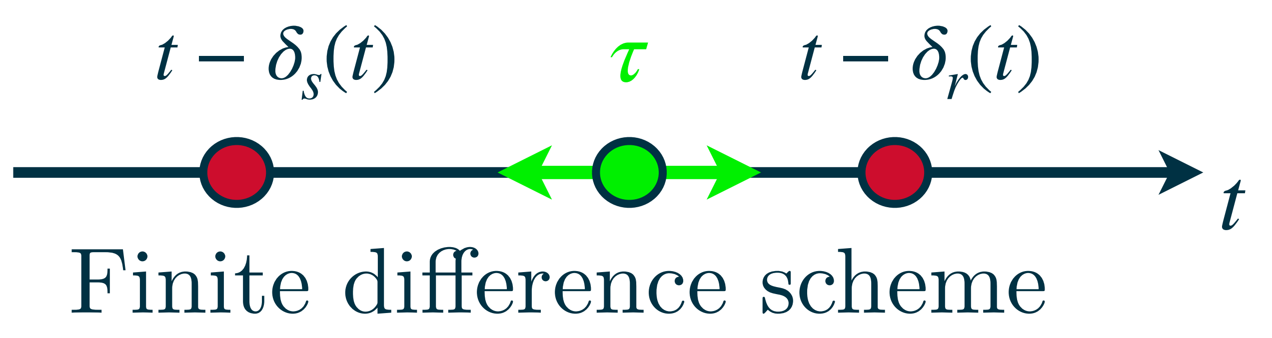
Error from $\frac{\mathrm{d}}{\mathrm{d}\tau} \approx \frac{\mathrm{d}}{\mathrm{d}t}$

Error from Taylor series truncation

LFA: SINGLE-LINK MEASUREMENTS

The <u>laser frequency shifts</u> between spacecraft are related to the time **derivatives** of the strain as:

$$y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))}$$



$\dot{H}_l(\tau)$ but... what is τ?

$$\delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_s(t)}{c}, \qquad \delta_r(t) = \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_r(t)}{c},$$

τ	Finite difference type	Single-link measurement (leading order)
$\tau = t-\delta_s(t)$	Backward	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\delta_s(t)\right) + \boxed{\mathcal{O}\left[\frac{L_l\dot{L}_l}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^2}{c^2}\ddot{H}_l\right]}$
$\tau = t-\delta_r(t)$	Forward	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\delta_r(t)\right) + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^2}{c^2}\ddot{H}_l\right]}$ Eq.(51) of [Babak+ 2021]
$\tau = t-\frac{\delta_s(t)+\delta_r(t)}{2}$	Central	$y_{slr}(t) = -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\frac{\delta_s(t)+\delta_r(t)}{2}\right) + \boxed{\mathcal{O}\left[\frac{L_l\dot{L}_l}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l\lVert\dot{\mathbf{R}}_{r,s}\rVert}{c^2}\dot{H}_l\right]} + \boxed{\mathcal{O}\left[\frac{L_l^3}{c^3}\dddot{H}_l\right]}$

Error from $\frac{\mathrm{d}}{\mathrm{d}\tau} \approx \frac{\mathrm{d}}{\mathrm{d}t}$

Error from Taylor series truncation

Large for high frequencies!

LFA: Michelson combinations

Laser frequency shifts: $y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t - \frac{\delta_s(t)+\delta_r(t)}{2}\right) \sim \dot{h}_{+,\times}$

- Laser frequency shifts: $y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t - \frac{\delta_s(t)+\delta_r(t)}{2}\right) \sim \dot{h}_{+,\times}$

- We apply a similar procedure to the Michelson combinations $X(t)$, $Y(t)$, and $Z(t)$:

$$
\begin{aligned}
X_{1.5}(t) = y_{1-32}&(t - L_{-2} - L_2 - L_3)\\
&-y_{1-32}(t - L_3)\\
&+y_{231}(t - L_2 - L_{-2})\\
&-y_{231}(t)\\
&+y_{123}(t - L_{-2})\\
&-y_{123}(t - L_{-2} - L_{-3} - L_3)\\
&+y_{3-21}(t)\\
&-y_{3-21}(t - L_{-3} - L_3)\,,
\end{aligned}
$$

$$
\begin{aligned}
X_{2.0}(t) = y_{1-32}&(t - L_{-2} - L_2 - L_3)\\
&-y_{1-32}(t - L_3)\\
&+y_{231}(t - L_2 - L_{-2})\\
&-y_{231}(t)\\
&+y_{123}(t - L_{-2})\\
&-y_{123}(t - L_{-2} - L_{-3} - L_3)\\
&+y_{3-21}(t)\\
&-y_{3-21}(t - L_{-3} - L_3)\,,\\
&+y_{1-32}(t - L_{-2} - L_2 - 2L_3 - L_{-3})\\
&-y_{1-32}(t - 2L_3 - L_{-3} - 2L_{-2} - 2L_2)\\
&+y_{231}(t - L_{-2} - L_2 - L_3 - L_{-3})\\
&-y_{231}(t - L_3 - L_{-3} - 2L_{-2} - 2L_2)\\
&+y_{123}(t - 2L_{-2} - L_2 - 2L_3 - 2L_{-3})\\
&-y_{123}(t - L_3 - L_{-3} - 2L_{-2} - L_2)\\
&+y_{3-21}(t - L_{-2} - L_2 - 2L_3 - 2L_{-3})\\
&-y_{3-21}(t - L_3 - L_{-3} - L_{-2} - L_2)
\end{aligned}
$$

LFA: Michelson combinations

- Laser frequency shifts: $y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\frac{\delta_s(t)+\delta_r(t)}{2}\right) \sim \dot{h}_{+,\times}$
- We apply a similar procedure to the Michelson combinations $X(t)$, $Y(t)$, and $Z(t)$:

Unequal and time-dependent delays

$$X_{1.5}(t) \approx \frac{4}{c}\left[L_3(t)\dot{y}_{123}\left(t-\frac{L_3(t)}{c}-\frac{L_2(t)}{2c}\right)-L_2(t)\dot{y}_{231}\left(t-\frac{L_2(t)}{c}-\frac{L_3(t)}{2c}\right)\right] \sim \ddot{h}_{+,\times}$$

$$X_{2.0}(t) \approx \frac{8}{c}\left[\frac{L_2(t)+L_3(t)}{c}\right]\left[L_3(t)\ddot{y}_{123}\left(t-\frac{2L_3(t)}{c}-\frac{3L_2(t)}{2c}\right)-L_2(t)\ddot{y}_{231}\left(t-\frac{2L_2(t)}{c}-\frac{3L_3(t)}{2c}\right)\right] \sim \dddot{h}_{+,\times}$$

$$\begin{aligned}X_{1.5}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3)\\ & -y_{1-32}(t-L_3)\\ & +y_{231}(t-L_2-L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t-L_{-2})\\ & -y_{123}(t-L_{-2}-L_{-3}-L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t-L_{-3}-L_3)\,,\end{aligned}$$

$$\begin{aligned}X_{2.0}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3)\\ & -y_{1-32}(t-L_3)\\ & +y_{231}(t-L_2-L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t-L_{-2})\\ & -y_{123}(t-L_{-2}-L_{-3}-L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t-L_{-3}-L_3)\,,\\ & +y_{1-32}(t-L_{-2}-L_2-2L_3-L_{-3})\\ & -y_{1-32}(t-2L_3-L_{-3}-2L_{-2}-2L_2)\\ & +y_{231}(t-L_{-2}-L_2-L_3-L_{-3})\\ & -y_{231}(t-L_3-L_{-3}-2L_{-2}-2L_2)\\ & +y_{123}(t-2L_{-2}-L_2-2L_3-2L_{-3})\\ & -y_{123}(t-L_3-L_{-3}-2L_{-2}-L_2)\\ & +y_{3-21}(t-L_{-2}-L_2-2L_3-2L_{-3})\\ & -y_{3-21}(t-L_3-L_{-3}-L_{-2}-L_2)\end{aligned}$$

LFA: Michelson combinations

- Laser frequency shifts: $y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t-\frac{\delta_s(t)+\delta_r(t)}{2}\right) \sim \dot{h}_{+,\times}$

- We apply a similar procedure to the Michelson combinations $X(t)$, $Y(t)$, and $Z(t)$:

Unequal and time-dependent delays

$$X_{1.5}(t) \approx \frac{4}{c}\left[L_3(t)\dot{y}_{123}\left(t-\frac{L_3(t)}{c}-\frac{L_2(t)}{2c}\right)-L_2(t)\dot{y}_{231}\left(t-\frac{L_2(t)}{c}-\frac{L_3(t)}{2c}\right)\right] \sim \ddot{h}_{+,\times}$$

$$X_{2.0}(t) \approx \frac{8}{c}\left[\frac{L_2(t)+L_3(t)}{c}\right]\left[L_3(t)\ddot{y}_{123}\left(t-\frac{2L_3(t)}{c}-\frac{3L_2(t)}{2c}\right)-L_2(t)\ddot{y}_{231}\left(t-\frac{2L_2(t)}{c}-\frac{3L_3(t)}{2c}\right)\right] \sim \dddot{h}_{+,\times}$$

Equal and constant delays

$$X_{1.5}(t) \approx \frac{4L}{c}\left[\dot{y}_{123}\left(t-\frac{3L}{2c}\right)-\dot{y}_{231}\left(t-\frac{3L}{2c}\right)\right]$$

$$X_{2.0}(t) \approx \frac{16L^2}{c^2}\left[\ddot{y}_{123}\left(t-\frac{7L}{2c}\right)-\ddot{y}_{231}\left(t-\frac{7L}{2c}\right)\right]$$

$$\begin{aligned}X_{1.5}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3)\\ & -y_{1-32}(t-L_3)\\ & +y_{231}(t-L_2-L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t-L_{-2})\\ & -y_{123}(t-L_{-2}-L_{-3}-L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t-L_{-3}-L_3)\,,\end{aligned}$$

$$\begin{aligned}X_{2.0}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3)\\ & -y_{1-32}(t-L_3)\\ & +y_{231}(t-L_2-L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t-L_{-2})\\ & -y_{123}(t-L_{-2}-L_{-3}-L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t-L_{-3}-L_3)\,,\\ & +y_{1-32}(t-L_{-2}-L_2-2L_3-L_{-3})\\ & -y_{1-32}(t-2L_3-L_{-3}-2L_{-2}-2L_2)\\ & +y_{231}(t-L_{-2}-L_2-L_3-L_{-3})\\ & -y_{231}(t-L_3-L_{-3}-2L_{-2}-2L_2)\\ & +y_{123}(t-2L_{-2}-L_2-2L_3-2L_{-3})\\ & -y_{123}(t-L_3-L_{-3}-2L_{-2}-L_2)\\ & +y_{3-21}(t-L_{-2}-L_2-2L_3-2L_{-3})\\ & -y_{3-21}(t-L_3-L_{-3}-L_{-2}-L_2)\end{aligned}$$

LFA: Michelson combinations

- Laser frequency shifts: $y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t - \frac{\delta_s(t)+\delta_r(t)}{2}\right) \sim \dot{h}_{+,\times}$

- We apply a similar procedure to the Michelson combinations $X(t)$, $Y(t)$, and $Z(t)$:

Unequal and time-dependent delays

$$X_{1.5}(t) \approx \frac{4}{c}\left[L_3(t)\dot{y}_{123}\left(t - \frac{L_3(t)}{c} - \frac{L_2(t)}{2c}\right) - L_2(t)\dot{y}_{231}\left(t - \frac{L_2(t)}{c} - \frac{L_3(t)}{2c}\right)\right] \sim \ddot{h}_{+,\times}$$

$$X_{2.0}(t) \approx \frac{8}{c}\left[\frac{L_2(t)+L_3(t)}{c}\right]\left[L_3(t)\ddot{y}_{123}\left(t - \frac{2L_3(t)}{c} - \frac{3L_2(t)}{2c}\right) - L_2(t)\ddot{y}_{231}\left(t - \frac{2L_2(t)}{c} - \frac{3L_3(t)}{2c}\right)\right] \sim \dddot{h}_{+,\times}$$

