

Proto-neutron star oscillations in core-collapse supernovae

Dimitra Tseneklidou

University of Valencia

14th Iberian GWs Meeting

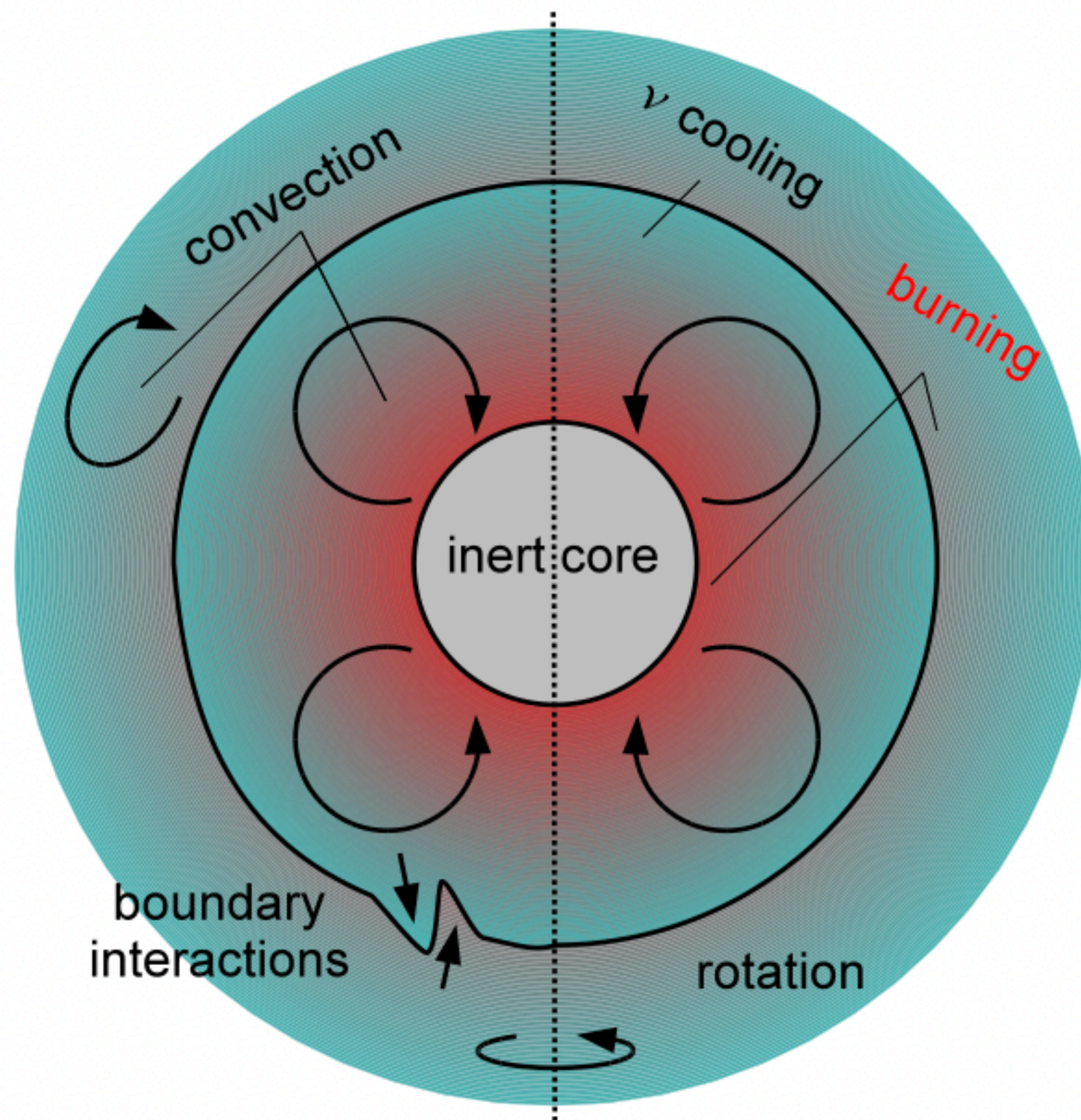
June 25th 2025, Madrid

Collaborators: R. Luna, P. Cerdá-Durán, A. Torres-Forné

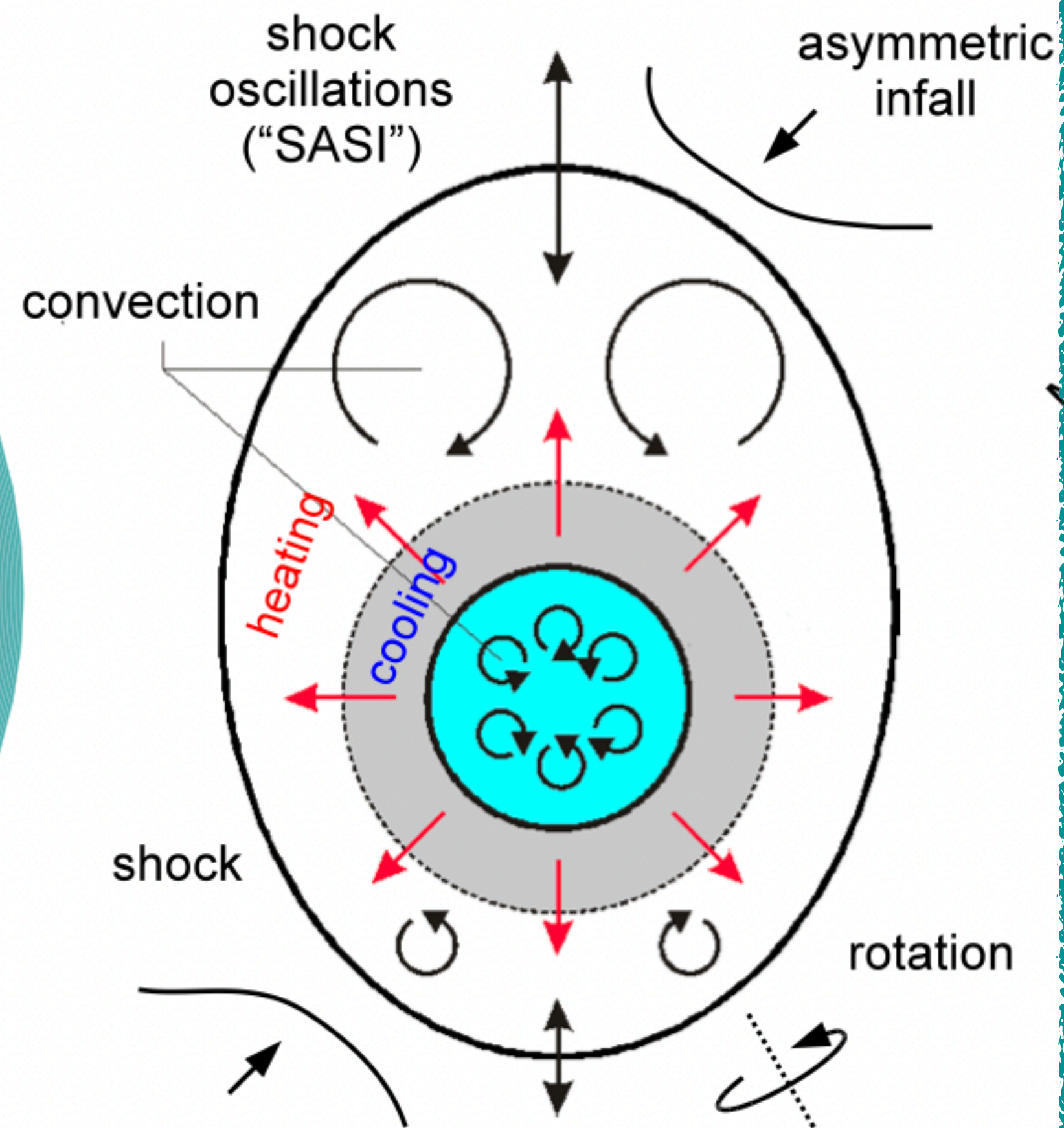
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Anatomy of a CCSN



a) C, Ne, O, and Si core and shell burning (years to hours)



b) Core collapse and shock revival (hundreds of milliseconds)

GW emission from CCSNe

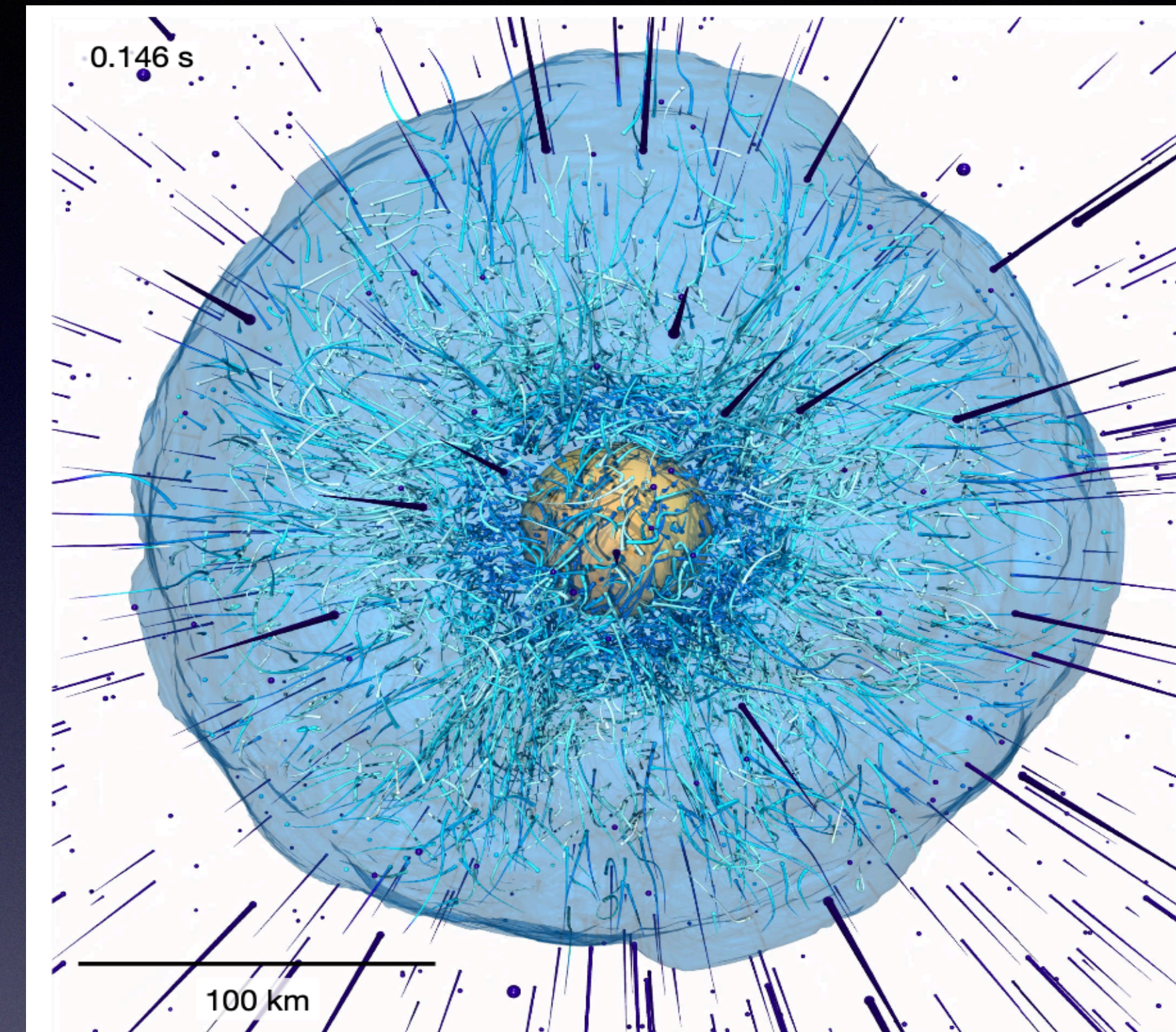
Burrows et al (2021)

Proto-neutron star (PNS) phase
(before explosion):

- Duration: $0.1 - 1$ s
- PNS mass grows: $0.5 M_{\odot} \rightarrow 1.2 - 2 M_{\odot}$
- PNS shrinks: $30 \text{ km} \rightarrow 10 \text{ km}$
- PNS cools down



Convection, SASI



Reviews:

- Janka, Melson, and Summa, Ann. Rev. Nucl. Part. Sci. 66 341 (2016)
- Mueller, Liv. Rev. Comp. Astr. 6:3 (2020)
- Mezzacappa, Endeve, Messer, and Bruenn, Liv. Rev. Comp. Astr. 6:4 (2020)
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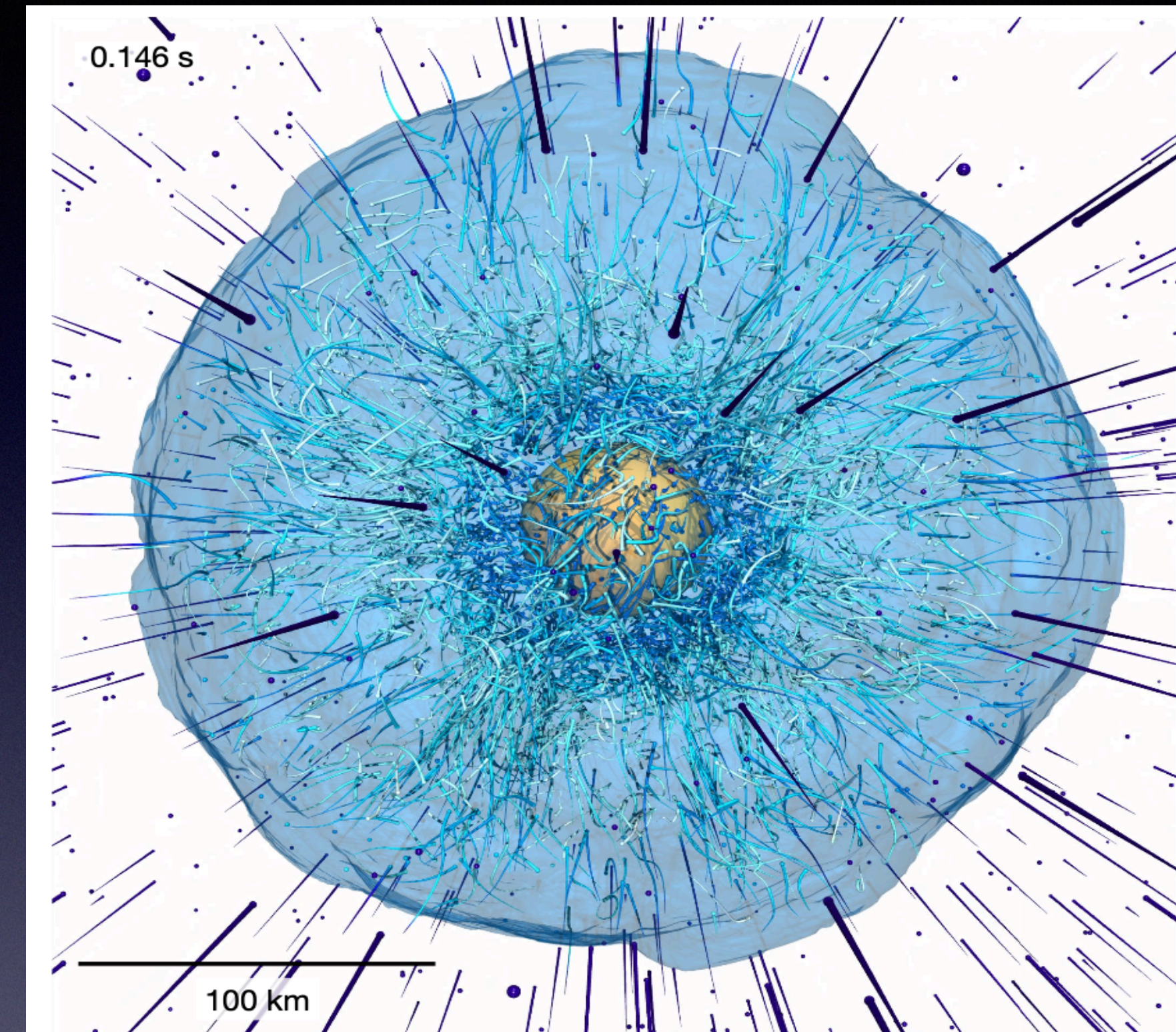
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Excitation of PNS normal modes