Equal and constant delays

$$X_{1.5}(t) \approx \frac{4L}{c}\left[\dot{y}_{123}\left(t - \frac{3L}{2c}\right) - \dot{y}_{231}\left(t - \frac{3L}{2c}\right)\right]$$

$$X_{2.0}(t) \approx \frac{16L^2}{c^2}\left[\ddot{y}_{123}\left(t - \frac{7L}{2c}\right) - \ddot{y}_{231}\left(t - \frac{7L}{2c}\right)\right]$$

BUT the approximation is not accurate enough for high frequencies

Hybridize!

$$\begin{aligned}X_{1.5}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3)\\ & -y_{1-32}(t - L_3)\\ & +y_{231}(t - L_2 - L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t - L_{-2})\\ & -y_{123}(t - L_{-2} - L_{-3} - L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t - L_{-3} - L_3)\,,\end{aligned}$$

$$\begin{aligned}X_{2.0}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3)\\ & -y_{1-32}(t - L_3)\\ & +y_{231}(t - L_2 - L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t - L_{-2})\\ & -y_{123}(t - L_{-2} - L_{-3} - L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t - L_{-3} - L_3)\,,\\ & +y_{1-32}(t - L_{-2} - L_2 - 2L_3 - L_{-3})\\ & -y_{1-32}(t - 2L_3 - L_{-3} - 2L_{-2} - 2L_2)\\ & +y_{231}(t - L_{-2} - L_2 - L_3 - L_{-3})\\ & -y_{231}(t - L_3 - L_{-3} - 2L_{-2} - 2L_2)\\ & +y_{123}(t - 2L_{-2} - L_2 - 2L_3 - 2L_{-3})\\ & -y_{123}(t - L_3 - L_{-3} - 2L_{-2} - L_2)\\ & +y_{3-21}(t - L_{-2} - L_2 - 2L_3 - 2L_{-3})\\ & -y_{3-21}(t - L_3 - L_{-3} - L_{-2} - L_2)\end{aligned}$$

Hybrid Response

- Deviations from the LFA appear as the binary evolves toward the late inspiral and merger-ringdown phases.
- We combine the LFA approach with the full LISA response
- We use the LFA for the long inspiral and the full response for latest part of the waveform.
- Given a hybridization frequency $f_{\rm Hyb}$, we estimate the matching time with $t_{\rm Hyb} \approx t_c - f_{\rm Hyb}^{-8/3} \times \frac{5}{(8\pi)^{8/3}} \left(\frac{G\mathscr{M}_c}{c^3}\right)^{-5/3}$

Hybrid response

PhenomXPy
[García-Quirós+ 2025]

Hybrid Response

- Deviations from the LFA appear as the binary evolves toward the late inspiral and merger-ringdown phases.
- We combine the LFA approach with the full LISA response
- We use the LFA for the long inspiral and the full response for latest part of the waveform.
- Given a hybridization frequency $f_{\rm Hyb}$, we estimate the matching time with $t_{\rm Hyb} \approx t_c - f_{\rm Hyb}^{-8/3} \times \frac{5}{(8\pi)^{8/3}} \left(\frac{G\mathscr{M}_c}{c^3}\right)^{-5/3}$
- Hybridization is done at the level of the links y_{slr} and $X(t)$, $Y(t)$, $Z(t)$:

Hybrid response

PhenomXPY
[García-Quirós+ 2025]

$$y_{slr}(t) = \Sigma(t_{\rm LFA}; t_{\rm Hyb}, \sigma, +1) \times \boxed{y_{slr}^{\rm LFA}(t_{\rm LFA})} + \Sigma(t_{\rm Full}; t_{\rm Hyb}, \sigma, -1) \times \boxed{y_{slr}^{\rm Full}(t_{\rm Full})}$$

$$\boxed{y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t - \frac{\delta_s(t)+\delta_r(t)}{2}\right)}$$

$$\boxed{y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))}}$$

$$\Sigma(t; t_{\rm Hyb}, \sigma, \epsilon) = \left[1 + e^{\varepsilon(t-t_{\rm Hyb})/\sigma}\right]^{-1}$$

$$X(t) = \Sigma(t_{\rm LFA}; t_{\rm Hyb}, \sigma, +1) \times \boxed{X^{\rm LFA}(t_{\rm LFA})} + \Sigma(t_{\rm Full}; t_{\rm Hyb}, \sigma, -1) \times \boxed{X^{\rm Full}(t_{\rm Full})}$$

$$\boxed{\begin{aligned} X_{1.5}(t) &= \frac{4}{c}\left[L_3(t)\dot{y}_{123}\left(t - \frac{L_3(t)}{c} - \frac{L_2(t)}{2c}\right) - L_2(t)\dot{y}_{231}\left(t - \frac{L_2(t)}{c} - \frac{L_3(t)}{2c}\right)\right] \\ X_{1.5}(t) &= \frac{4L}{c}\left[\dot{y}_{123}\left(t-\frac{3L}{2c}\right) - \dot{y}_{231}\left(t - \frac{3L}{2c}\right)\right]\end{aligned}}$$

$$\boxed{\begin{aligned} X_{1.5}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3) \\ & - y_{1-32}(t - L_3) \\ & + y_{231}(t - L_2 - L_{-2}) \\ & - y_{231}(t) \\ & + y_{123}(t - L_{-2}) \\ & - y_{123}(t - L_{-2} - L_{-3} - L_3) \\ & + y_{3-21}(t) \\ & - y_{3-21}(t - L_{-3} - L_3), \end{aligned}}$$

Hybrid Response

- Deviations from the LFA appear as the binary evolves toward the late inspiral and merger-ringdown phases.
- We combine the LFA approach with the full LISA response
- We use the LFA for the long inspiral and the full response for latest part of the waveform.

Hybrid response

PHENOMXPY
[García-Quirós+ 2025]

- Given a hybridization frequency $f_{\rm Hyb}$, we estimate the matching time with $t_{\rm Hyb} \approx t_c - f_{\rm Hyb}^{-8/3} \times \frac{5}{(8\pi)^{8/3}} \left(\frac{G\mathcal{M}_c}{c^3}\right)^{-5/3}$
- Hybridization is done at the level of the links y_{slr} and $X(t)$, $Y(t)$, $Z(t)$:

$$y_{slr}(t) = \Sigma(t_{\rm LFA}; t_{\rm Hyb}, \sigma, +1) \times \boxed{y_{slr}^{\rm LFA}(t_{\rm LFA})} + \Sigma(t_{\rm Full}; t_{\rm Hyb}, \sigma, -1) \times \boxed{y_{slr}^{\rm Full}(t_{\rm Full})}$$

$$\boxed{y_{slr}(t) \approx -\frac{L_l(t)}{2c}\dot{H}_l\left(t - \frac{\delta_s(t) + \delta_r(t)}{2}\right)}$$

$$\boxed{y_{slr}(t) = \frac{H_l\left(t - \delta_s(t)\right) - H_l\left(t - \delta_r(t)\right)}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t))}}$$

$$\Sigma(t; t_{\rm Hyb}, \sigma, \epsilon) = \left[1 + e^{\varepsilon(t - t_{\rm Hyb})/\sigma}\right]^{-1}$$

$$X(t) = \Sigma(t_{\rm LFA}; t_{\rm Hyb}, \sigma, +1) \times \boxed{X^{\rm LFA}(t_{\rm LFA})} + \Sigma(t_{\rm Full}; t_{\rm Hyb}, \sigma, -1) \times \boxed{X^{\rm Full}(t_{\rm Full})}$$

$$\boxed{\begin{aligned} X_{1.5}(t) &= \frac{4}{c}\left[L_3(t)\dot{y}_{123}\left(t - \frac{L_3(t)}{c} - \frac{L_2(t)}{2c}\right) - L_2(t)\dot{y}_{231}\left(t - \frac{L_2(t)}{c} - \frac{L_3(t)}{2c}\right)\right] \\ X_{1.5}(t) &= \frac{4L}{c}\left[\dot{y}_{123}\left(t - \frac{3L}{2c}\right) - \dot{y}_{231}\left(t - \frac{3L}{2c}\right)\right] \end{aligned}}$$

$$\boxed{\begin{aligned} X_{1.5}(t) = {} & y_{1-32}(t - L_{-2} - L_2 - L_3) \\ & - y_{1-32}(t - L_3) \\ & + y_{231}(t - L_2 - L_{-2}) \\ & - y_{231}(t) \\ & + y_{123}(t - L_{-2}) \\ & - y_{123}(t - L_{-2} - L_{-3} - L_3) \\ & + y_{3-21}(t) \\ & - y_{3-21}(t - L_{-3} - L_3)\,, \end{aligned}}$$

- Relative portion of the waveform projected by the LFA: 95% (Full: 5%)

Response validation: test case

- PyIMRPhenomTHM [García-Quirós+ 2025] with
$$(l,m)=\{(2,\pm 2),(2,\pm 1),(3,\pm 3),(4,\pm 4),(5,\pm 5)\}$$
- Good agreement for the Hybrid and LFA during the early inspiral phase.
- Larger errors for LFA in the merger-ringdown.
- Hybrid = Full response in the merger-ringdown.
- LFA(inspiral) + Full(merger-ringdown) = Hybrid
- One order-of-magnitude improvement with unequal and time-dependent delays w.r.t equal and constant delays.

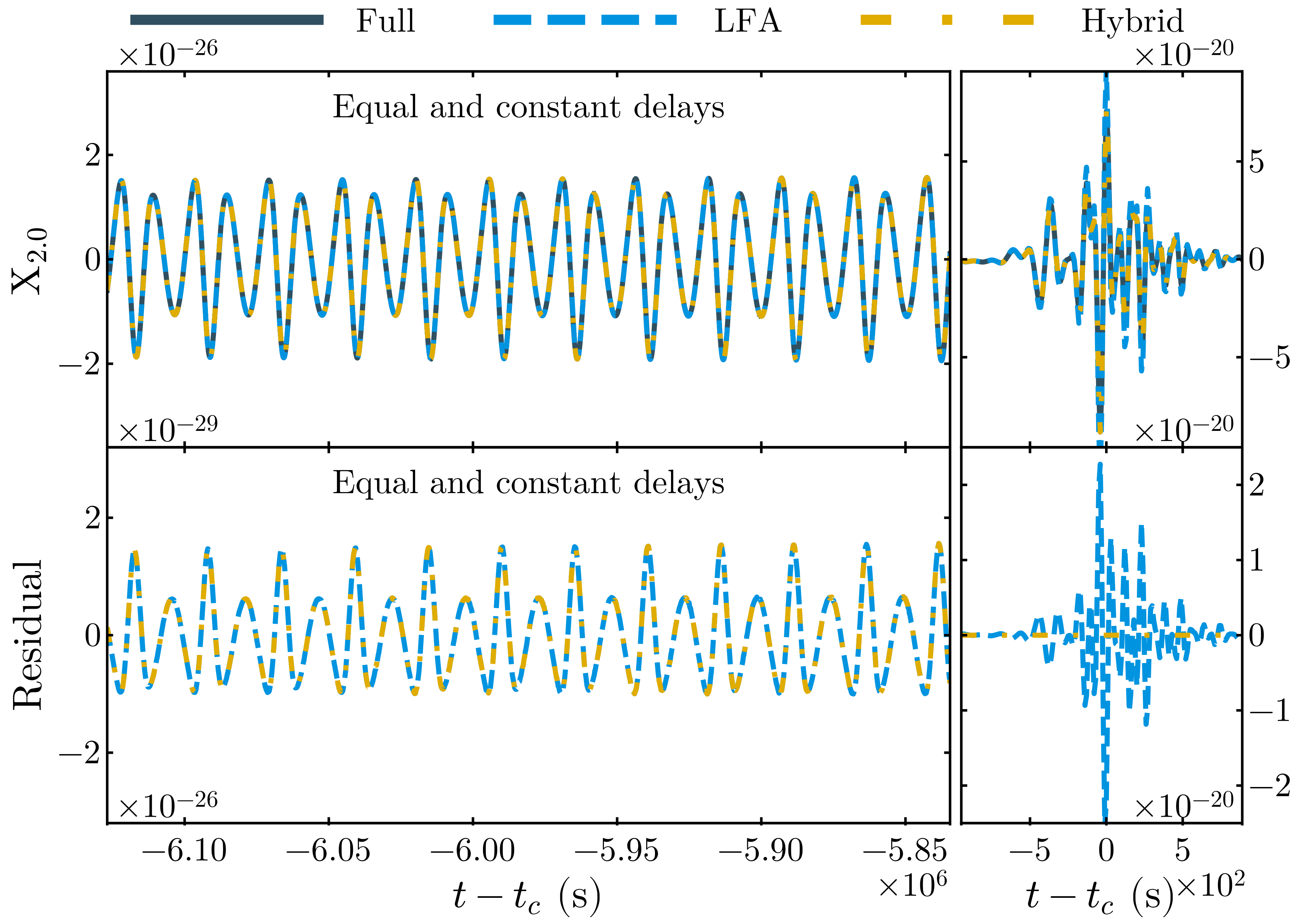


Figure: $(M = 3\times 10^6\,\mathrm{M}_\odot,\ q=4,\ \chi_1=\chi_2=0.8)$

Response validation: test case

- PyIMRPhenomTHM [García-Quirós+ 2025] with
$$(l, m) = \{(2, \pm 2), (2, \pm 1), (3, \pm 3), (4, \pm 4), (5, \pm 5)\}$$
- Good agreement for the Hybrid and LFA during the early inspiral phase.
- Larger errors for LFA in the merger-ringdown.
- Hybrid = Full response in the merger-ringdown.
- LFA(inspiral) + Full(merger-ringdown) = Hybrid
- One order-of-magnitude improvement with unequal and time-dependent delays w.r.t equal and constant delays.