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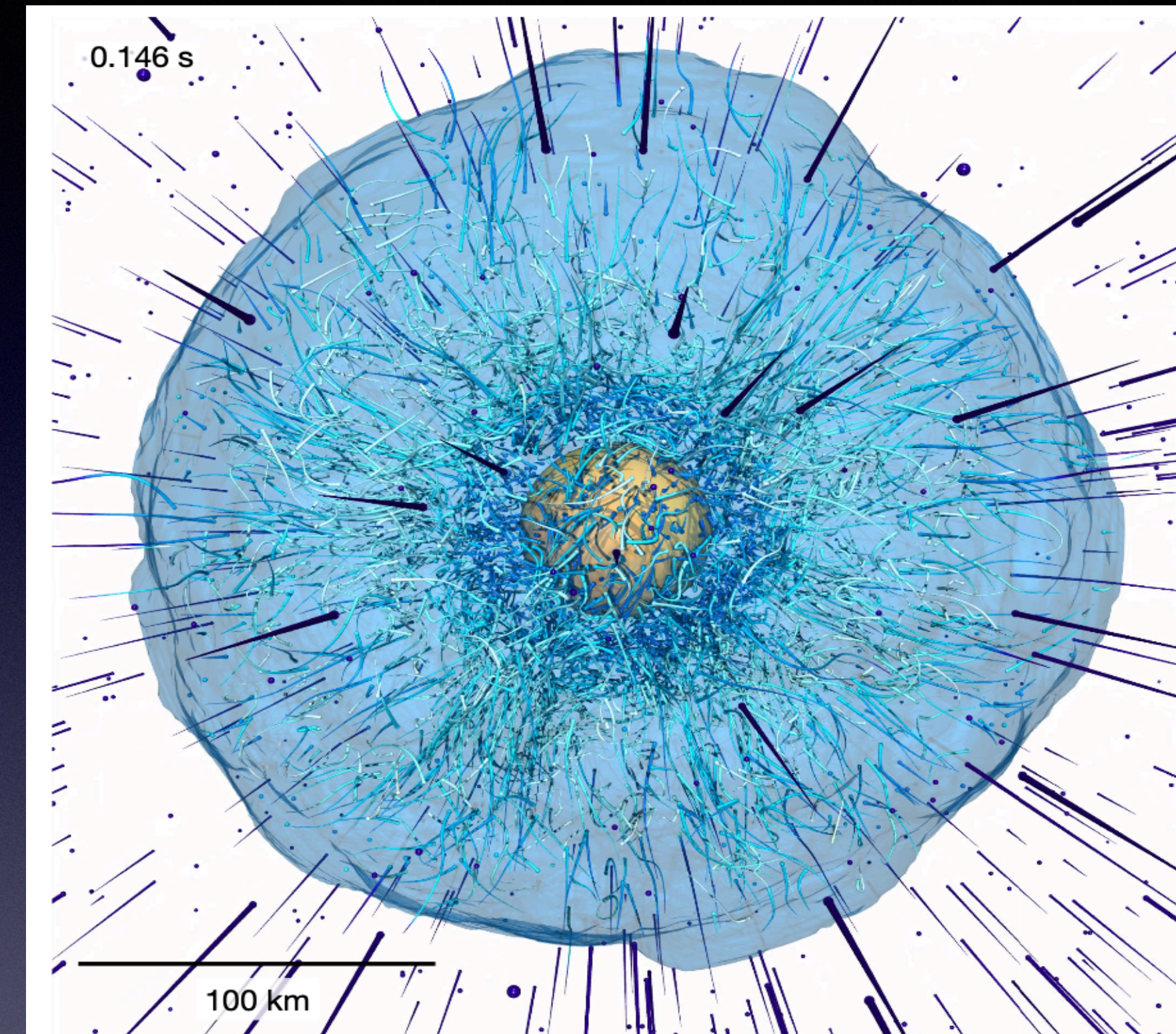
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Convection, SASI

Excitation of PNS normal modes

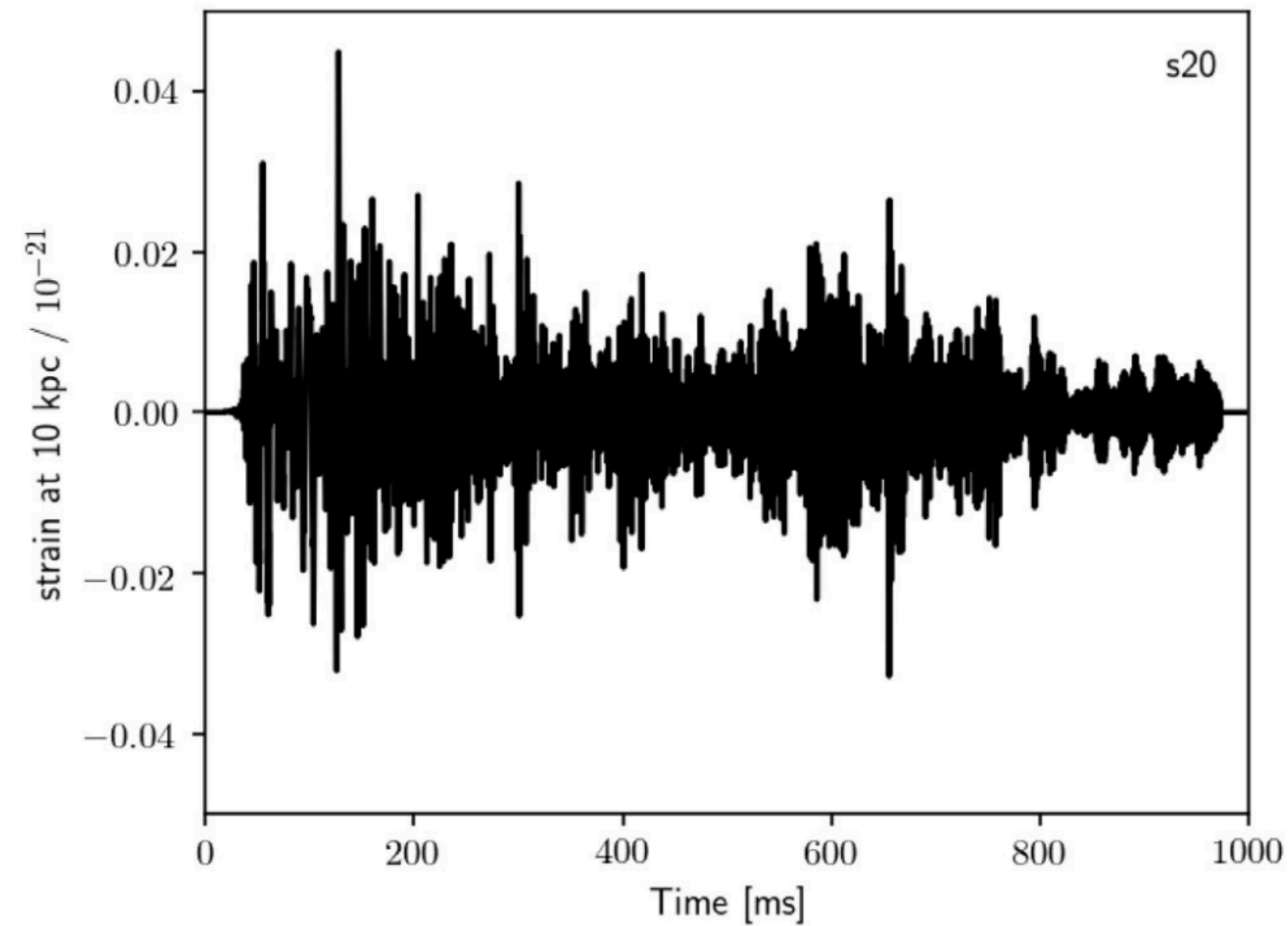
MAIN MECHANISM OF GW EMISSION!!



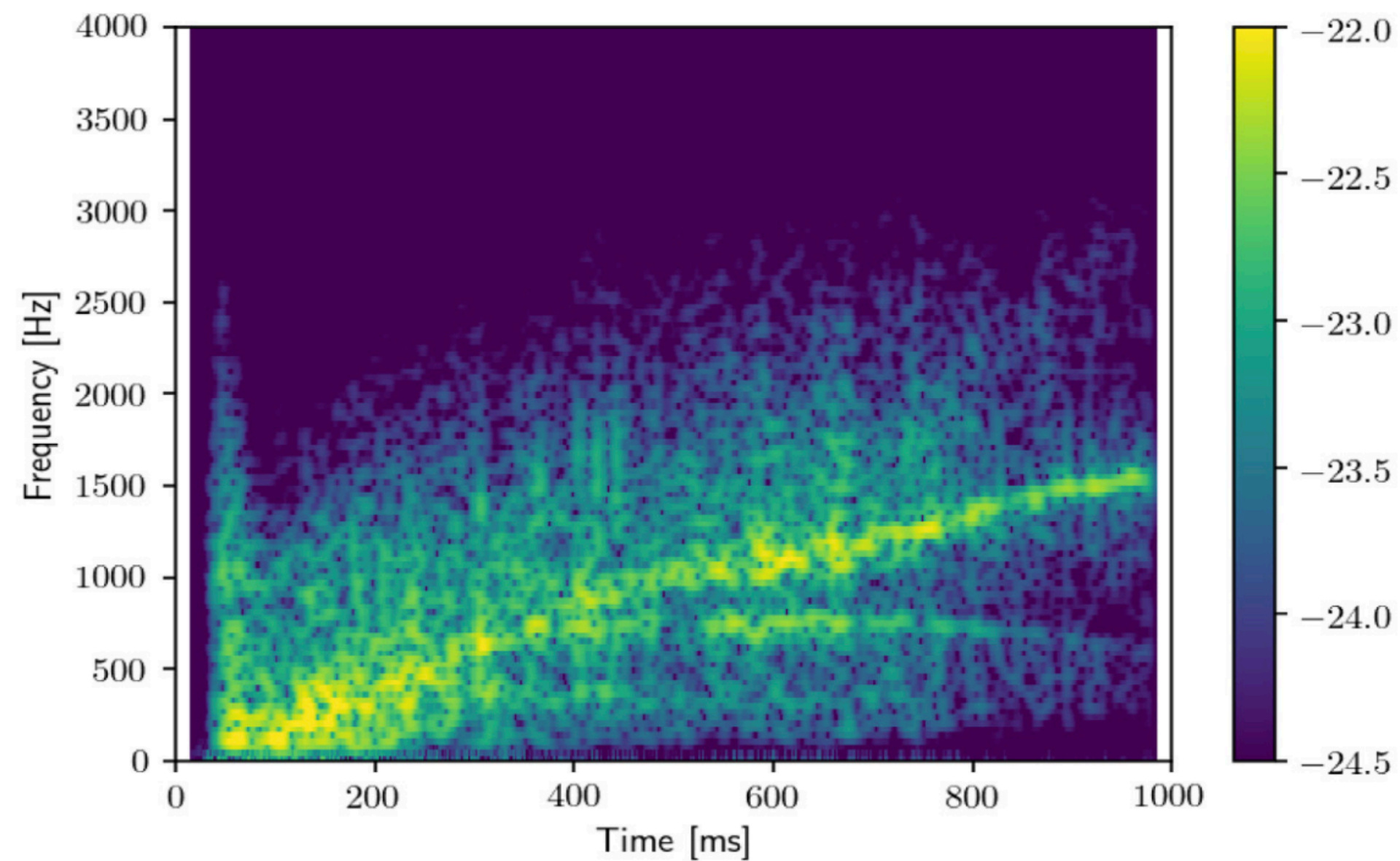
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GW signal from CCSNe



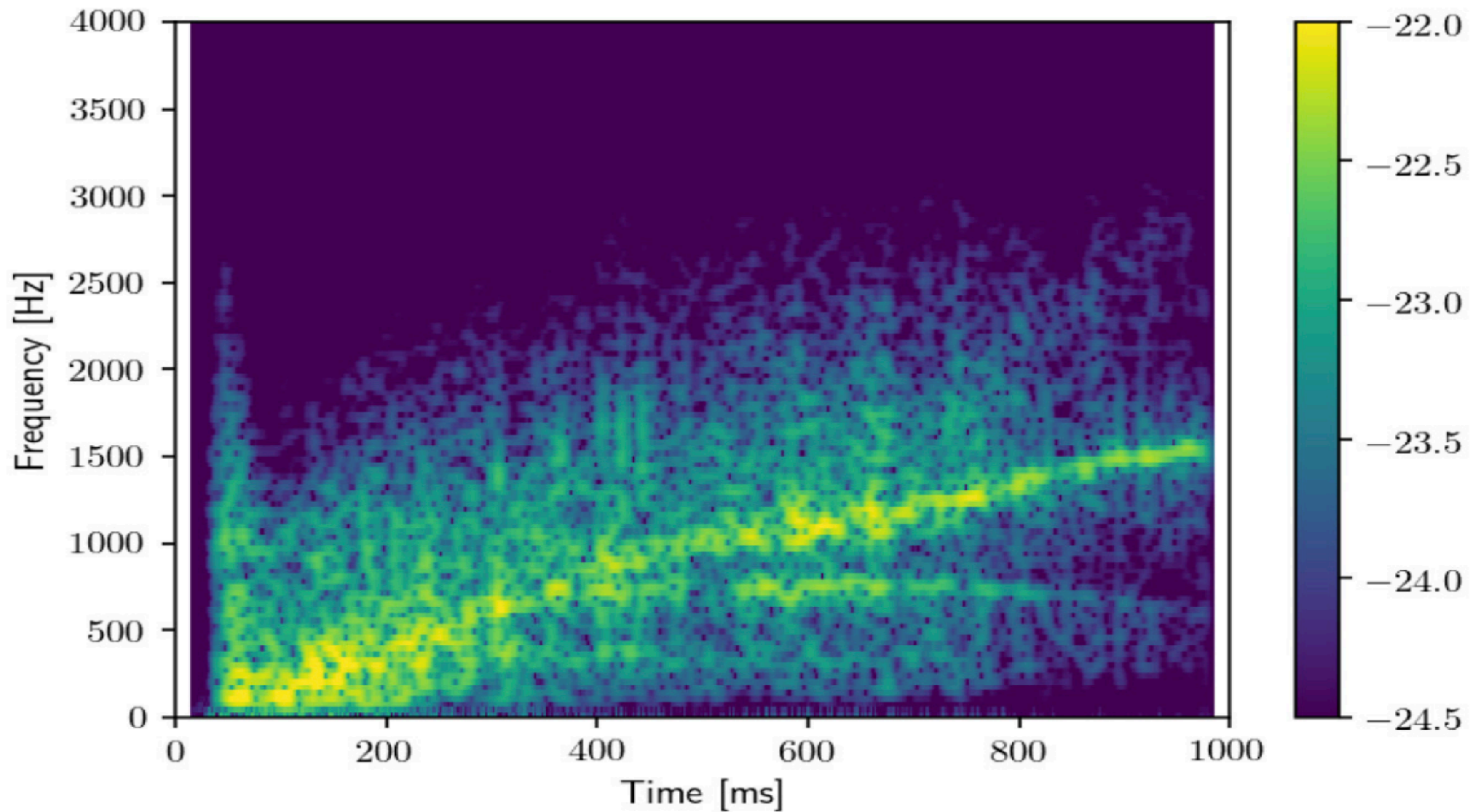
Highly stochastic!!



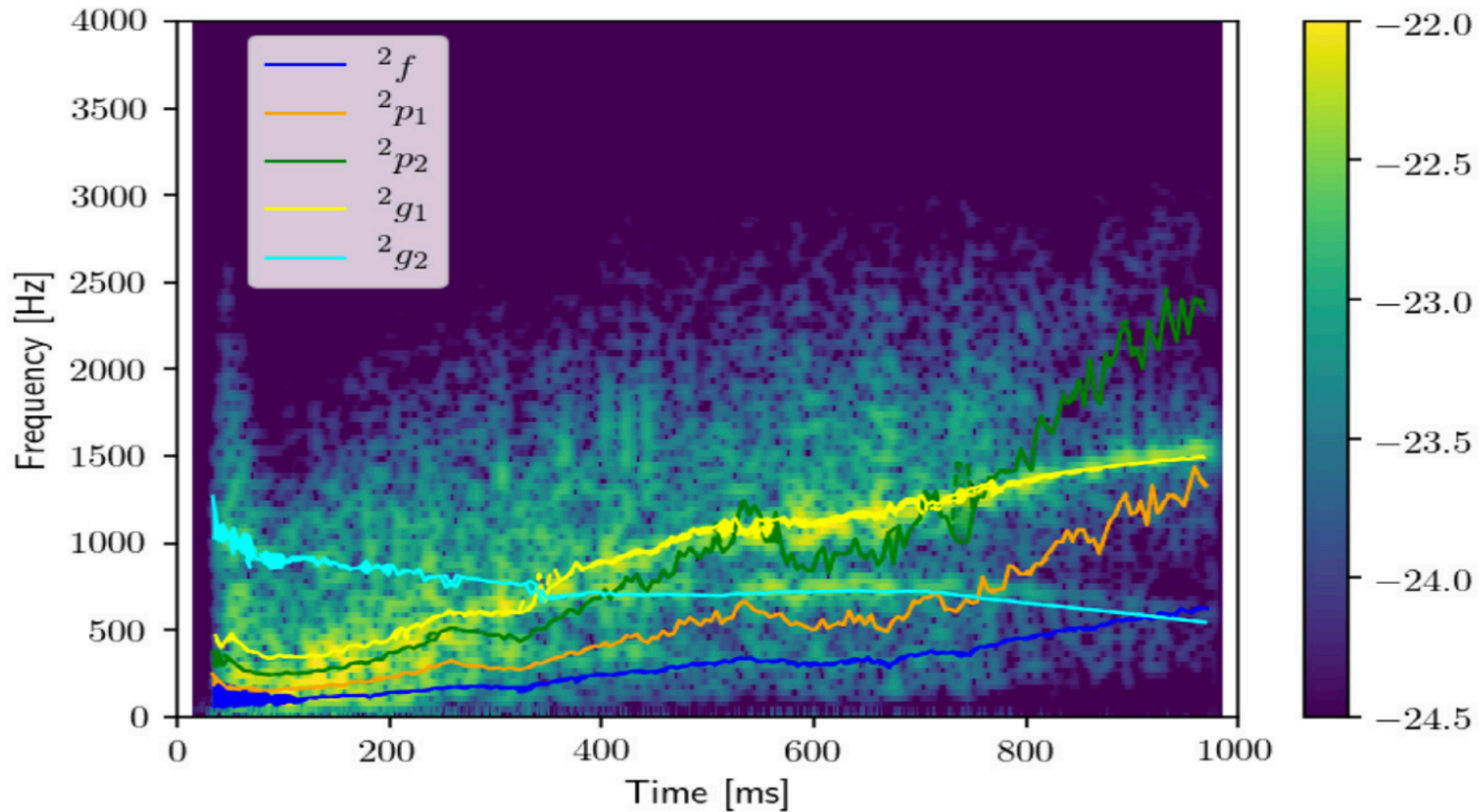
Spectrograms?

Torres-Forné et al. (2019)

Oscillation modes

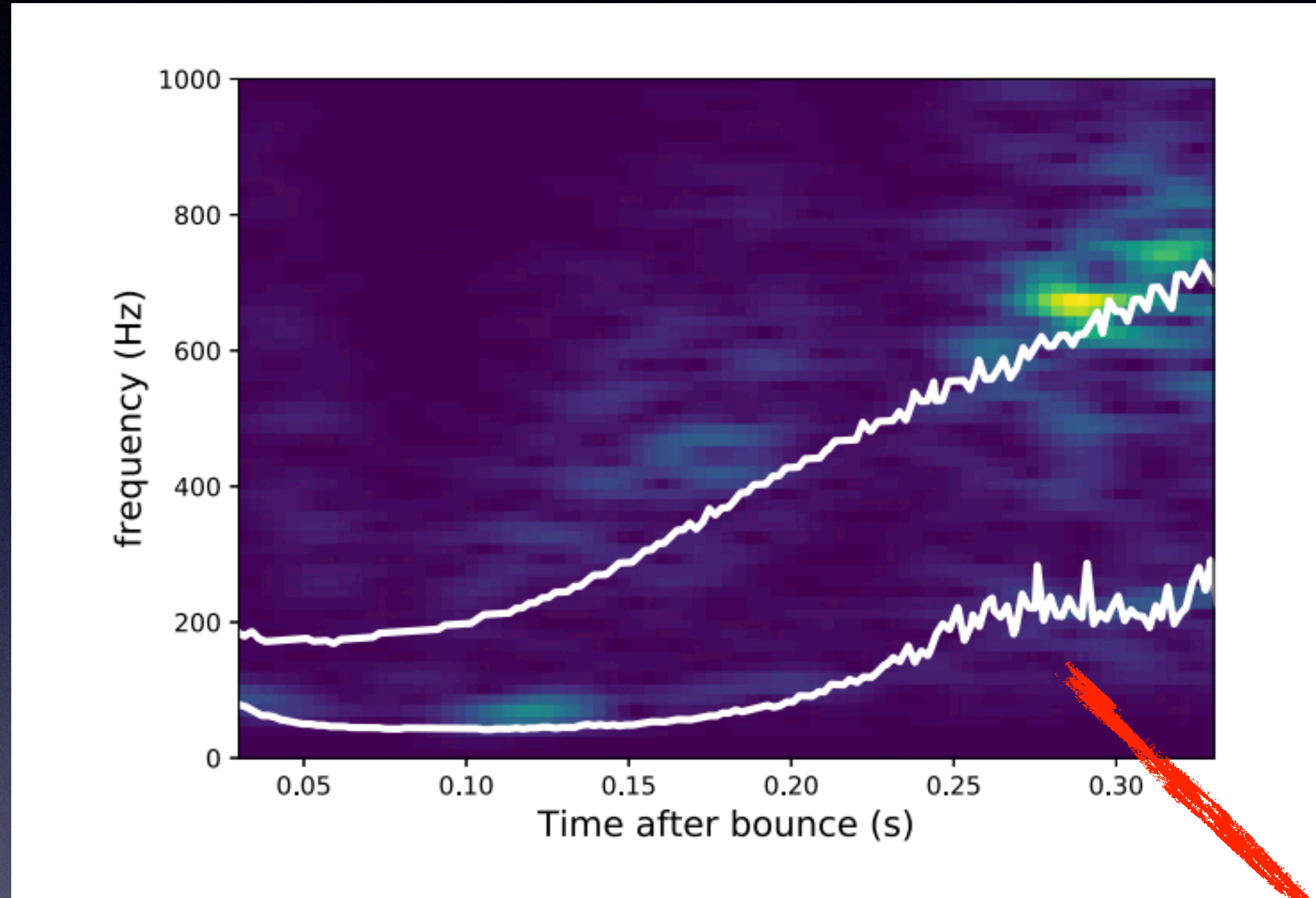


Oscillation modes

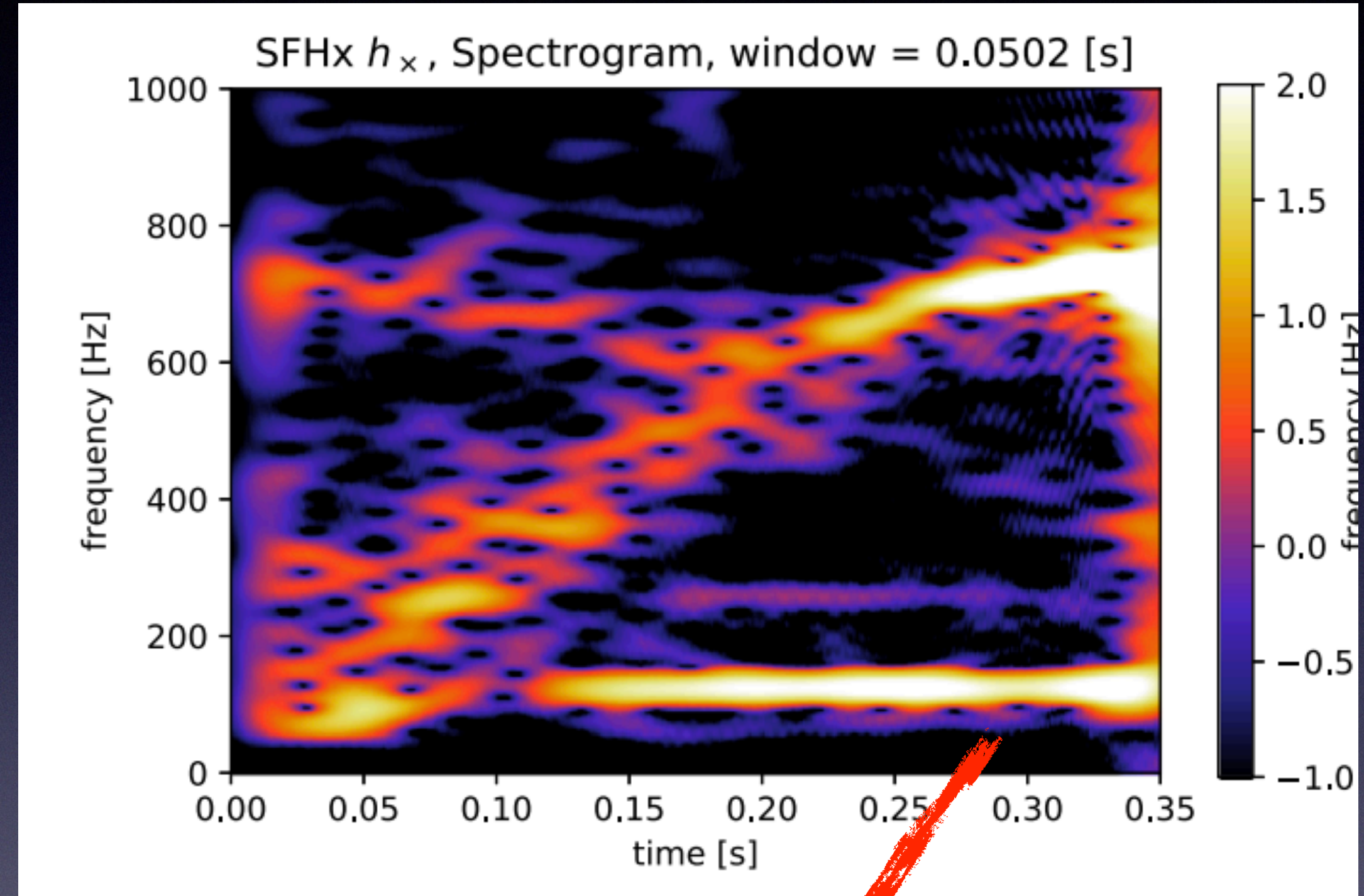


BUT ... there is more!

Powell et al. (2021)



Kawahara et al. (2018)



SASI

standing Accretion Shock instability

Asteroseismology

Recipe

- General Relativistic Hydrodynamic Equations
- Linear perturbation equations of a star in hydrostatic equilibrium
- Solve the eigenvalue problem
- Use data from CCSNe simulations
- Classify the modes
- Infer properties of the star (universal relations)

Asteroseismology

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Eigenvalue Problem

The system of equations:

σ : eigenfrequencies

$$[\mathbf{A}] \cdot [\eta] = \sigma [\mathbf{B}] \cdot [\eta]$$

η : perturbed variables matrix

A, B : coefficient matrices

+ Rankine - Hugoniot Boundary Conditions at the shock

* Spectral methods

* Physics Informed Neural Networks (PINNs)

Spectral Methods

- ▶ Chebyshev collocation method
- ▶ Differentiation matrix D
- ▶ Multi-domain MN ; M : # of domains, N : # of points
- ▶ Discretised system of S equations $\rightarrow S \times (MN + 1)$ matrices

ADVANTAGES

Exponential convergence - fast!

Small grids

Discretised equations in matrix form

All modes (also complex!) are computed at once

Regularity of the solution is ensured

Cases of study

$$l=2$$

1. p- and g-modes in a sphere

- > Ideal fluid with $c_s^2, \Gamma_1 \rightarrow \text{constant}$ and $v \rightarrow 0$
- > Buoyant region
- > Varying Brunt–Vaisälä frequency $\mathcal{N}^2 \equiv \frac{\alpha^2}{\psi^4} \mathcal{G}_\alpha \mathcal{B}_\alpha$

2. Plane Waves in an 1D flow

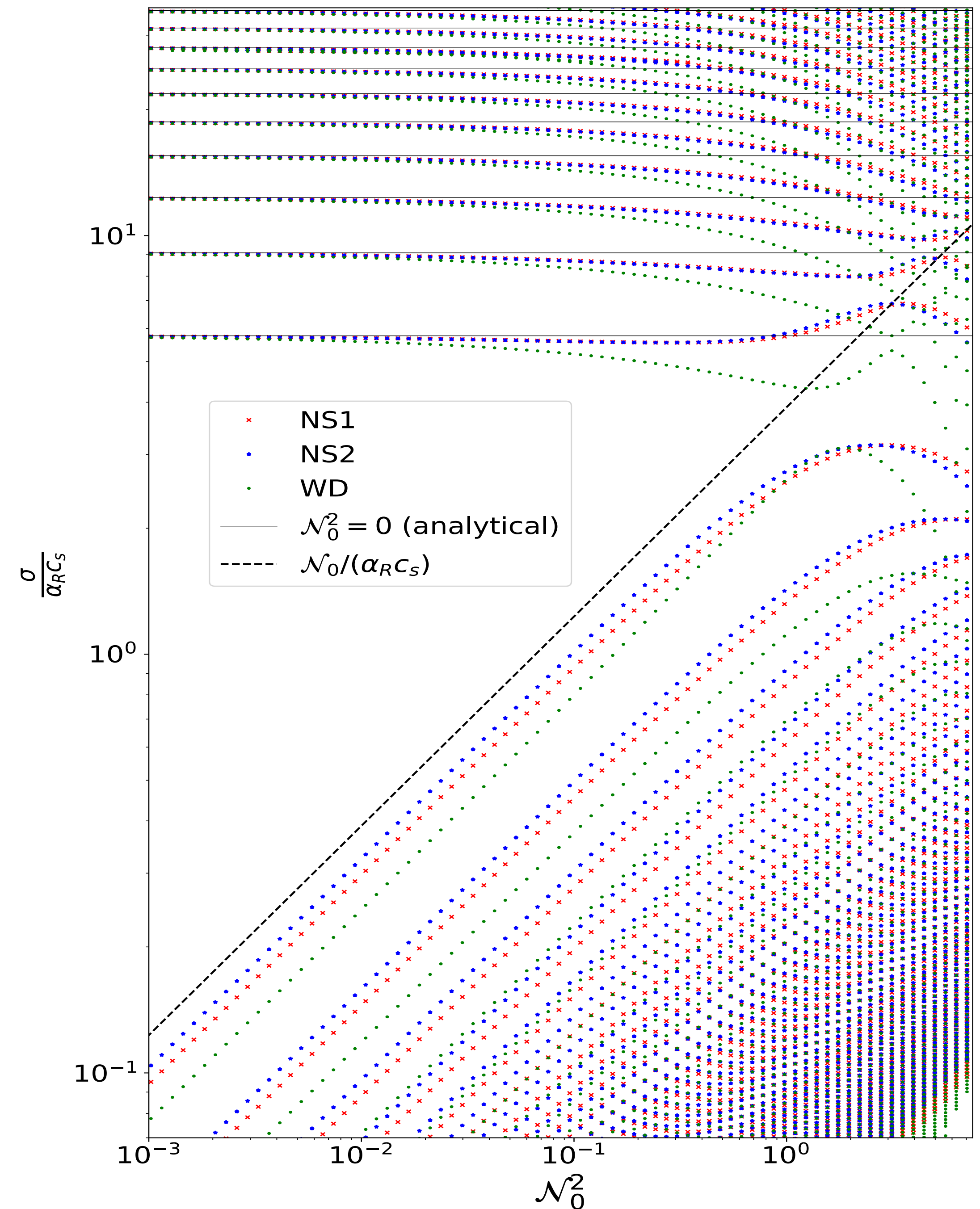
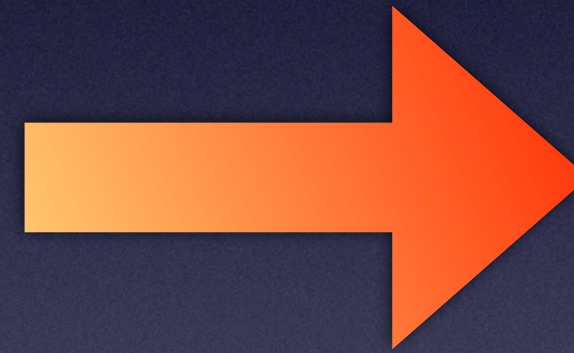
3. Accretion flow in a fluid sphere