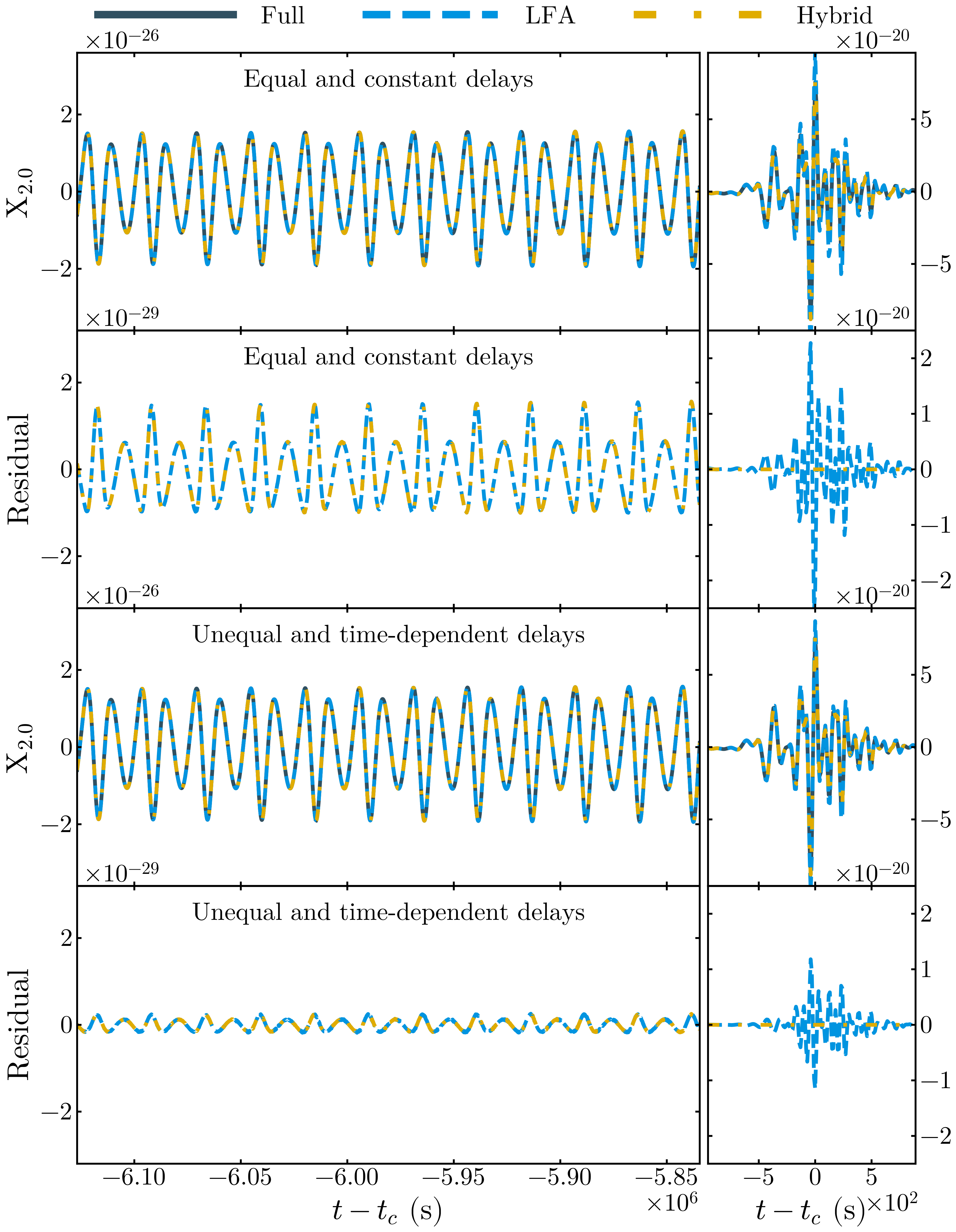
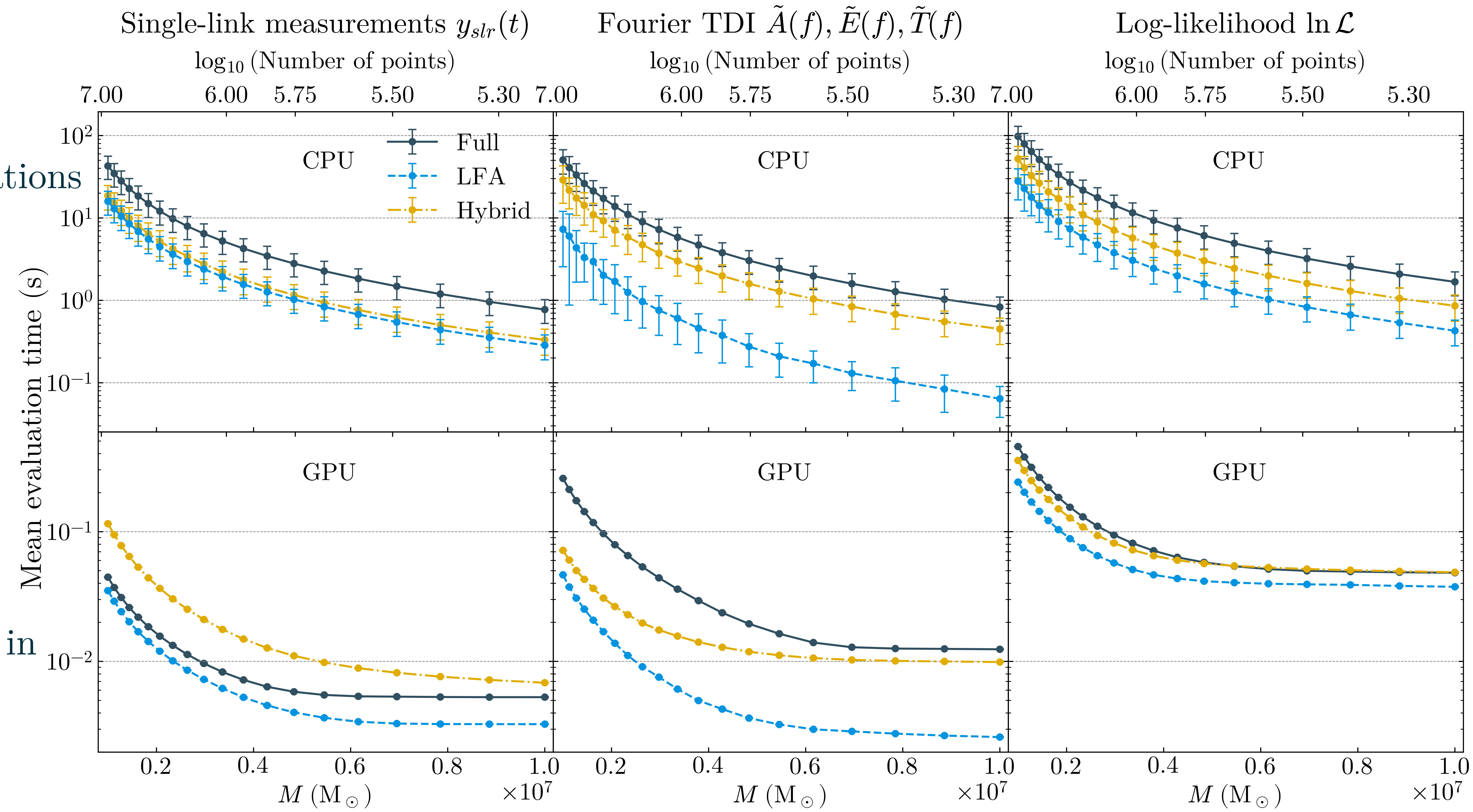


Figure: $(M = 3 \times 10^6\,\mathrm{M}_\odot,\ q = 4,\ \chi_1 = \chi_2 = 0.8)$

Response validation: timing benchmarks

- Averaged over 100 realizations
  - $q \in [1,10]$
  - $\chi_{1,2} \in [-1,1]$
- Fixed $\delta t = 5$ s.
- Speed up factor in CPU:
  - Hybrid ~ 2
  - LFA $\sim 3-5$
- Data-transfer overheads in GPUs for the Hybrid