- > $\mathcal{N}^2 = 0$
- > Sound speed, pressure, density $\rightarrow$ constant
- > Lapse $\rightarrow$ general
- > Velocity $< c$ and subsonic

Case I: Eigenfrequencies

Models

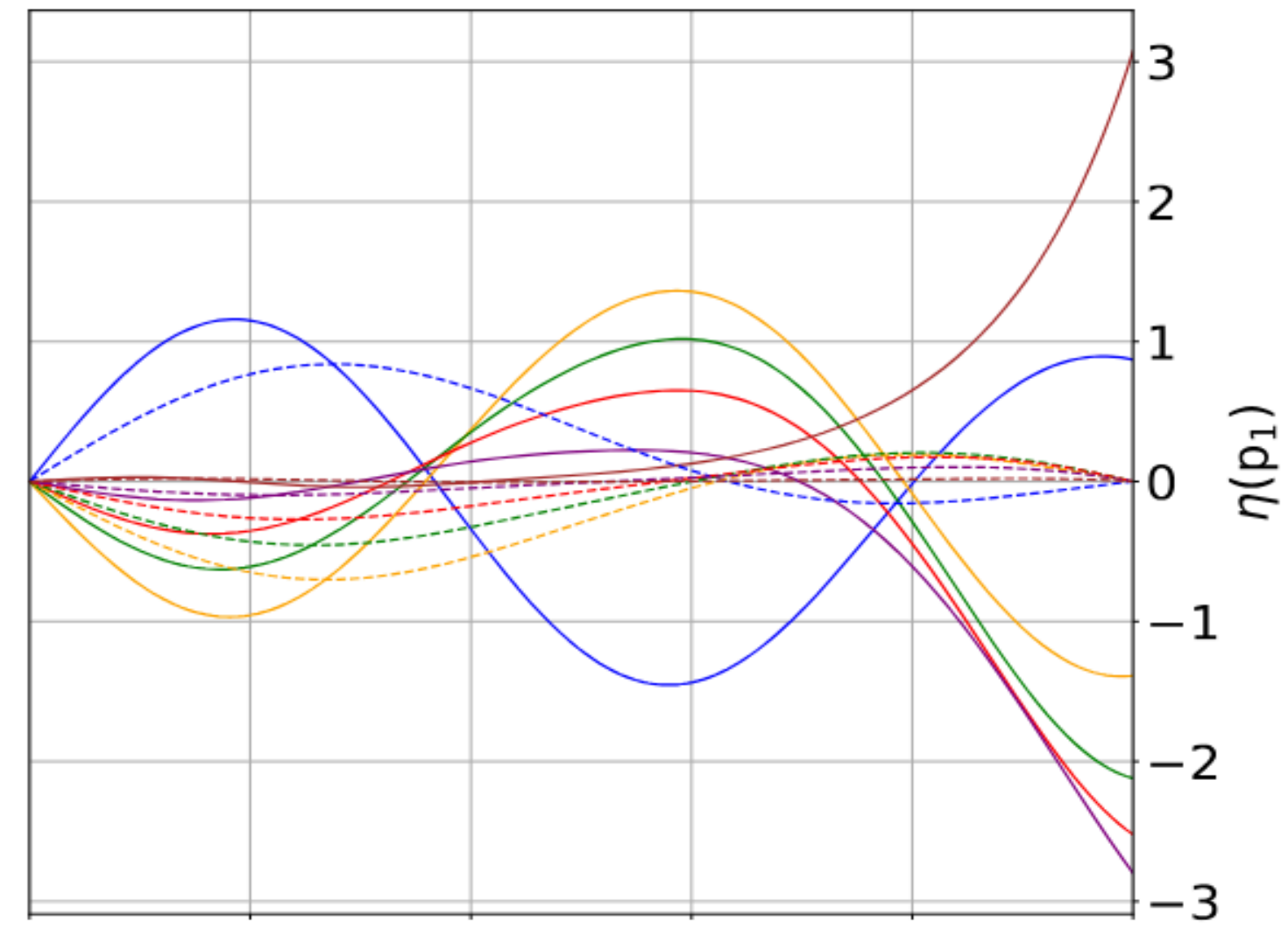
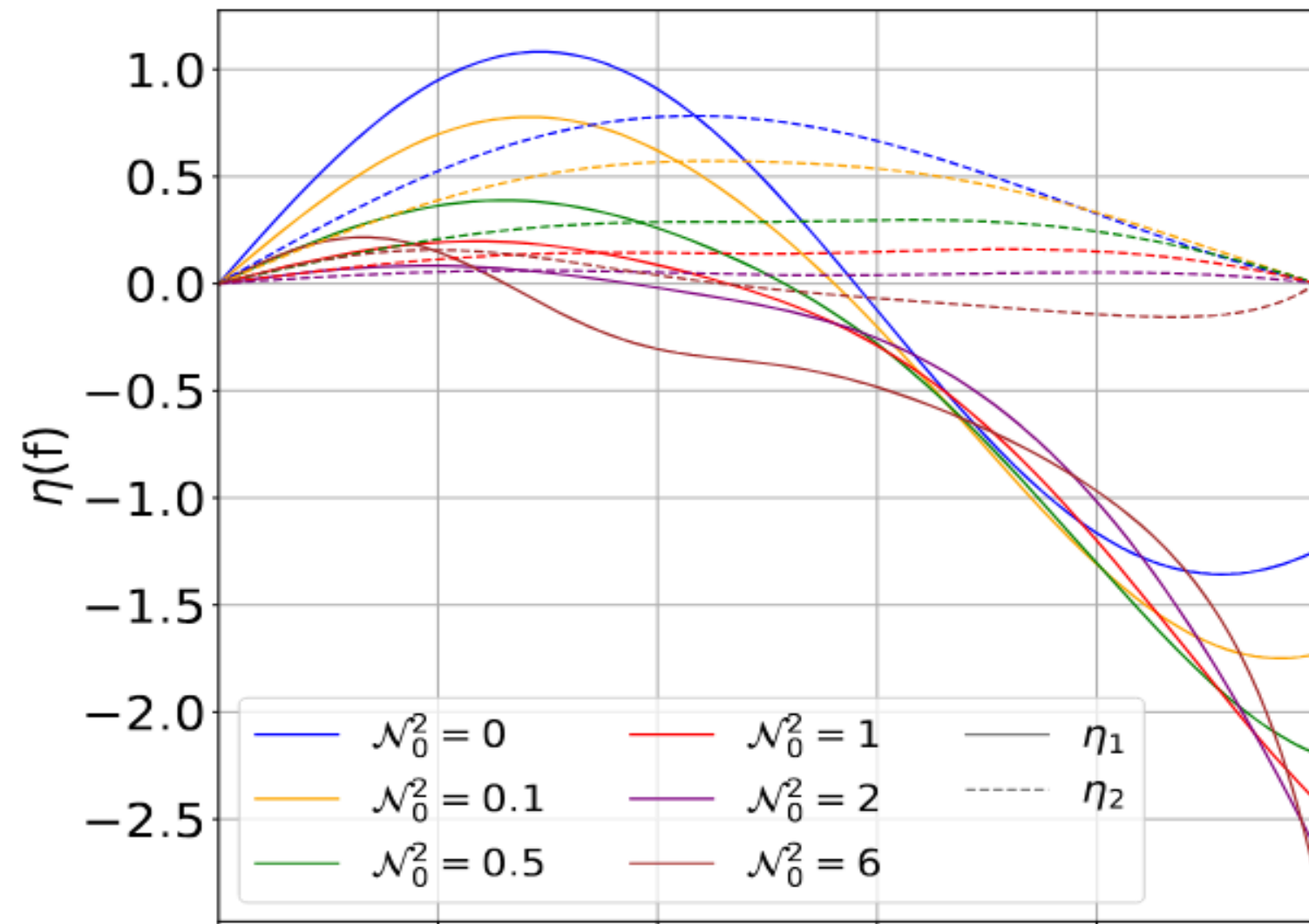
model	M ($M_{\odot}$)	R(km)	Γ_1	c_s^2	α_R
NS1	1.4	10	2	0.1	0.81
NS2	2.0	10	2	0.1	0.74
WD	1.2	$5 \cdot 10^3$	4/3	0.1	0.84



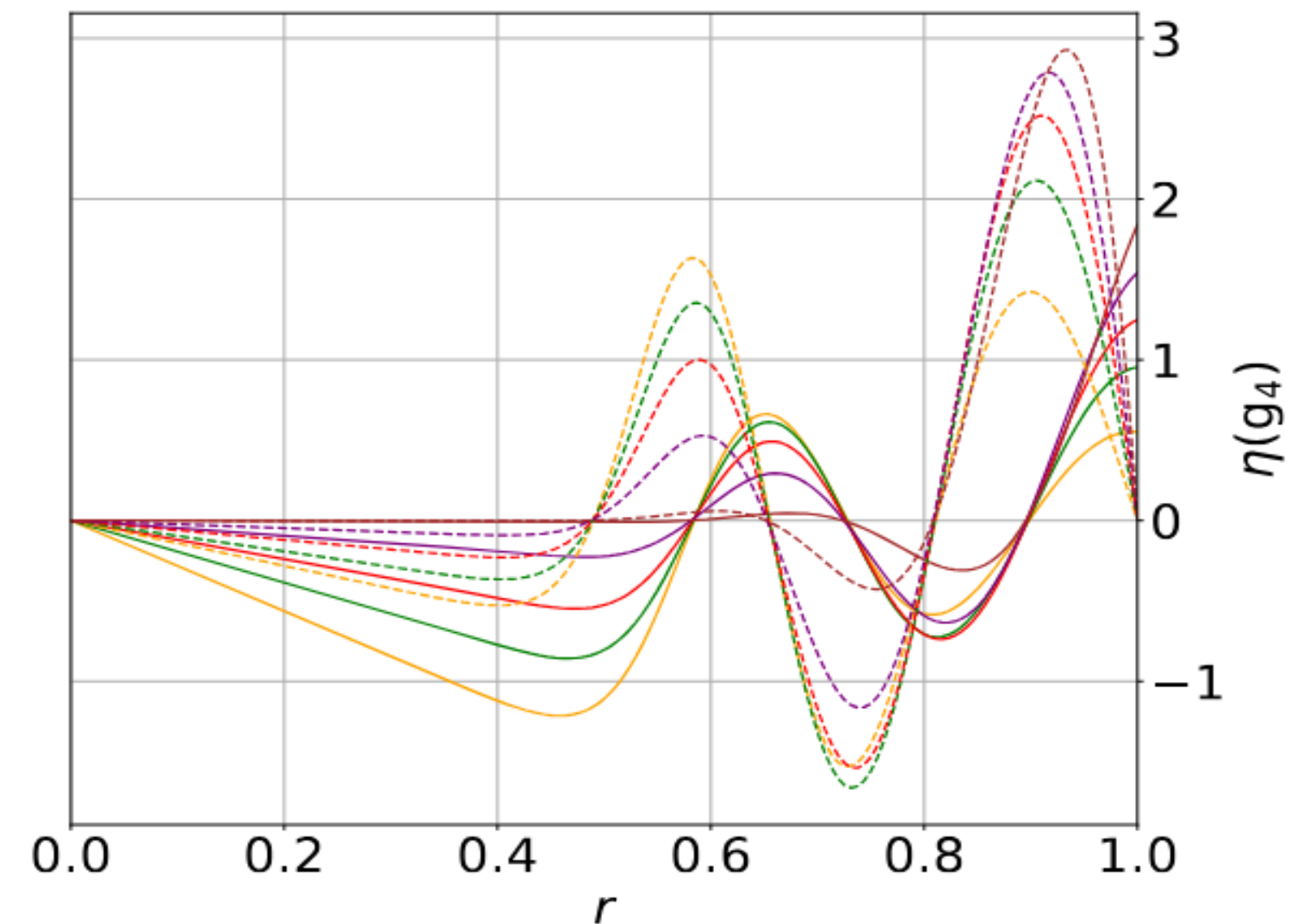
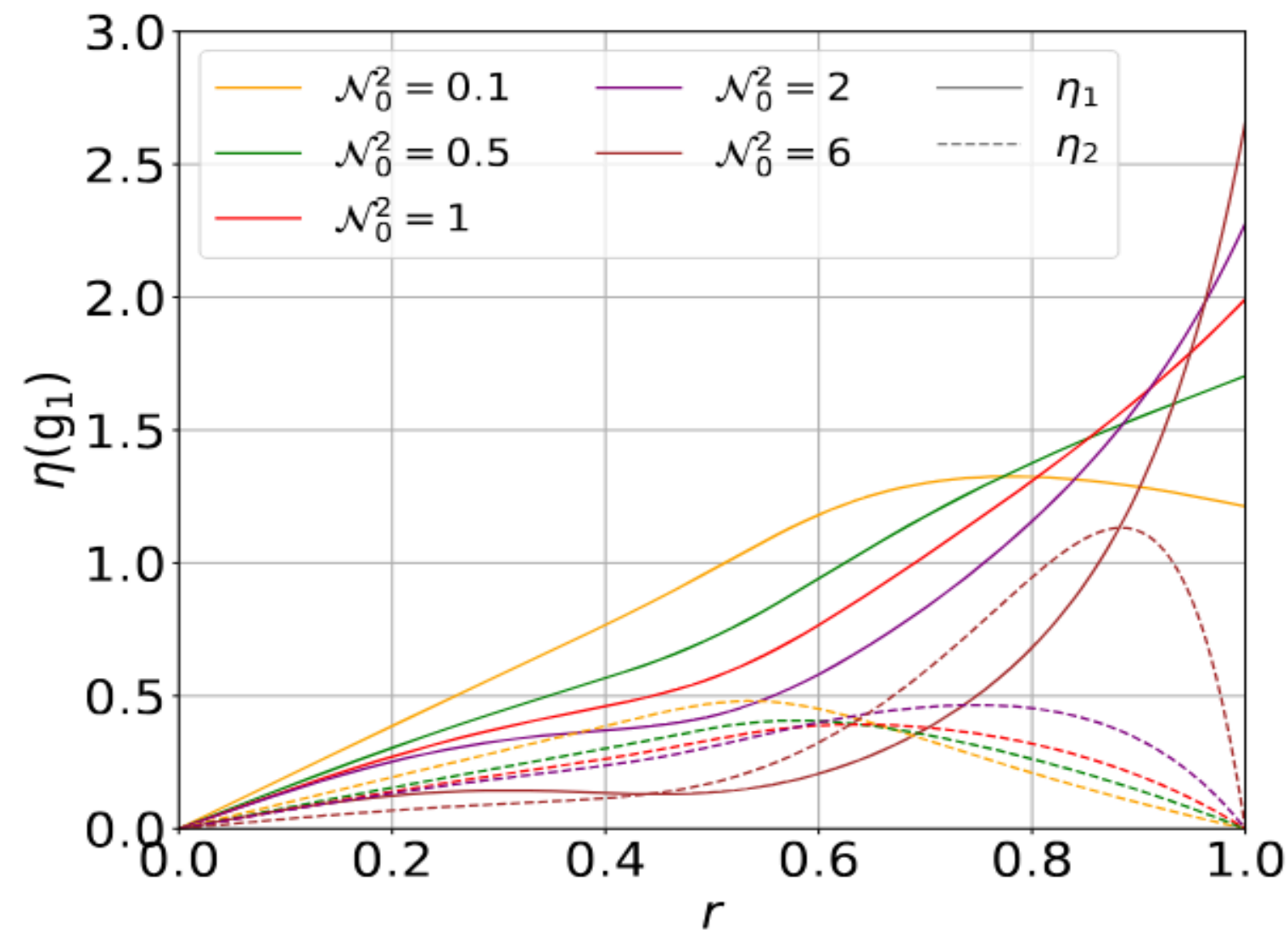
DT et al. (2025)
arXiv:2503.16317

Case I: Eigenfunctions

p-modes

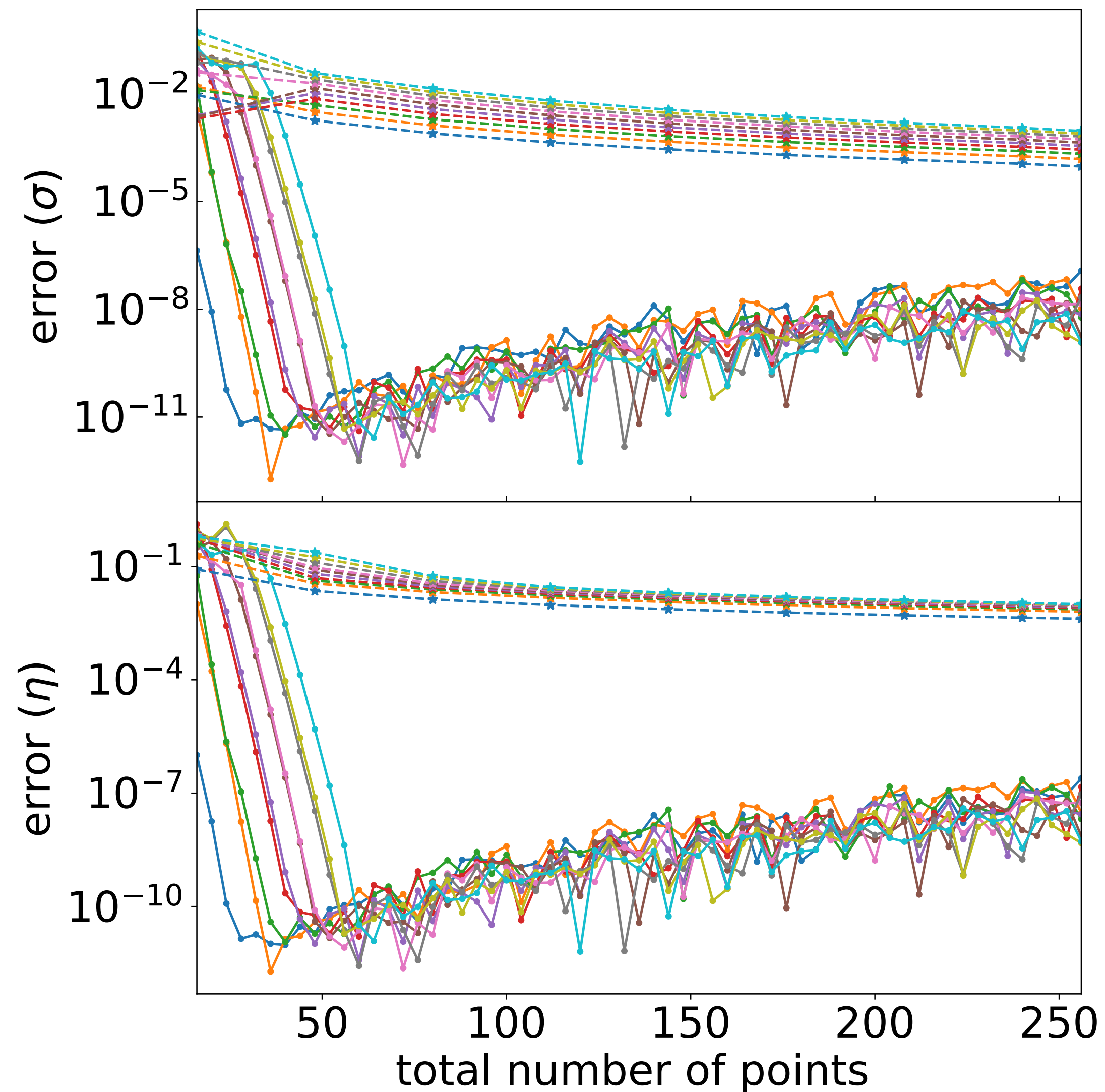


g-modes



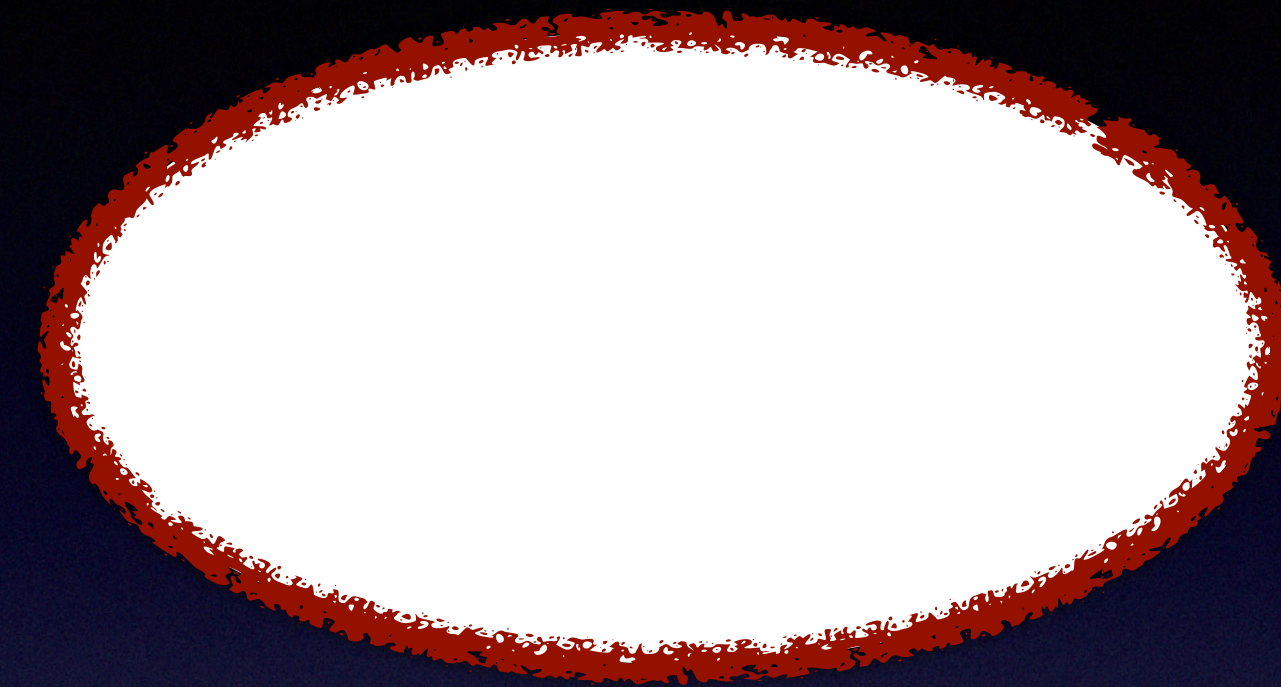
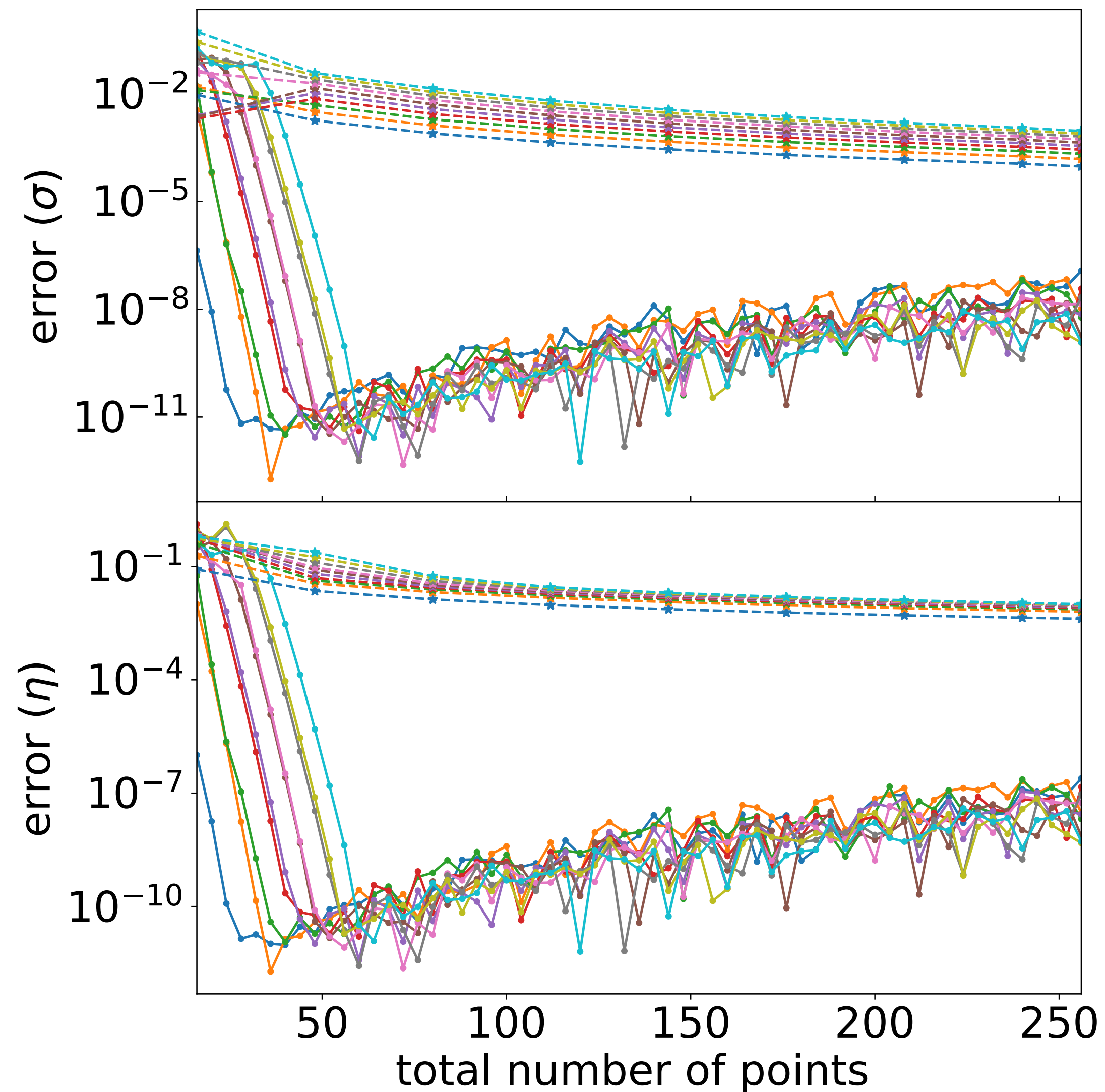
Case I: Convergence

- $N^2 = 0$
- Comparison to analytical solutions



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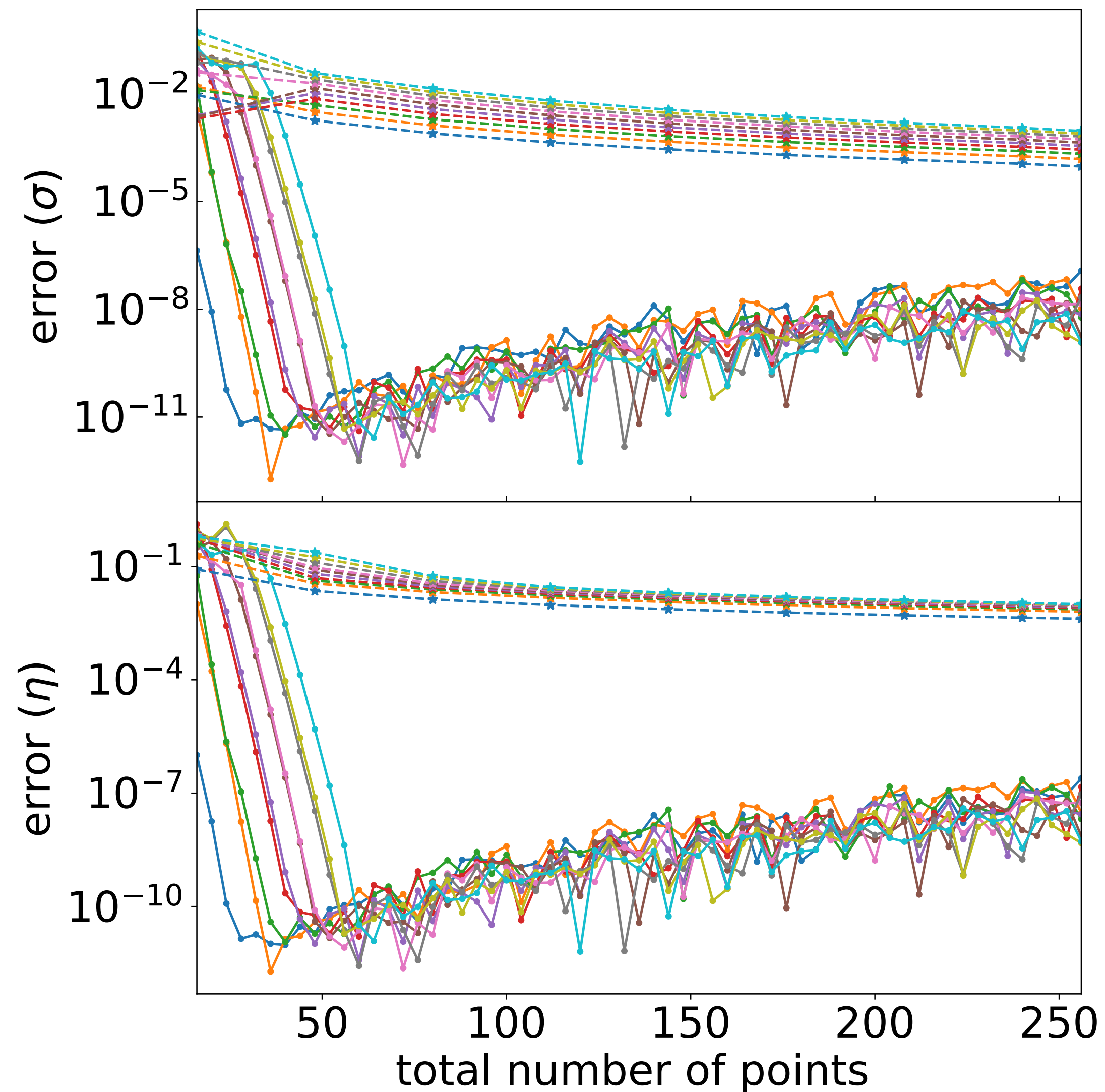
- ▶ $N^2 = 0$
- ▶ Comparison to analytical solutions



f (5.76)	p_6 (25.01)
p_1 (9.10)	p_7 (28.17)
p_2 (12.32)	p_8 (31.32)
p_3 (15.51)	p_9 (34.47)
p_4 (18.69)	Spectral Methods
p_5 (21.85)	GREAT