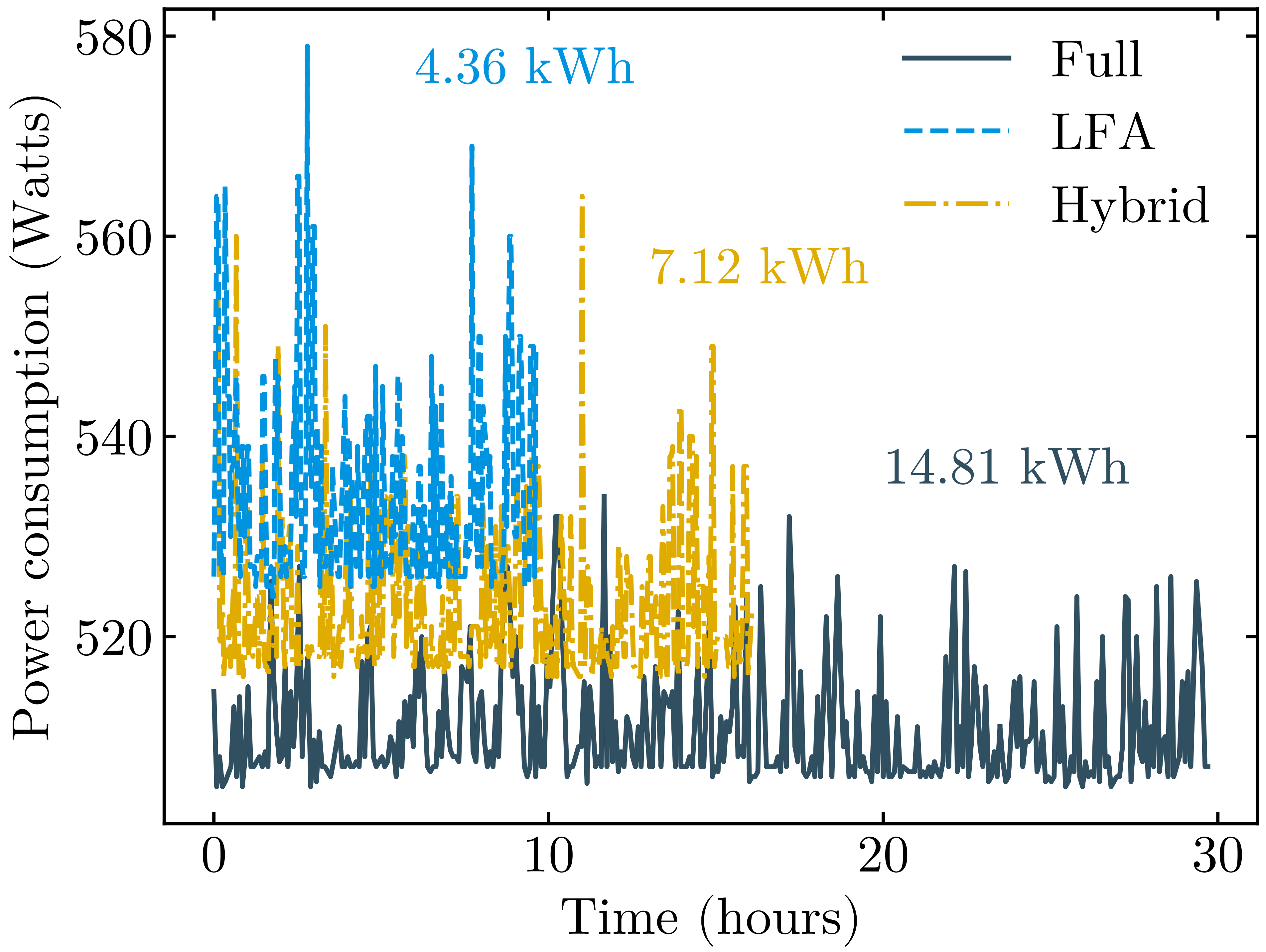


CPU times: 1 node × 1 core

GPU times: NVIDIA Hopper H100 64GB HBM2

Response validation: power consumption

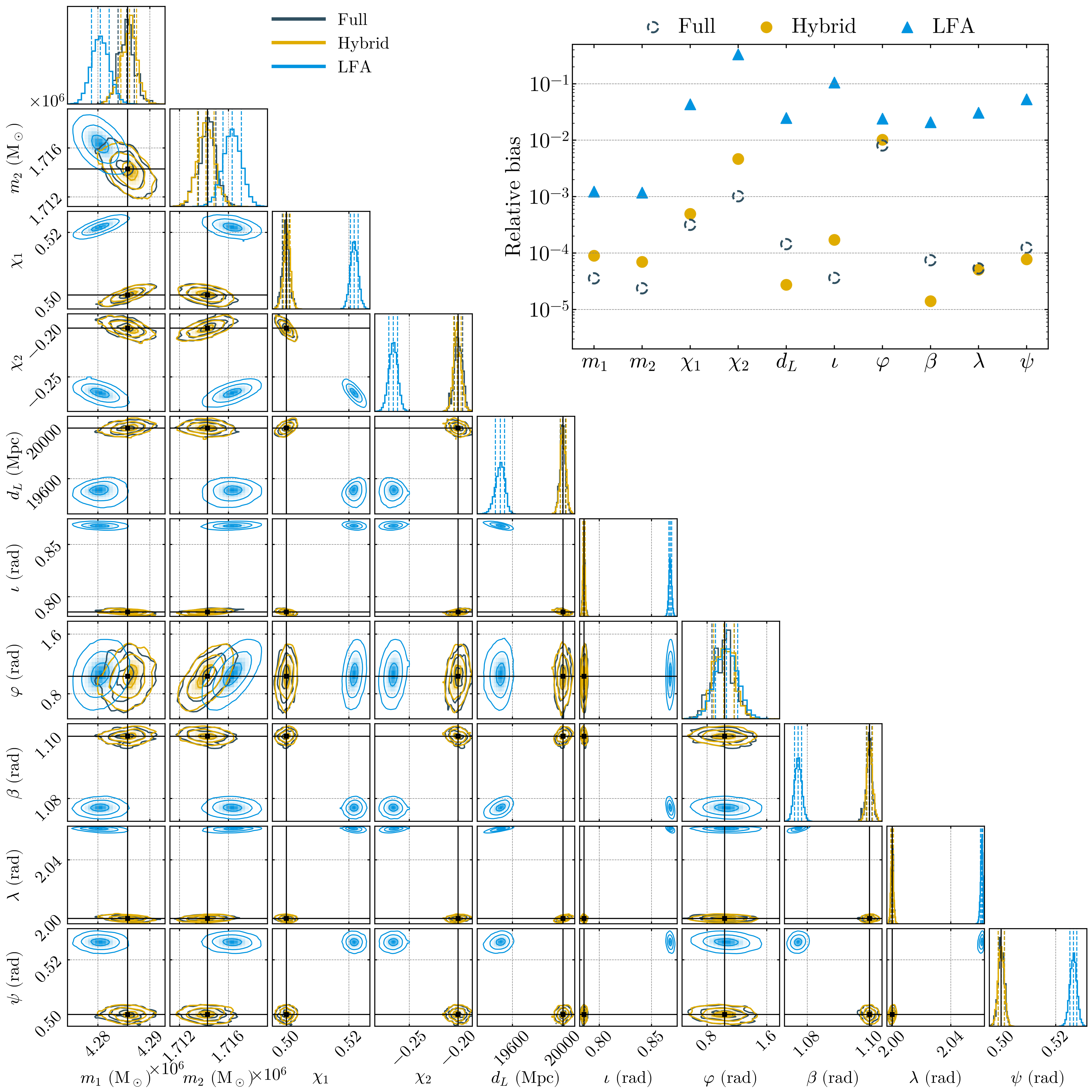
- Higher instantaneous power consumption for LFA and Hybrid.
- Consistent speedups.
- Hybrid consumes 1/2 of the energy of the Full.
- LFA consumes 1/3 of the energy of the Full.
- These factors are applicable also to CO_2 emission.



RESULTS: QUASI-CIRCULAR PE

- Zero-noise IMR injection with 2.0 TDI Full response
- Recovery with different responses with same delay config.
- Fisher initialization.
- $f_{\rm min} = 10^{-4}\,$Hz, $f_{\rm max} = 0.02\,$Hz.
- Sampler: Bilby `ptemcee`
- SNR of the injection $= 1875$
  - `nwalkers` $= 28$, `ntemps` $= 8$, `npool` $= 112$

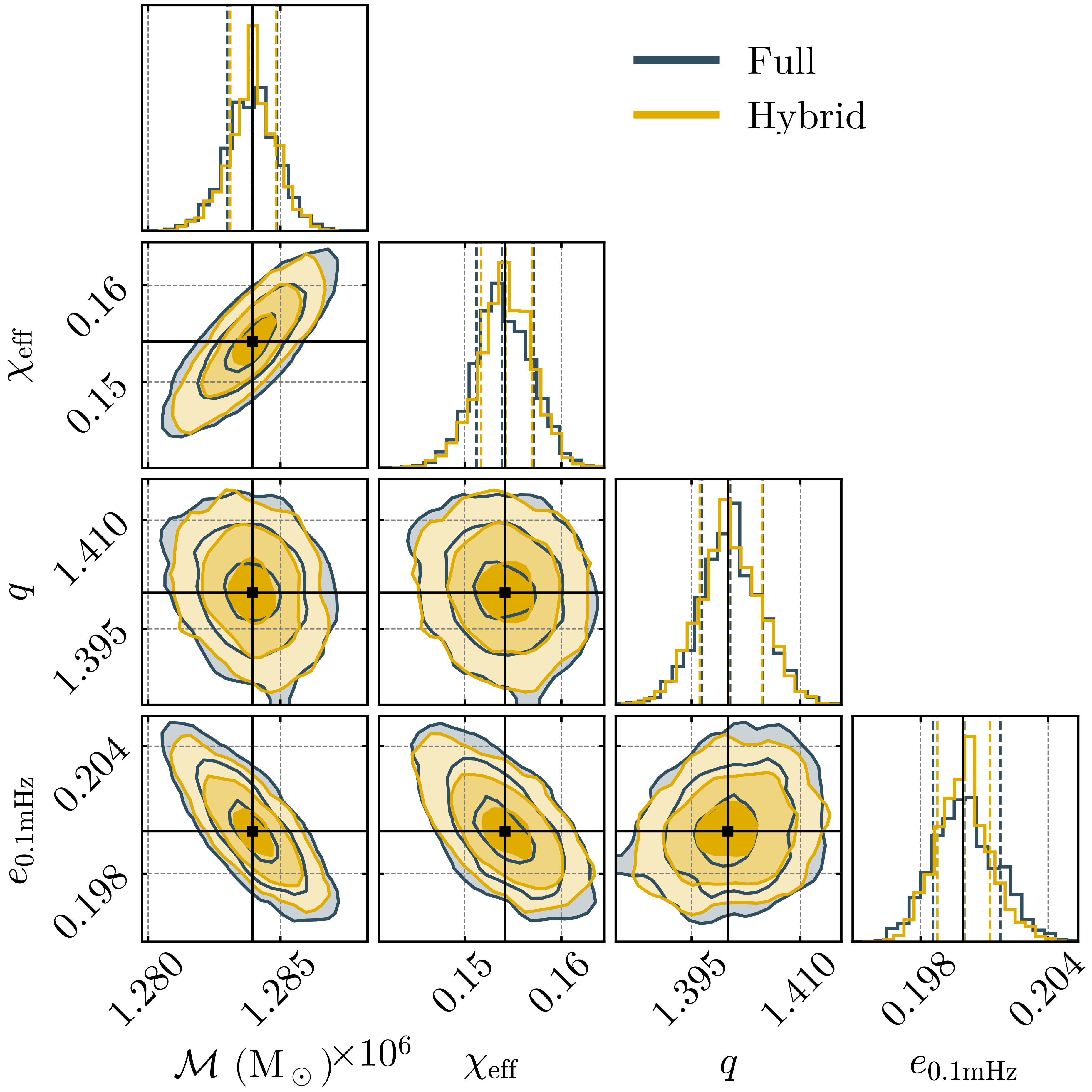
Parameter	Injected value	Response for the recovery		
		Full	Hybrid	LFA
m_1 (10^6 M$_\odot$)	4.285714	$4.2856^{+0.0014}_{-0.0017}$	$4.2861^{+0.0013}_{-0.0017}$	$4.2805^{+0.0017}_{-0.0017}$
m_2 (10^6 M$_\odot$)	1.714286	$1.7143^{+0.0006}_{-0.0008}$	$1.7142^{+0.0007}_{-0.0007}$	$1.7163^{+0.0008}_{-0.0008}$
χ_1	0.5	$0.4998^{+0.0011}_{-0.0011}$	$0.5002^{+0.0010}_{-0.0011}$	$0.5217^{+0.0013}_{-0.0012}$
χ_2	-0.2	$-0.200^{+0.004}_{-0.004}$	$-0.201^{+0.004}_{-0.004}$	$-0.266^{+0.004}_{-0.005}$
d_L (Mpc)	20000	19997^{+22}_{-19}	20001^{+24}_{-21}	19504^{+33}_{-39}
ι (rad)	0.785398	$0.7854^{+0.0010}_{-0.0009}$	$0.7853^{+0.0011}_{-0.0010}$	$0.8679^{+0.0014}_{-0.0013}$
φ (rad)	1.035	$1.027^{+0.138}_{-0.161}$	$1.024^{+0.137}_{-0.136}$	$1.06^{+0.15}_{-0.15}$
β (rad)	1.1	$1.0999^{+0.0009}_{-0.0009}$	$1.1000^{+0.0010}_{-0.0008}$	$1.0770^{+0.0011}_{-0.0011}$
λ (rad)	2.0	$2.0001^{+0.0007}_{-0.0008}$	$1.9999^{+0.0010}_{-0.0009}$	$2.0611^{+0.0006}_{-0.0009}$
ψ (rad)	0.5	$0.5000^{+0.0012}_{-0.0011}$	$0.5000^{+0.0011}_{-0.0011}$	$0.5265^{+0.0012}_{-0.0013}$
$\ln \mathcal{L}$		$1759645.5^{+2.2}_{-2.8}$	$1759644.2^{+2.0}_{-2.6}$	$1407784.6^{+1.8}_{-2.5}$
Sampling time		3h 18m (112 cores)	1h 31m (112 cores)	9d 8h 25m (GPU)
# samples		11592	13300	11732

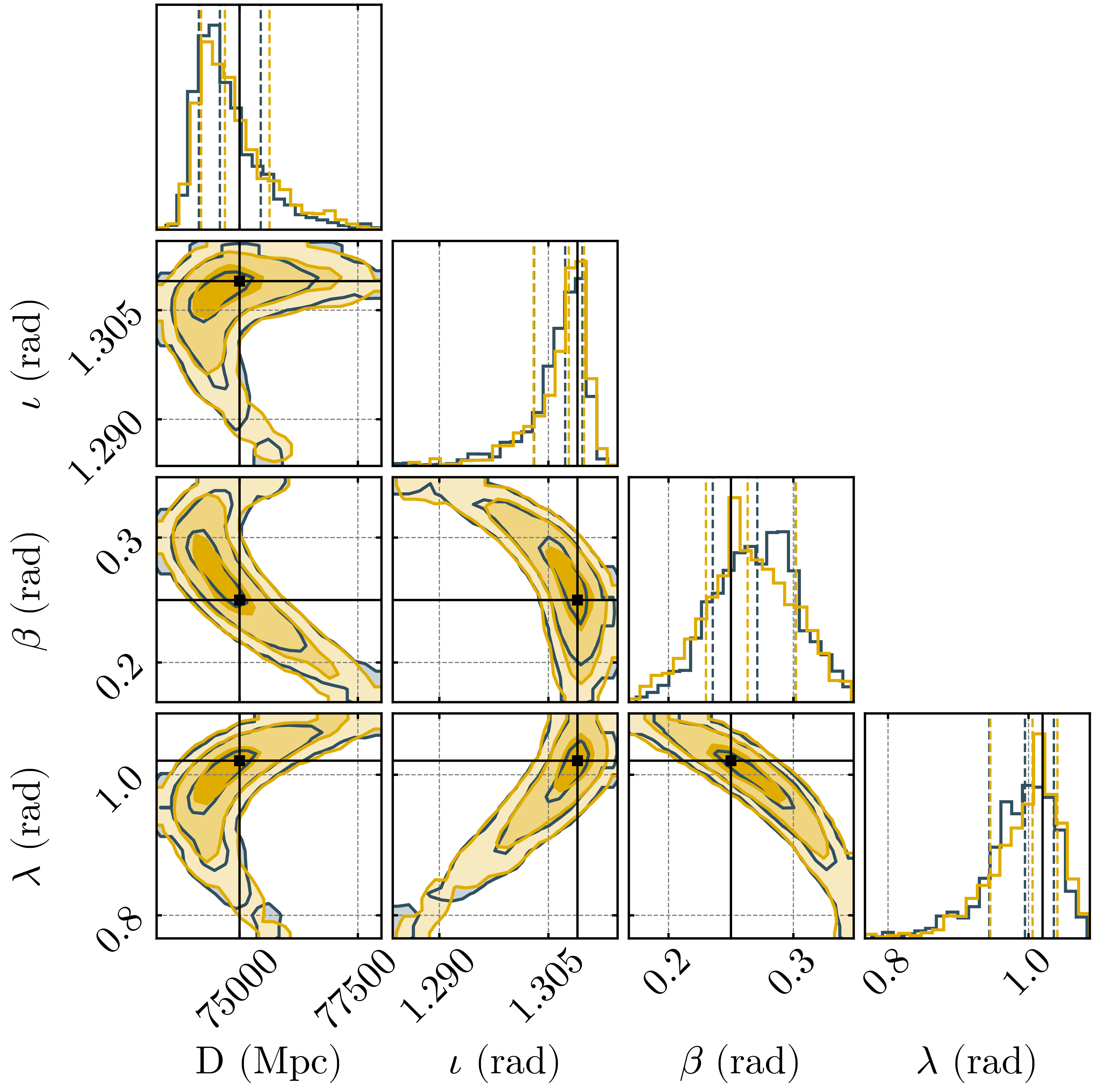


- IMRPhenomTEHM [Planas, Ramos-Buades+ 2025]
- 12-dimensional parameter space: $m_1, m_2, \chi_1, \chi_2, \mathrm{D}, e_{\mathrm{gw}}, l_{\mathrm{gw}}, \iota, \phi, \beta, \lambda, \psi$

Hybrid response performance (112 cores × 1 node)

$M = 3 \times 10^6 \mathrm{M}_\odot, \quad q = 1.4, \quad \mathrm{SNR} = 620$

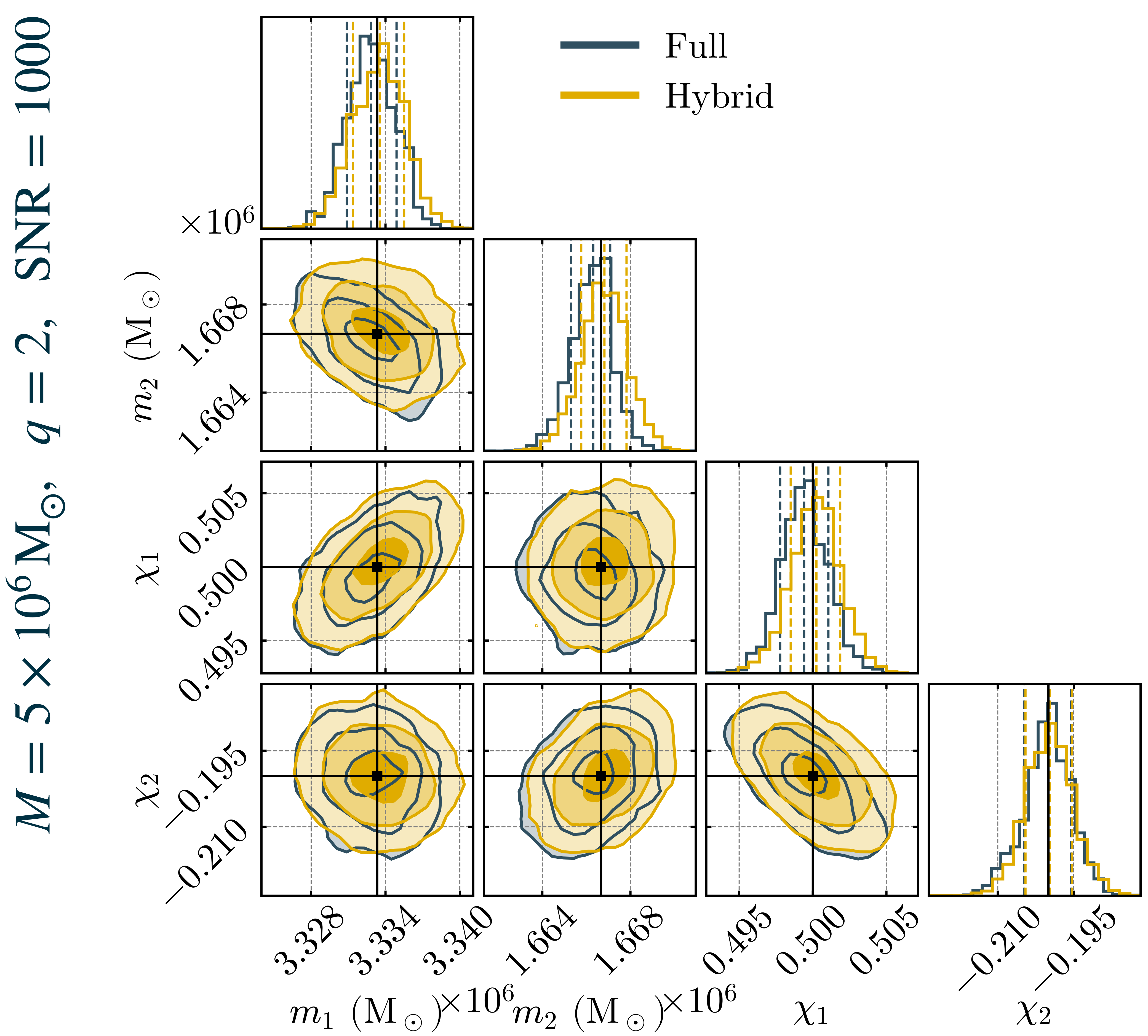




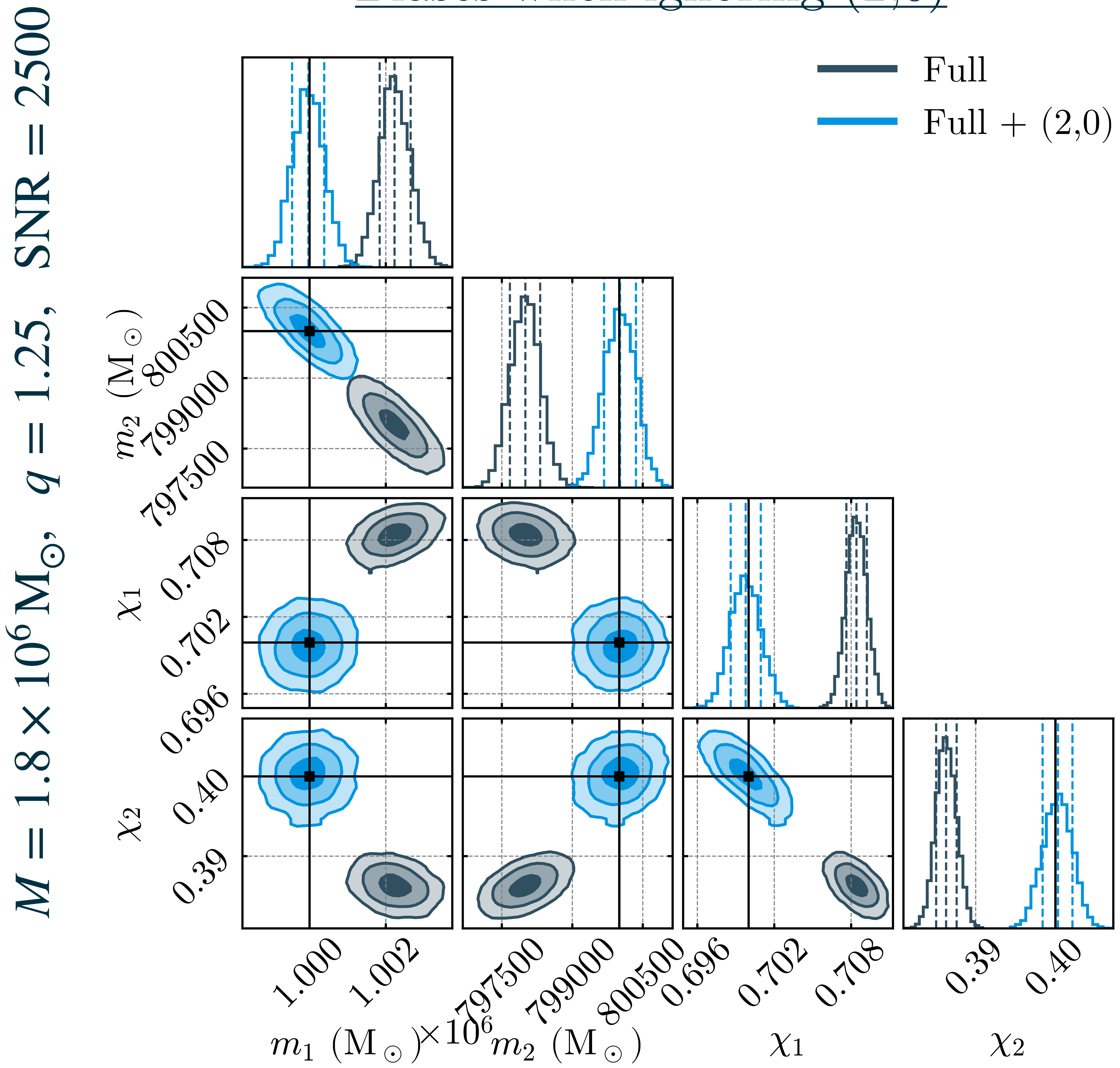
Results: GW memory PE

- IMRPhenomTHM + (2,0) [Rosselló-Sastre, Husa, Bera 2024]
- 10-dimensional parameter space: $m_1, m_2, \chi_1, \chi_2, \mathrm{D}, \iota, \phi, \beta, \lambda, \psi$