Case I: Convergence

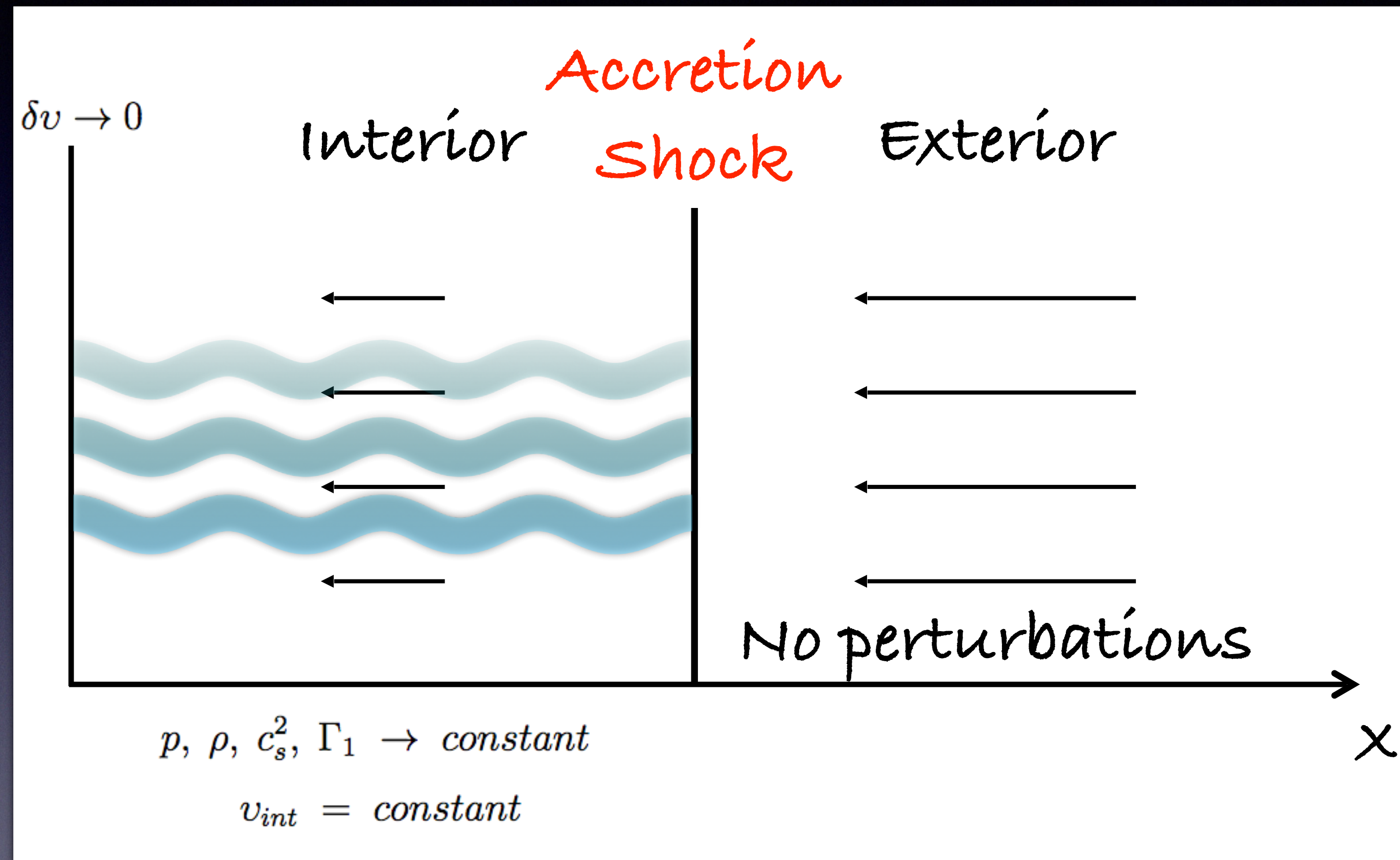
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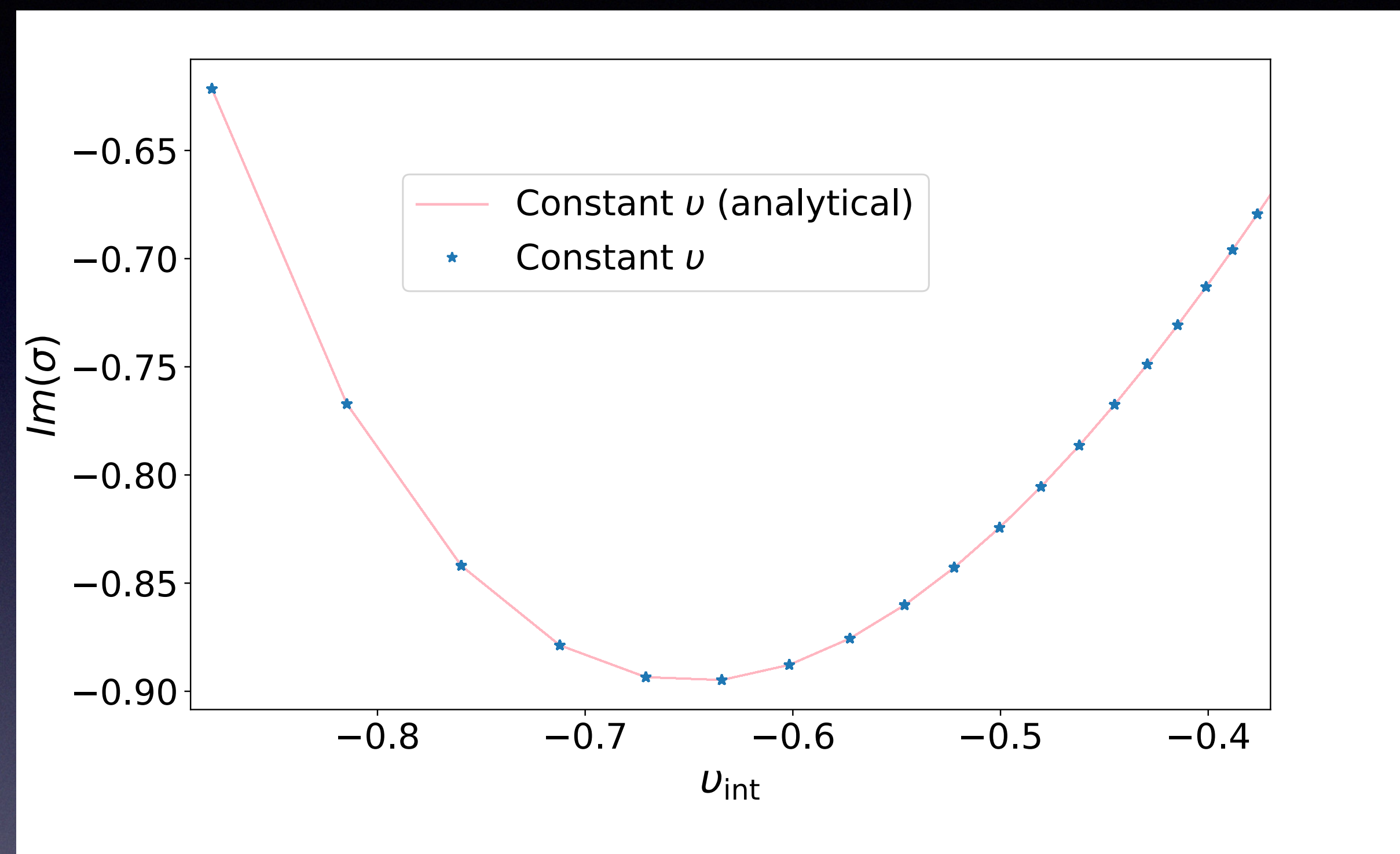
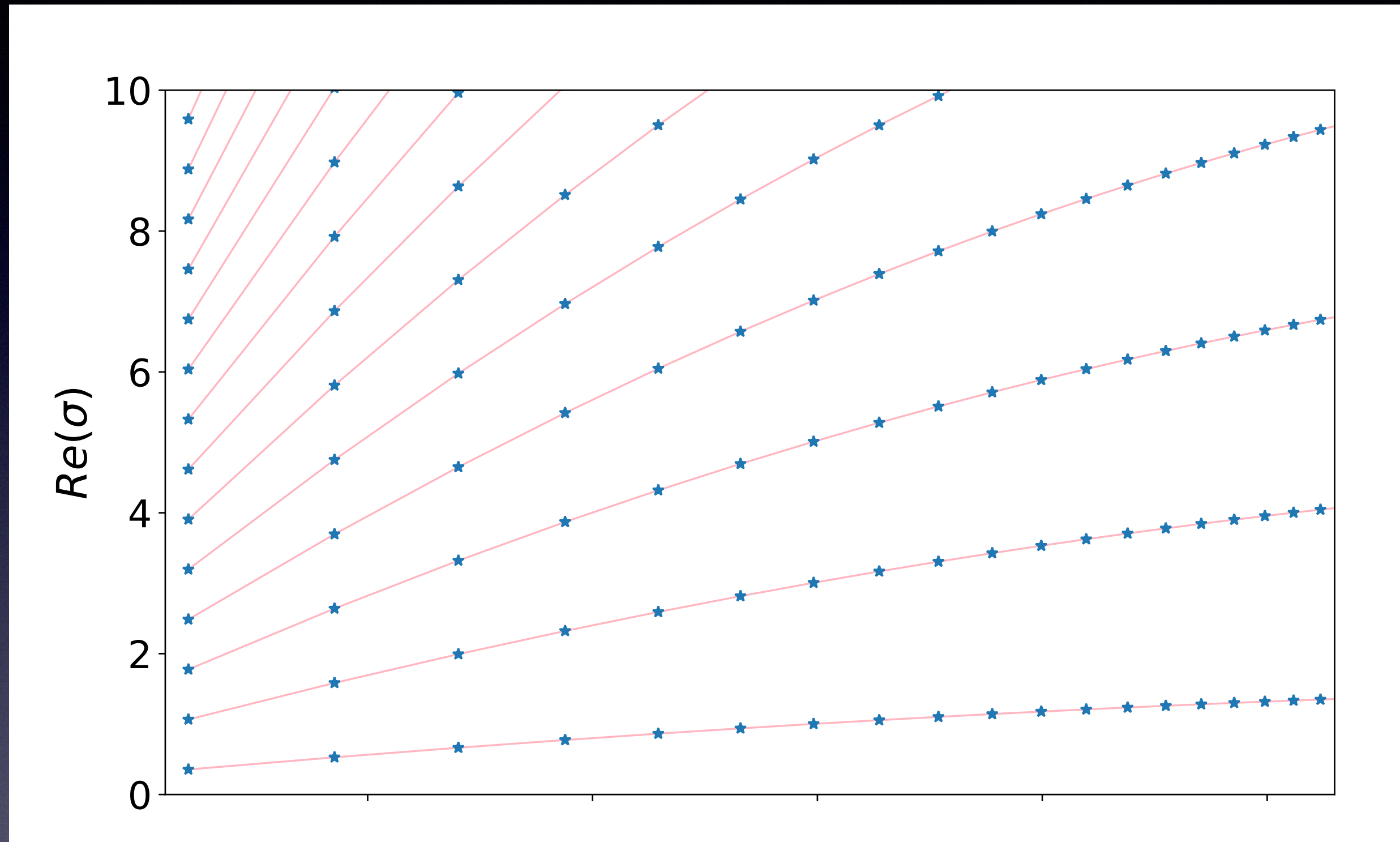
Exponential
convergence!

Case II: Plane Waves

Newtonian
analysis



Case II: Eigenfrequencies

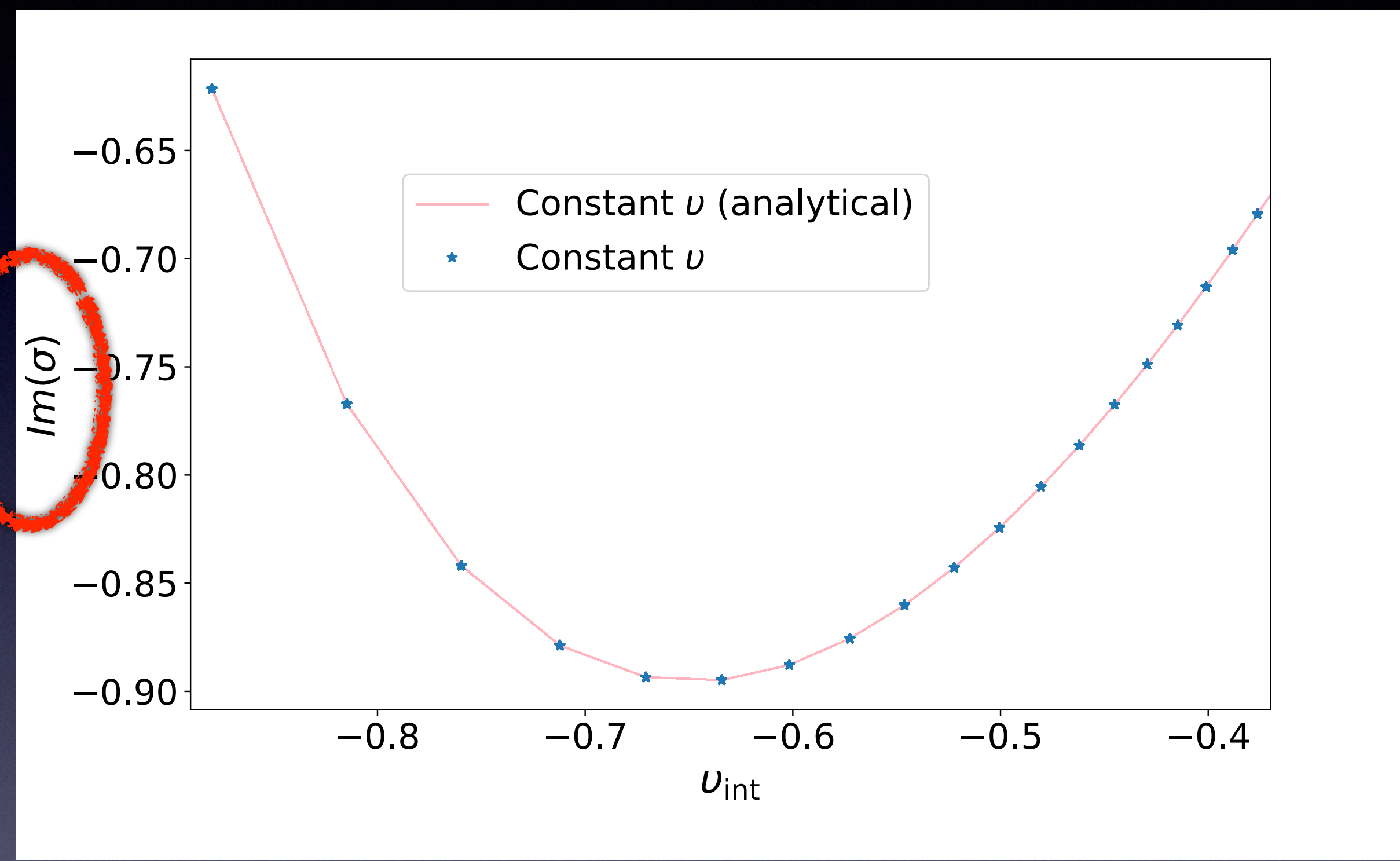
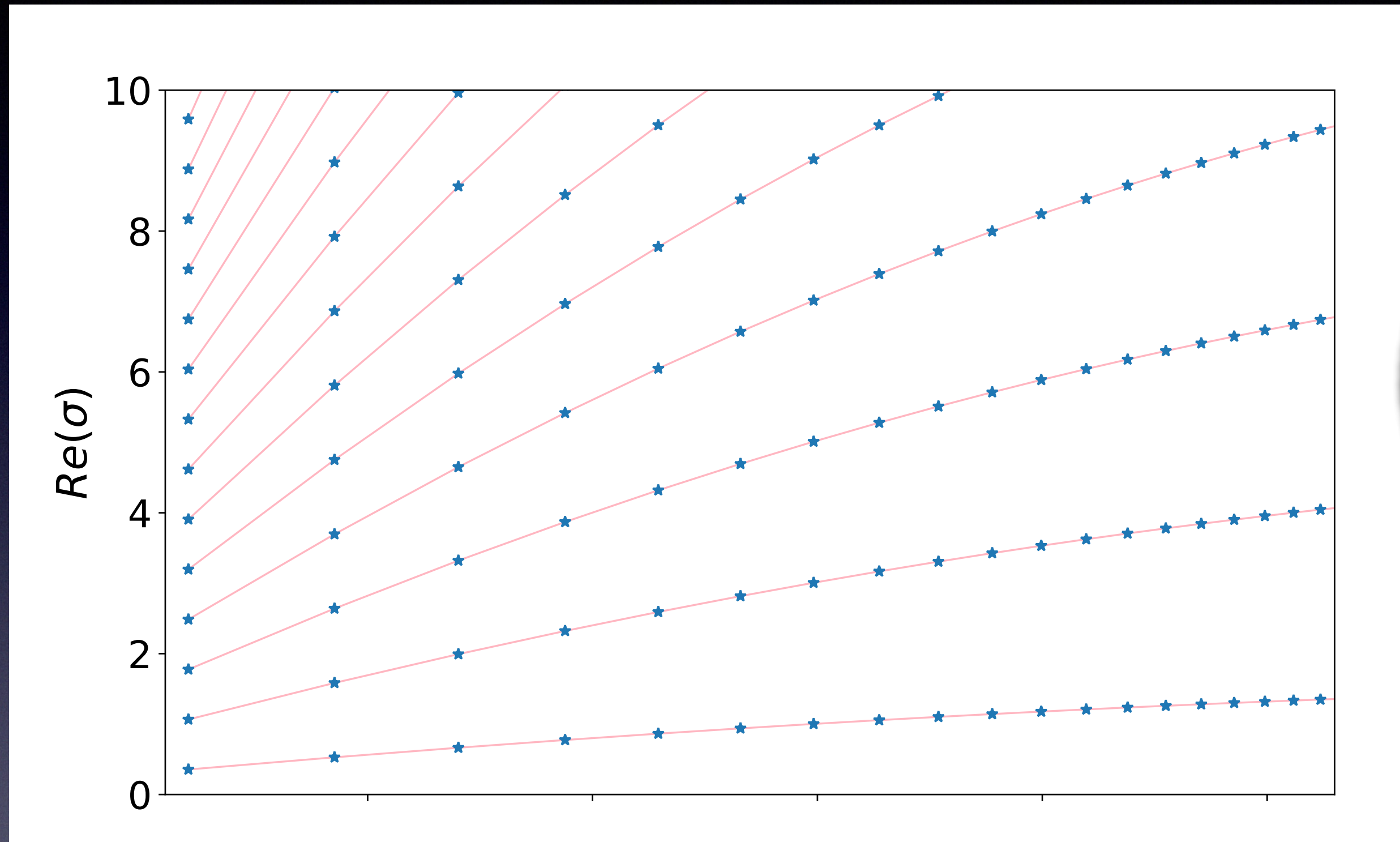