Hybrid response performance (112 cores × 1 node)



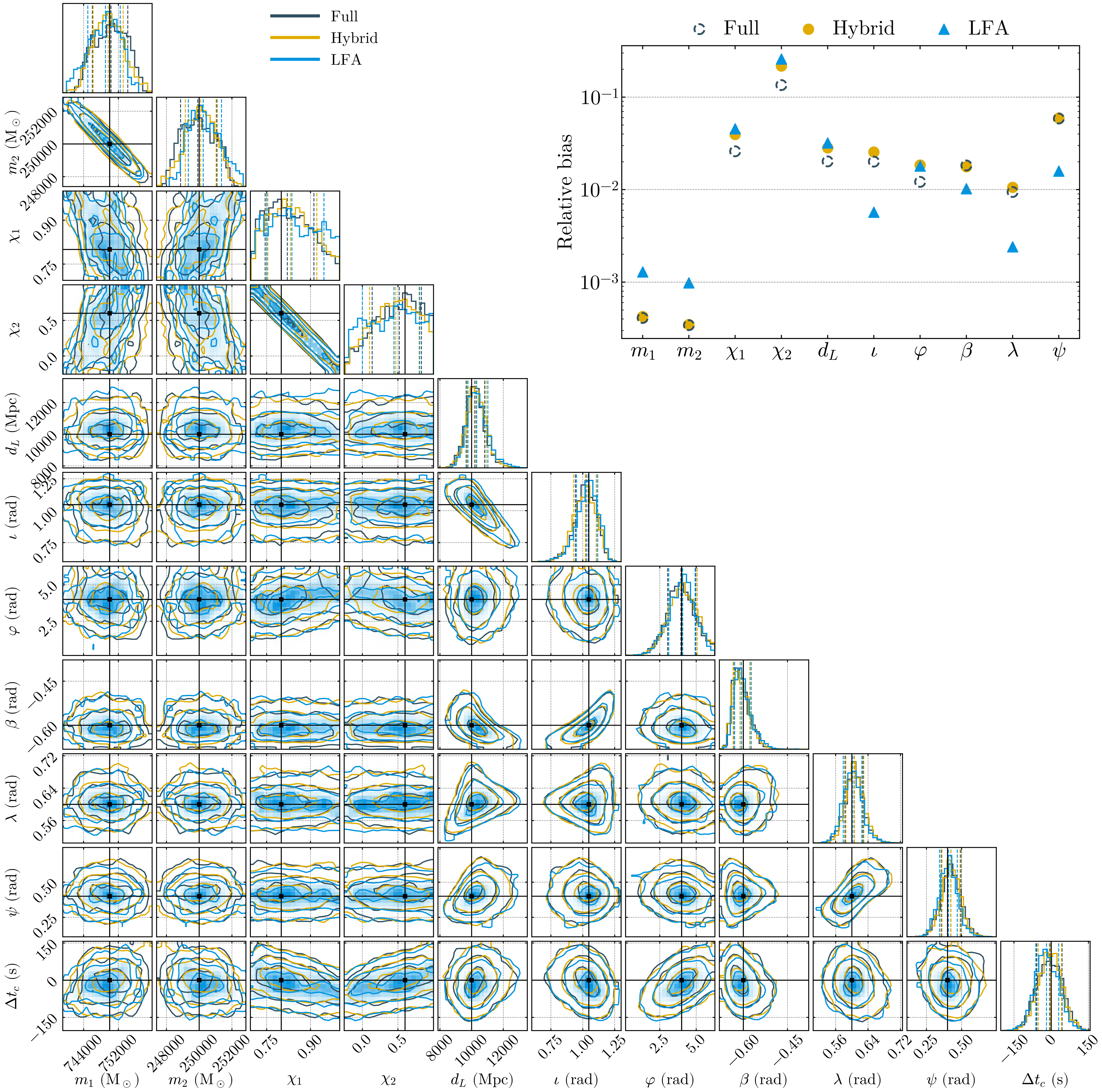
Biases when ignoring (2,0)



RESULTS: PRE-MERGER PE

- Injection with the full response, recovery with different responses
- $f_{\text{min}} = 10^{-4}\,\text{Hz}$, $f_{\text{max}} = 10^{-3}\,\text{Hz}$.
- SNR of the injection = 90
- $\sim$ 10 hours before the merger.

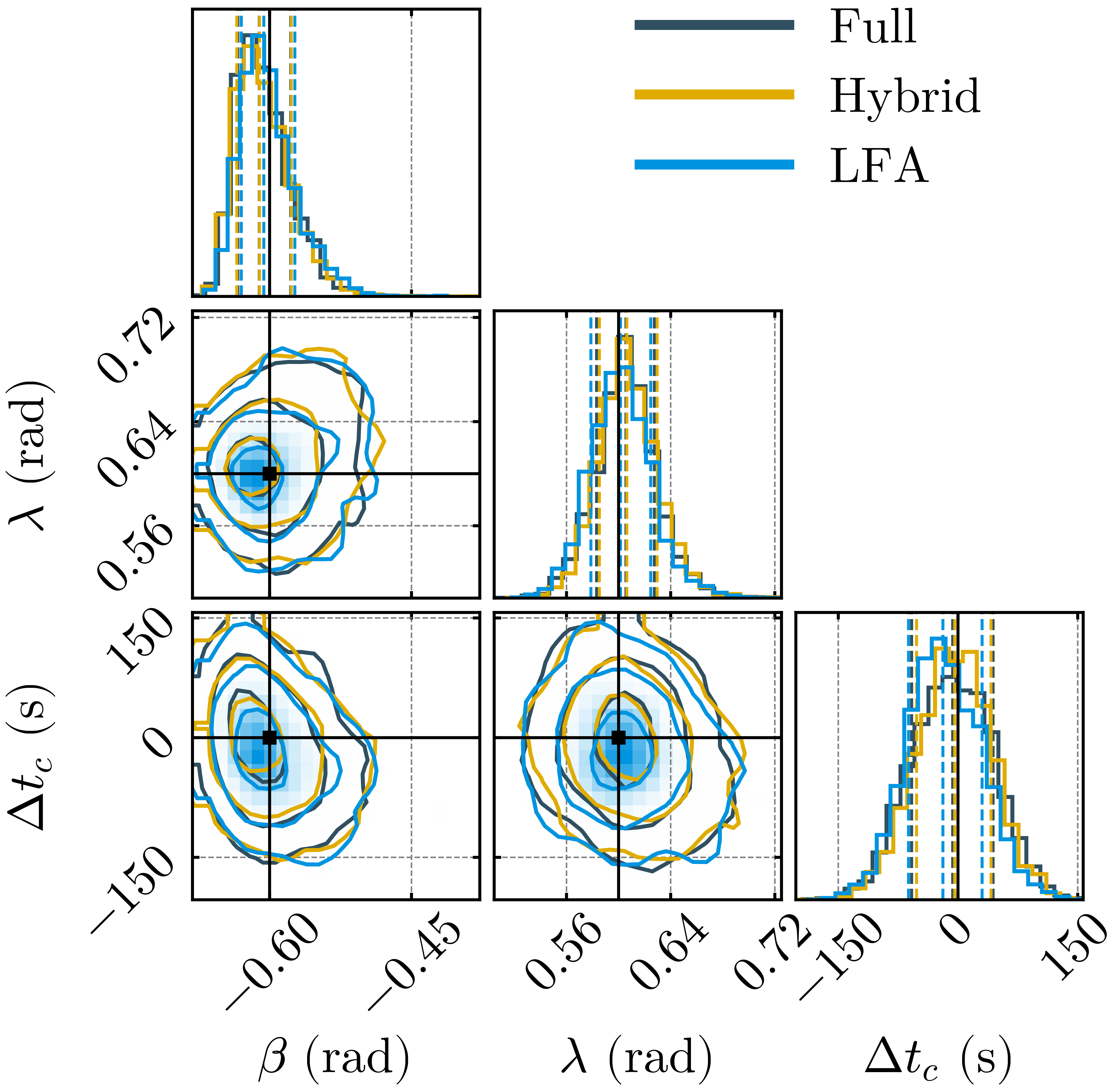
Parameter	**Injected value**	**10 hours before merger**		
		Full	Hybrid	LFA
m_1 (10^5 M$_\odot$)	7.5000	$7.503^{+0.039}_{-0.043}$	$7.497^{+0.034}_{-0.035}$	$7.490^{+0.034}_{-0.041}$
m_2 (10^5 M$_\odot$)	2.5000	$2.499^{+0.012}_{-0.011}$	$2.5009^{+0.0095}_{-0.0093}$	$2.502^{+0.011}_{-0.009}$
χ_1	0.80	$0.821^{+0.091}_{-0.076}$	$0.832^{+0.090}_{-0.077}$	$0.84^{+0.11}_{-0.09}$
χ_2	0.60	$0.52^{+0.31}_{-0.38}$	$0.47^{+0.33}_{-0.37}$	$0.44^{+0.36}_{-0.44}$
d_L (Mpc)	10000	10200^{+620}_{-550}	10280^{+640}_{-560}	10320^{+700}_{-550}
ι (rad)	1.0472	$1.026^{+0.080}_{-0.085}$	$1.020^{+0.085}_{-0.093}$	$1.041^{+0.075}_{-0.091}$
φ (rad)	4.0	$4.0^{+1.0}_{-0.9}$	$4.1^{+1.0}_{-1.0}$	$4.07^{+0.85}_{-0.95}$
λ (rad)	0.6	$0.606^{+0.022}_{-0.023}$	$0.606^{+0.023}_{-0.021}$	$0.601^{+0.023}_{-0.023}$
β (rad)	−0.6	$-0.611^{+0.033}_{-0.023}$	$-0.611^{+0.034}_{-0.024}$	$-0.606^{+0.033}_{-0.024}$
ψ (rad)	0.4	$0.423^{+0.071}_{-0.068}$	$0.424^{+0.065}_{-0.063}$	$0.406^{+0.063}_{-0.063}$
Δt_c (s)	0	-7^{+51}_{-51}	-4^{+46}_{-48}	-19^{+49}_{-43}
$\ln \mathcal{L}$		$4129.9^{+1.8}_{-2.3}$	$4129.8^{+1.8}_{-2.3}$	$4130.1^{+1.7}_{-2.4}$
Sampling time (GPU)		1d 20h 30min	1d 5h 25min	18h 41min
# samples		10836	10248	10780



Results: pre-merger PE

- Injection with the full response, recovery with different responses
- $f_{\min} = 10^{-4}$ Hz, $f_{\max} = 10^{-3}$ Hz.
- SNR of the injection = 90
- $\sim$ 10 hours before the merger.

Parameter	Injected value	10 hours before merger		
		Full	Hybrid	LFA
m_1 (10^5 M$_\odot$)	7.5000	$7.503^{+0.039}_{-0.043}$	$7.497^{+0.034}_{-0.035}$	$7.490^{+0.034}_{-0.041}$
m_2 (10^5 M$_\odot$)	2.5000	$2.499^{+0.012}_{-0.011}$	$2.5009^{+0.0095}_{-0.0093}$	$2.502^{+0.011}_{-0.009}$
χ_1	0.80	$0.821^{+0.091}_{-0.076}$	$0.832^{+0.090}_{-0.077}$	$0.84^{+0.11}_{-0.09}$
χ_2	0.60	$0.52^{+0.31}_{-0.38}$	$0.47^{+0.33}_{-0.37}$	$0.44^{+0.36}_{-0.44}$
d_L (Mpc)	10000	10200^{+620}_{-550}	10280^{+640}_{-560}	10320^{+700}_{-550}
ι (rad)	1.0472	$1.026^{+0.080}_{-0.085}$	$1.020^{+0.085}_{-0.093}$	$1.041^{+0.075}_{-0.091}$
φ (rad)	4.0	$4.0^{+1.0}_{-0.9}$	$4.1^{+1.0}_{-1.0}$	$4.07^{+0.85}_{-0.95}$
λ (rad)	0.6	$0.606^{+0.022}_{-0.023}$	$0.606^{+0.023}_{-0.021}$	$0.601^{+0.023}_{-0.023}$
β (rad)	-0.6	$-0.611^{+0.033}_{-0.023}$	$-0.611^{+0.034}_{-0.024}$	$-0.606^{+0.033}_{-0.024}$
ψ (rad)	0.4	$0.423^{+0.071}_{-0.068}$	$0.424^{+0.065}_{-0.063}$	$0.406^{+0.063}_{-0.063}$
Δt_c (s)	0	-7^{+51}_{-51}	-4^{+46}_{-48}	-19^{+49}_{-43}
$\ln \mathcal{L}$		$4129.9^{+1.8}_{-2.3}$	$4129.8^{+1.8}_{-2.3}$	$4130.1^{+1.7}_{-2.4}$
Sampling time (GPU)		1d 20h 30min	1d 5h 25min	18h 41min
# samples		10836	10248	10780



Conclusions

- Simplified time-domain expressions for the TDI mechanism by using finite differences.
- We have tested the hybrid response against the full response with benchmarks and injection-recovery studies.
- LFA accelerates the evaluation of the long inspiral and its application on deep alerts has been tested with pre-merger PE.
- First instance to use different prescriptions for the low- and high-frequency regime and combine them in the same response. Applications on the Global Fit.
- Main take-home messages:
  - LISA computes derivatives of the GW polarizations, emphasizing short-timescale structure of waveforms.
  - Hybrid response for signal including merger-ringdown.
  - LFA response for pre-merger signals. (low-frequency content)

Check out the paper! arXiv:2505.09600

Acknowledgments

Jorge Valencia is supported by the Spanish Ministry of Universities (FPU22/02211).