- ★ Analytical and numerical solutions agree
- ★ **p-modes** with **decreasing frequency**

- ★ Frequencies have imaginary part
- ★ **Stable** modes

DT et al. (2025)
arXiv:2503.16317

Case II: Eigenfrequencies

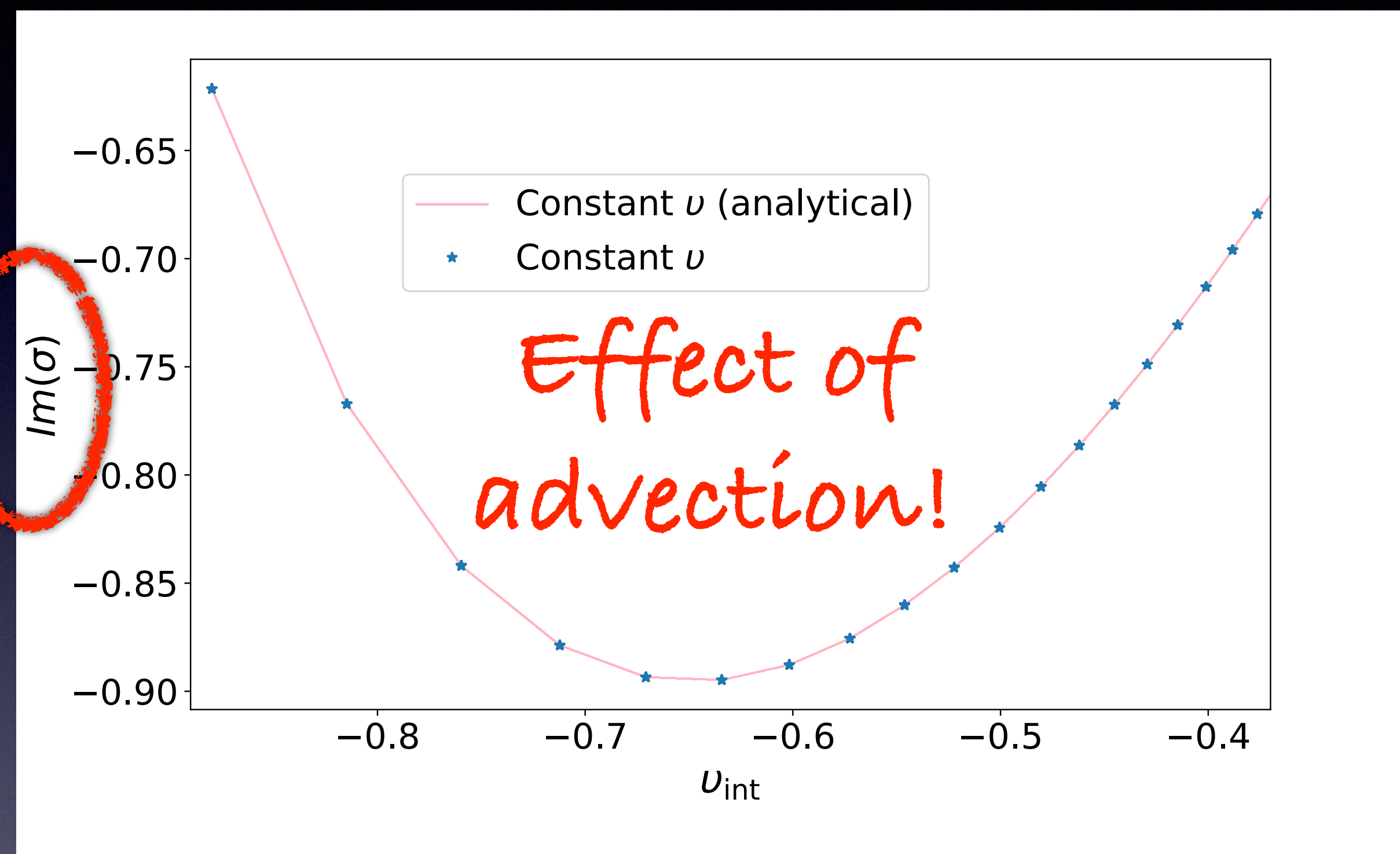
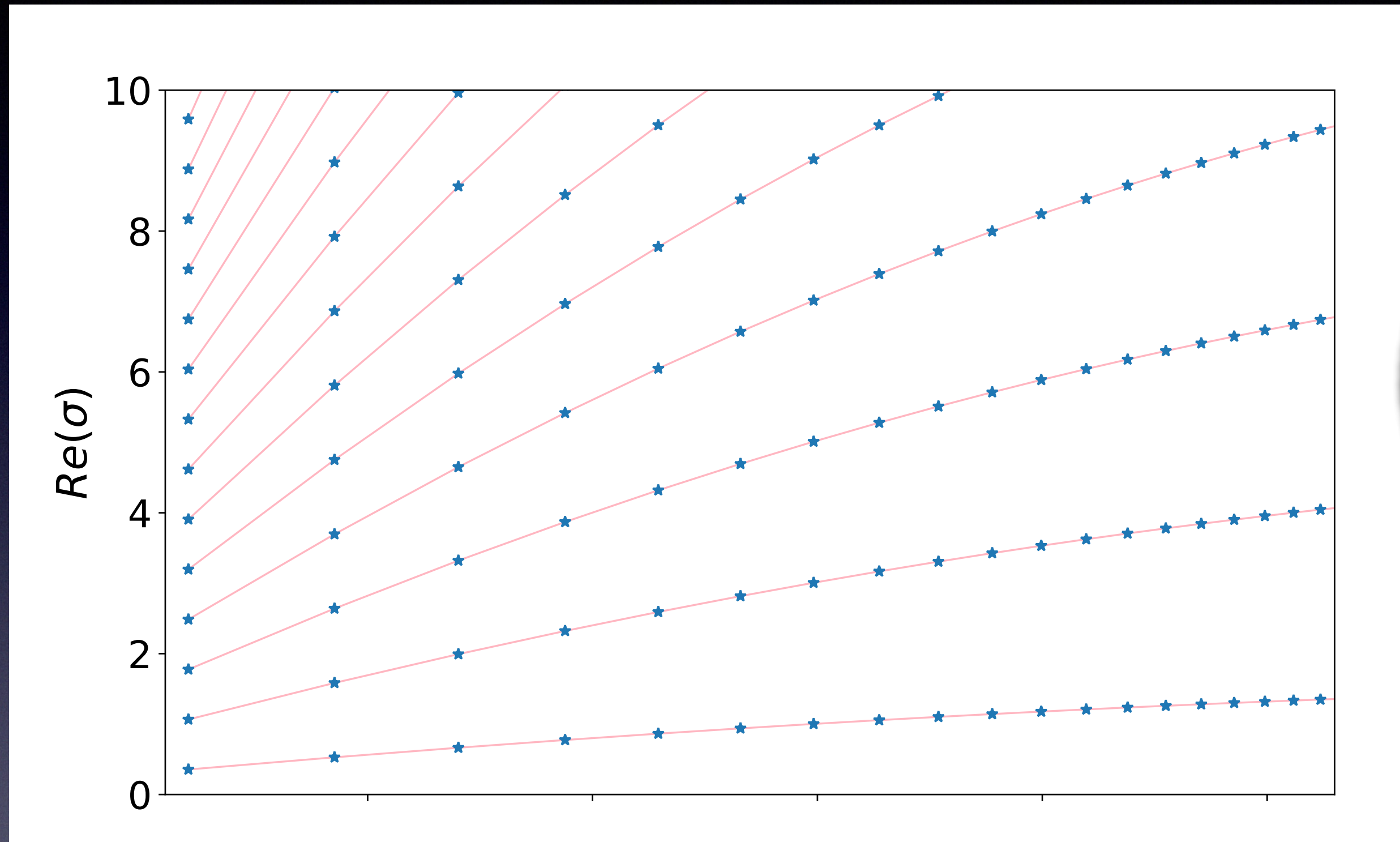


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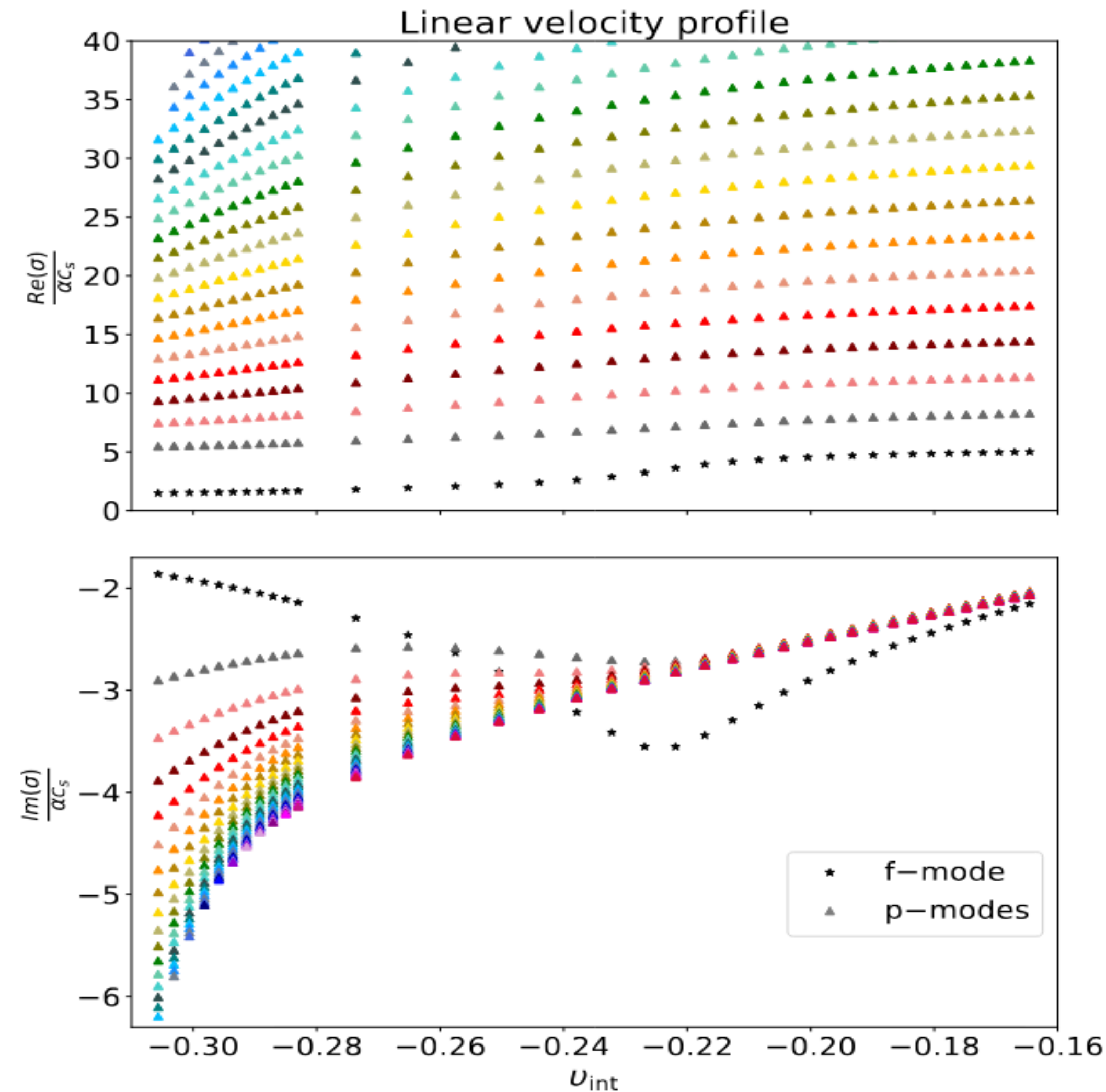
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Case III: Eigenfrequencies