We thankfully acknowledge the computer resources at MareNostrum and the technical support provided by Barcelona Supercomputing Center (BSC) through Grants No. AECT-2024-3-0033 and No. AECT-2025-1- 0043 from the Red Española Supercomputación (RES).

This work was supported by the Universitat de les Illes Balears (UIB); the Spanish Agencia Estatal de Investigación grants PID2022-138626NB-I00, RED2022-134204-E, RED2022-134411-T, funded by MICIU/AEI/ 10.13039/501100011033, by the ESF+ and the ERDF/EU; and the Comunitat Autònoma de les Illes Balears through the Conselleria d'Educació i Universitats with funds from the European Union - NextGenerationEU/PRTR-C17.I1 (SINCO2022/6719) and from the European Union - European Regional Development Fund (ERDF) (SINCO2022/18146).

Backup slides

Low-Frequency Approximation (LFA) response

The **low-wavelength approximation** corresponds to frequencies $f \ll c/(2\pi L) = 0.019\,\mathrm{Hz}$, where timescales much smaller than $(0.019\,\mathrm{Hz})^{-1} \sim 50\,\mathrm{s}$ can be ignored. Consequently, inter-spacecraft time-delays of $L/c \sim 8\,\mathrm{s}$ can be neglected [Marsat+ 2020].

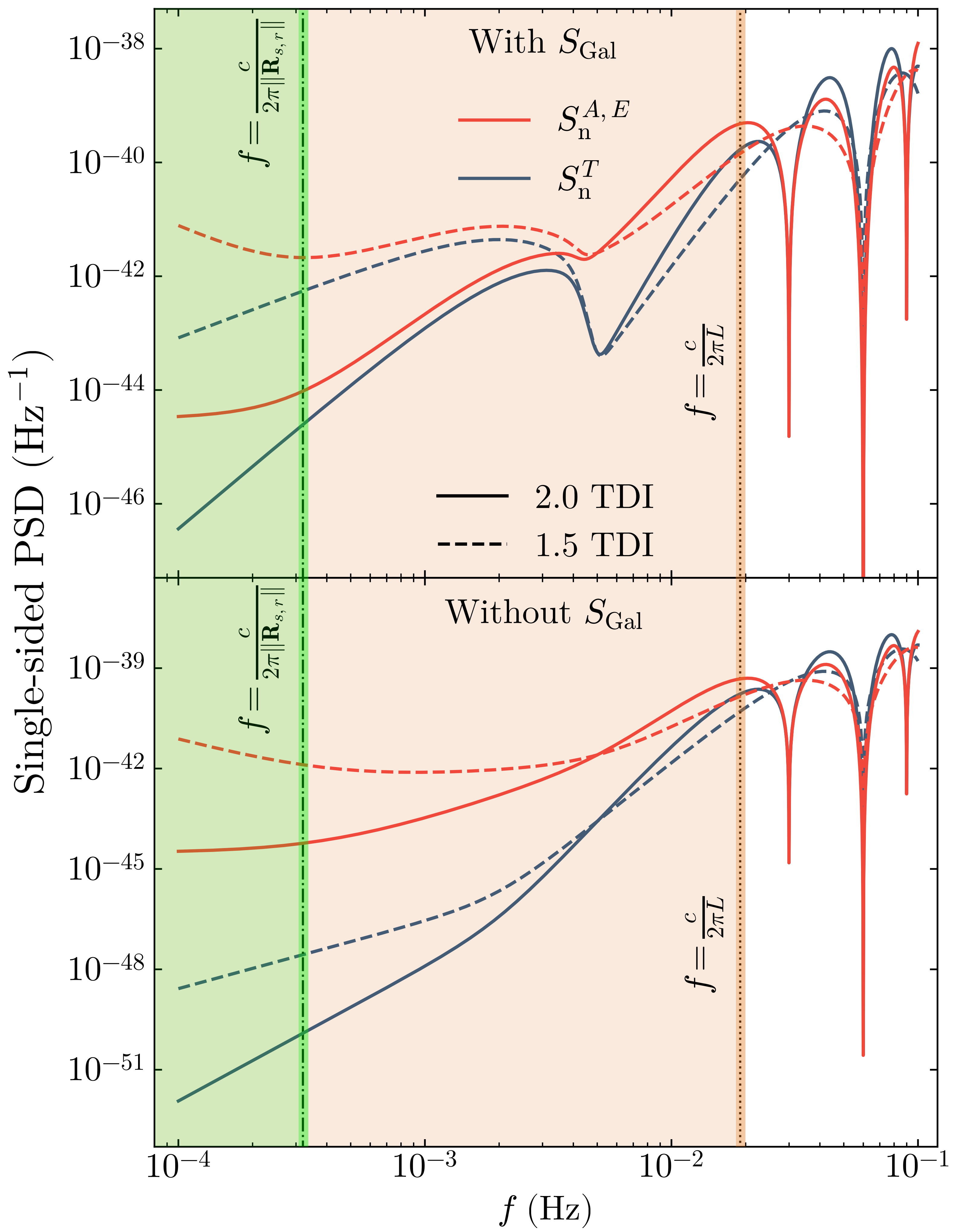
In this limit:

$$H_l(t - L/c) \approx H_l(t) - (L_l/c)\dot{H}_l(t) + \mathcal{O}\left[(L/c)^2\right]$$

since L/c is small compared with the timescale of the system.

$$y_{slr}(t) = \frac{H_l\left(t-\delta_s(t)\right) - H_l\left(t-\delta_r(t)\right)}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_l(t))} \qquad \delta_s(t) = \frac{L_l(t)}{c} + \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_s(t)}{c}, \qquad \delta_r(t) = \frac{\hat{\mathbf{k}}\cdot\mathbf{R}_r(t)}{c},$$

However, calculating y_{slr} requires the evaluation of H_l at δ_s and δ_r, whose magnitudes ($\sim 8\,\mathrm{min}$) are larger than $50\,\mathrm{s}$. For $f \ll c/(2\pi\|\mathbf{R}_{r,s}\|) = 3.2\times10^{-4}\,\mathrm{Hz}$, we can safely ignore timescales much smaller than $(3.2\times10^{-4}\,\mathrm{Hz})^{-1} \sim 52\,\mathrm{min}$ and expand retarded quantities in powers of L_l/c, $\|\mathbf{R}_{r,s}\|/c$ or a combination of both.



$f_L = 0.019\,\mathrm{Hz}$
$f_R = 3.2\times10^{-4}\,\mathrm{Hz}$ in [Marsat+ 2018]

Fourier-domain response comparison

- What is the best choice for ϵ? $\epsilon = 1/2$

$$\epsilon = 1/2$$

$$\tau = t - \frac{\delta_s(t) + \delta_r(t)}{2}$$

$$y_{slr}(t) = -\frac{L_l(t)}{2c} \sum_{m=0}^{\infty} \frac{(-1)^m[(\epsilon-1)^m - \epsilon^m]}{m!} \left[\frac{L_l(t)}{c}\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l(t)\right)\right]^{m-1} \frac{\mathrm{d}^m}{\mathrm{d}t^m} H_l(\tau_\epsilon),$$

$$y_{slr}(t) = -\frac{L_l(t)}{2c} \sum_{m=0}^{\infty} \frac{1}{(2m+1)!} \left[\frac{L_l(t)}{2c}\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l\right)\right]^{2m} \frac{\mathrm{d}^{2m+1}}{\mathrm{d}t^{2m+1}} H_l\left(t - \frac{L_l(t)}{2c} - \frac{\hat{\mathbf{k}} \cdot \left(\mathbf{R}_s(t) + \mathbf{R}_r(t)\right)}{2c}\right)$$

- We compare this expression with the Fourier-domain LISA response in [Marsat+ 2018, 2020]:

  - Rigid equilateral LISA constellation and $L_l(t) = L$

$$y_{slr}(t) = -\frac{L}{2c} \sum_{m=0}^{\infty} \frac{1}{(2m+1)!} \left[\frac{L}{2c}\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l\right)\right]^{2m} \frac{\mathrm{d}^{2m+1}}{\mathrm{d}t^{2m+1}} H_l\left(t - \frac{L}{2c} - \frac{\hat{\mathbf{k}} \cdot \left(\mathbf{R}_s + \mathbf{R}_r\right)}{2c}\right)$$

  - Same Fourier conventions.

$$\mathscr{F}\left[\frac{\mathrm{d}^m}{\mathrm{d}t^m} x(t)\right] = (-2i\pi f)^m \tilde{x}(f)$$

$$\mathscr{F}[x(t - t_0)] = e^{+2i\pi f t_0} \tilde{x}(f)$$

$$\mathrm{sinc}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{(2m+1)!}$$

$$\mathscr{F}[y_{slr}(t)] = \frac{i\pi f L}{c} \mathrm{sinc}\left[\frac{\pi f L}{c}\left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_l\right)\right] \exp\left[\frac{i\pi f}{c}\left(L + \hat{\mathbf{k}} \cdot \left(\mathbf{R}_s + \mathbf{R}_r\right)\right)\right] \tilde{H}_l$$

LFA: Michelson combinations

We apply a similar procedure to the Michelson combinations $X(t)$, $Y(t)$, and $Z(t)$:

$$\begin{aligned}X_{1.5}(t) = {} & y_{1-32}(t-L_{-2}-L_2-L_3)\\ & -y_{1-32}(t-L_3)\\ & +y_{231}(t-L_2-L_{-2})\\ & -y_{231}(t)\\ & +y_{123}(t-L_{-2})\\ & -y_{123}(t-L_{-2}-L_{-3}-L_3)\\ & +y_{3-21}(t)\\ & -y_{3-21}(t-L_{-3}-L_3)\,,\end{aligned}$$

$$\begin{aligned}X_{1.5}(t) = {} & \left[\frac{L_{-3}(t)+L_3(t)}{c}\right]\left[\dot{y}_{123}\left(t-\frac{L_{-3}(t)+L_{-2}(t)+L_2(t)}{2c}\right)-\frac{L_{-2}(t)}{2c}\ddot{y}_{123}\left(t-\frac{L_{-3}(t)+L_{-2}(t)+L_2(t)}{2c}\right)\right]\\ & +\left[\frac{L_{-3}(t)+L_3(t)}{c}\right]\left[\dot{y}_{3-21}\left(t-\frac{L_{-3}(t)+L_{-2}(t)+L_2(t)}{2c}\right)+\frac{L_{-2}(t)}{2c}\ddot{y}_{3-21}\left(t-\frac{L_{-3}(t)+L_{-2}(t)+L_2(t)}{2c}\right)\right]\\ & -\left[\frac{L_{-2}(t)+L_2(t)}{c}\right]\left[\dot{y}_{1-32}\left(t-\frac{L_{-2}(t)+L_2(t)+L_3(t)}{2c}\right)-\frac{L_3(t)}{2c}\ddot{y}_{1-32}\left(t-\frac{L_{-2}(t)+L_2(t)+L_3(t)}{2c}\right)\right]\\ & -\left[\frac{L_{-2}(t)+L_2(t)}{c}\right]\left[\dot{y}_{231}\left(t-\frac{L_{-2}(t)+L_2(t)+L_3(t)}{2c}\right)+\frac{L_3(t)}{2c}\ddot{y}_{231}\left(t-\frac{L_{-2}(t)+L_2(t)+L_3(t)}{2c}\right)\right]\\ & +\mathcal{O}\left[\frac{L_l\dot{L}_l}{c^2}\dot{y}_{slr}\right] \quad \text{error from } \frac{\mathrm{d}}{\mathrm{d}\tau_{2,3}}\approx\frac{\mathrm{d}}{\mathrm{d}t}\\ & \left.\begin{aligned}&+\mathcal{O}\left[\frac{L_l^2\ddot{L}_l}{c^3}\dot{y}_{slr}\right]+\mathcal{O}\left[\frac{L_l^2\dot{L}\ddot{L}_l}{c^4}\dot{y}_{slr}\right]\\ &+\mathcal{O}\left[\frac{L_l^2\dot{L}_l}{c^3}\ddot{y}_{slr}\right]+\mathcal{O}\left[\frac{L_l^2\dot{L}_l^2}{c^4}\ddot{y}_{slr}\right]\end{aligned}\right\}\text{error from } \frac{\mathrm{d}^2}{\mathrm{d}\tau_{2,3}^2}\approx\frac{\mathrm{d}^2}{\mathrm{d}t^2}\\ & +\mathcal{O}\left[\frac{L_l^3\ddot{L}_l}{c^4}\ddot{y}_{slr}\right]+\mathcal{O}\left[\frac{L_l^3\dot{L}_l\ddot{L}_l}{c^5}\ddot{y}_{slr}\right]+\mathcal{O}\left[\frac{L_l^3}{c^3}\dddot{y}_{slr}\right] \quad \text{error from Taylor series truncation after } \frac{\mathrm{d}^3}{\mathrm{d}\tau^3}\approx\frac{\mathrm{d}^3}{\mathrm{d}t^3}.\end{aligned}$$

$\mathcal{O}\left(10^{-7}\dot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-16}\dot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-6}\ddot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-15}\ddot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-23}\dot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-13}\ddot{y}_{slr}\right)$

$\mathcal{O}\left(10^{-22}\ddot{y}_{slr}\right)$

$\mathcal{O}\left(10^{2}\dddot{y}_{slr}\right)$

This error term will be low in the early inspiral of a slowly varying signal, satisfying $\dddot{y}_{slr} \ll \ddot{y}_{slr} \ll \dot{y}_{slr}$

[Colpi+ 2023]

$\dot{L}_l < 12\,\mathrm{m\,s^{-1}}$

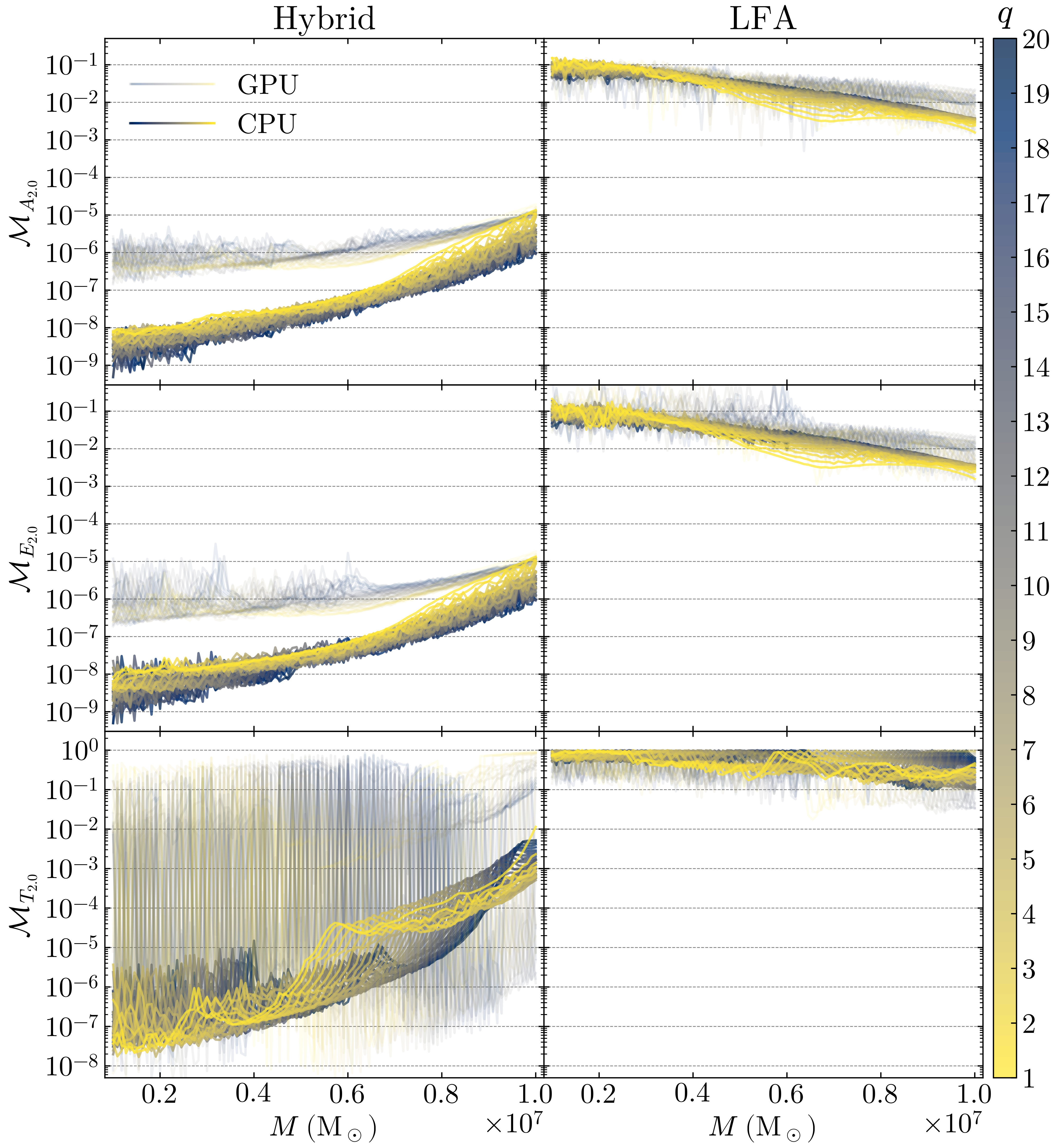
$\ddot{L}_l \sim 2\times10^{-9}\,\mathrm{m\,s^{-2}}$

Response validation: mismatches

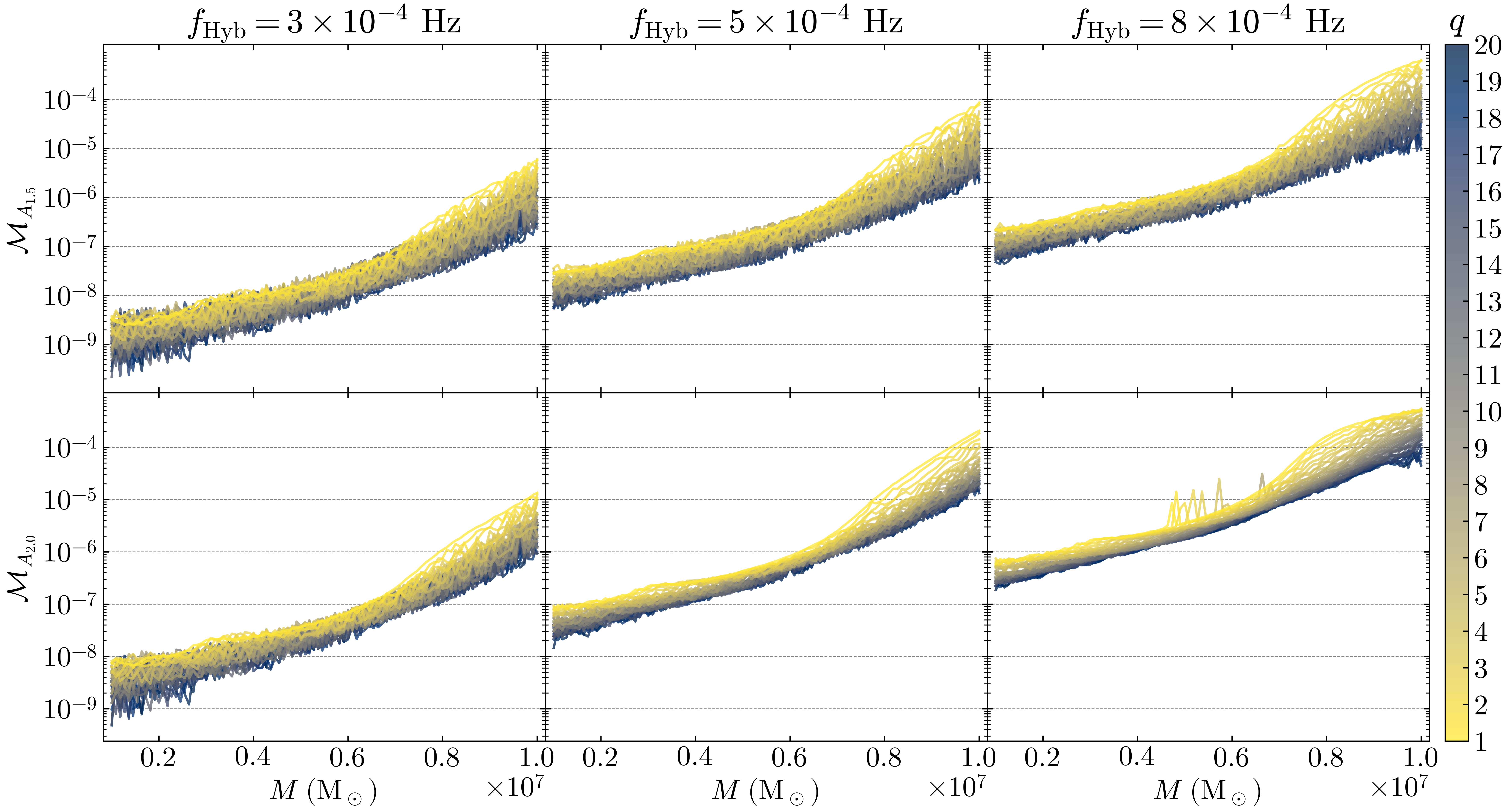
- Accuracy metric:

$$\mathcal{O}(x,y) = \frac{\langle x\,|\,y\rangle}{\sqrt{\langle x\,|\,x\rangle}\sqrt{\langle y\,|\,y\rangle}}, \quad \mathcal{M} = 1 - |\,\mathcal{O}\,|$$

- Mismatches across: $M \in [10^6, 10^7]\,\mathrm{M}_\odot$, $q \in [1,20]$
- We set $f_{\mathrm{min}} = 10^{-4}\,\mathrm{Hz}$, $f_{\mathrm{max}} = 0.05\,\mathrm{Hz}$ and $\delta t = 10\,\mathrm{s}$.
- $f_{\mathrm{Hyb}} = 3 \times 10^{-4}\,\mathrm{Hz}$.
- Unequal and time-dependent delays.
- Hybrid: $\mathcal{M}_{A_{2.0},E_{2.0}} < 10^{-5}$ & $\mathcal{M}_{T_{2.0}} < 10^{-2}$.
- LFA: $\mathcal{M}_{A_{2.0},E_{2.0}} < 10^{-1}$ & $10^{-1} < \mathcal{M}_{T_{2.0}} < 1$.
- CPU vs. GPU: cubic spline vs. linear interpolation.



Mismatches changing f_{Hyb}



Jorge Valencia, jorge.valencia@uib.es

Eccentricity