STABLE

Real part
of the frequency

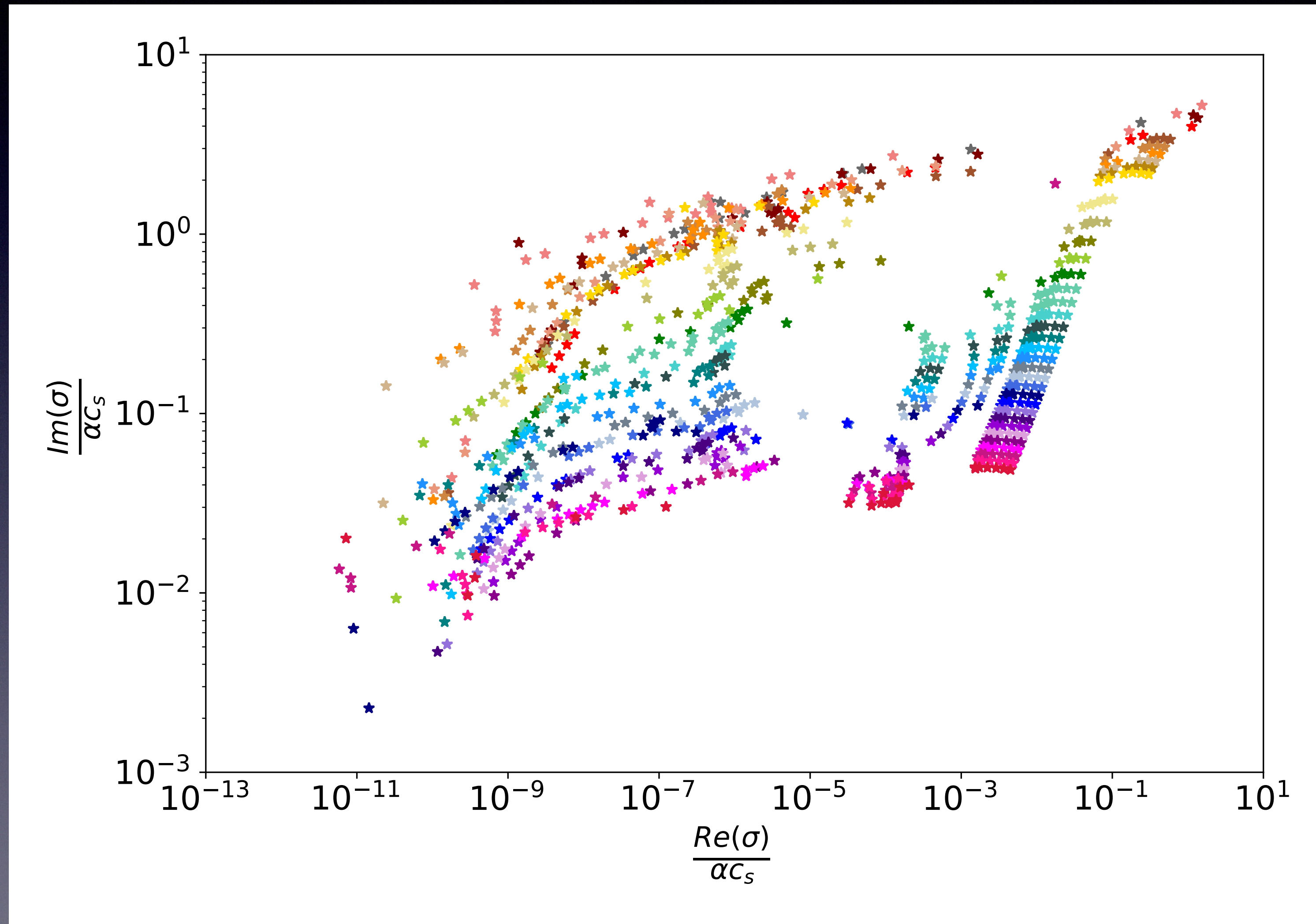
Imaginary part
of the frequency



Case III: Eigenfrequencies

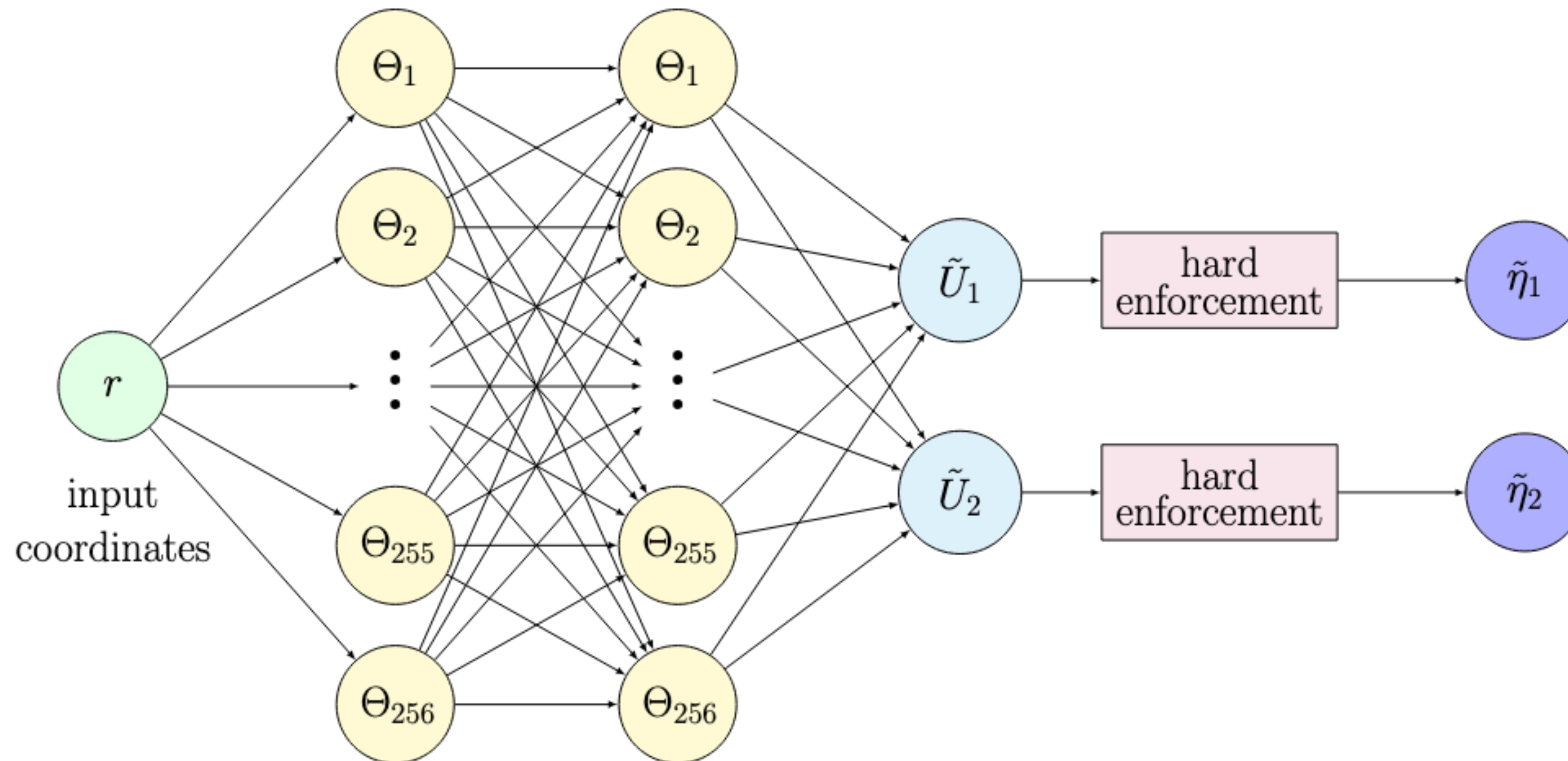
UNSTABLE

Colour scale:
interior velocity



DT et al. (2025)
arXiv:2503.16317

Implementing PINNs



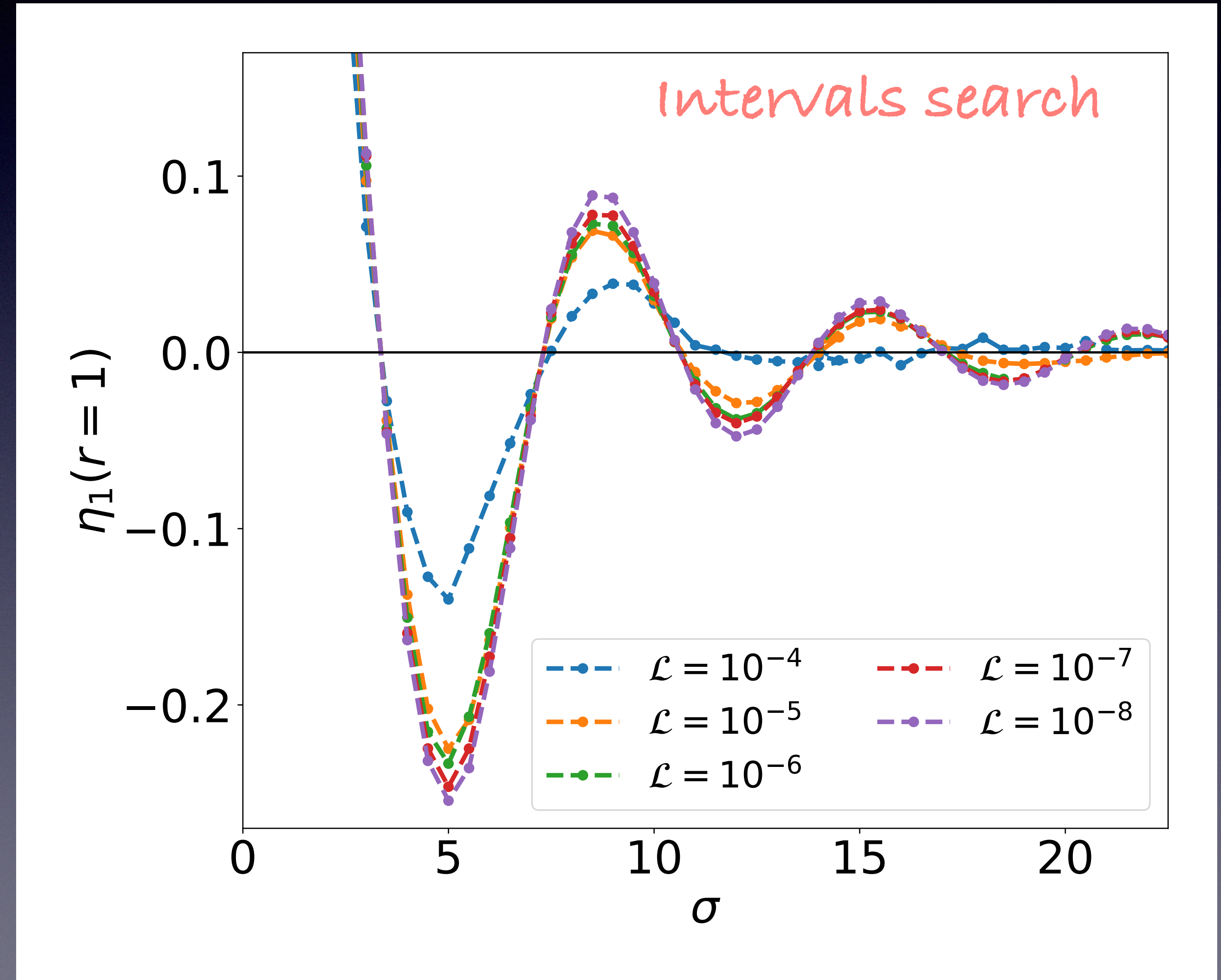
DT et al. (2025)
(arXiv:2504.12183)

PINNs I

- Static sphere of radius unity, density and sound speed
- $N^2 = 0$
- Analytical solutions

1. Find the intervals containing eigenvalues (s-PINN)

DT et al. (2025)
(arXiv:2504.12183)



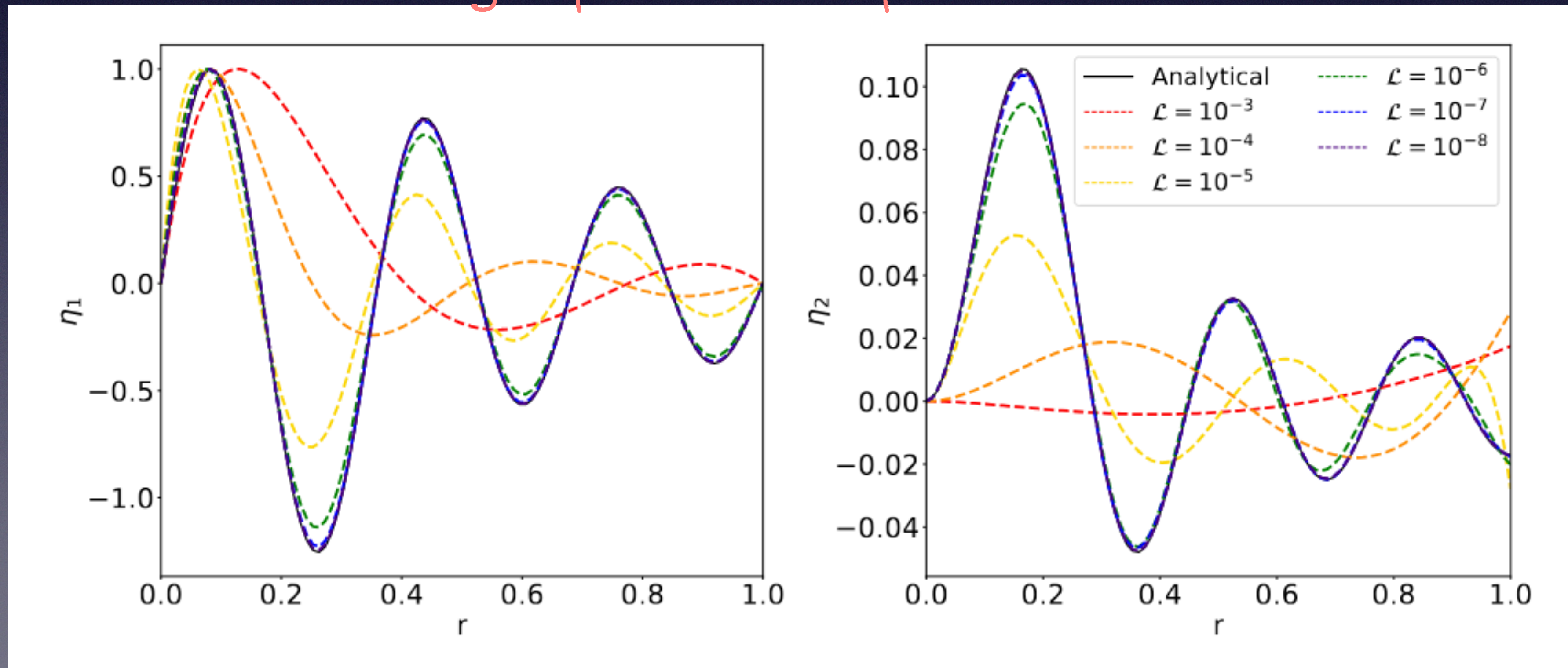
PINNs II

2. Calculate the eigenvalues using:

a. Bisection method (s-PINN) **VS**

b. f-PINN: the frequency as a parameter

Eigenfunctions of the 5th mode



Summary

Asteroseismology of CCSNe is very important because it could allow us to infer properties of the PNS directly from GW observations

- ✓ Included for the first time accretion flow + standing accretion shock
- ✓ Introduced two eigenvalue solvers: Spectral Methods + PINNs

Future work

- ▶ Realistic cases → data from CCSNe simulations (SASI?)
- ▶ Full GR cases (non-Cowling)
- ▶ Extension to BNS (Alberto Revilla Peña, Ruxandra Bondarescu)

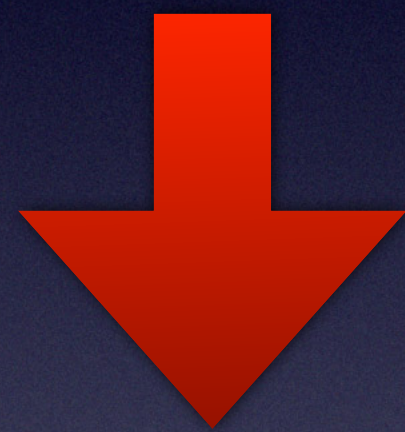
Thank you for
your attention!



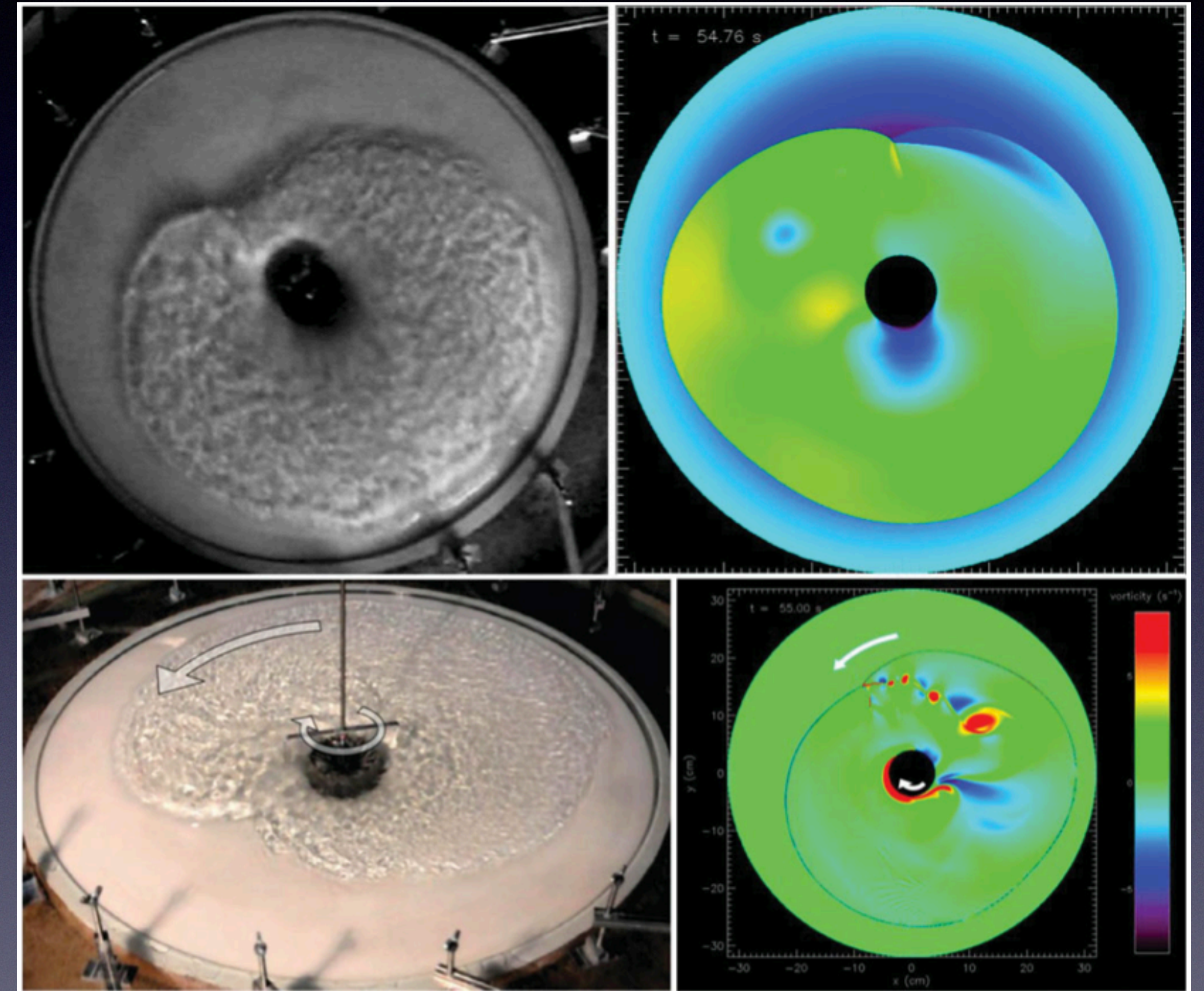
Extra material

Standing Accretion Shock Instability

Instability



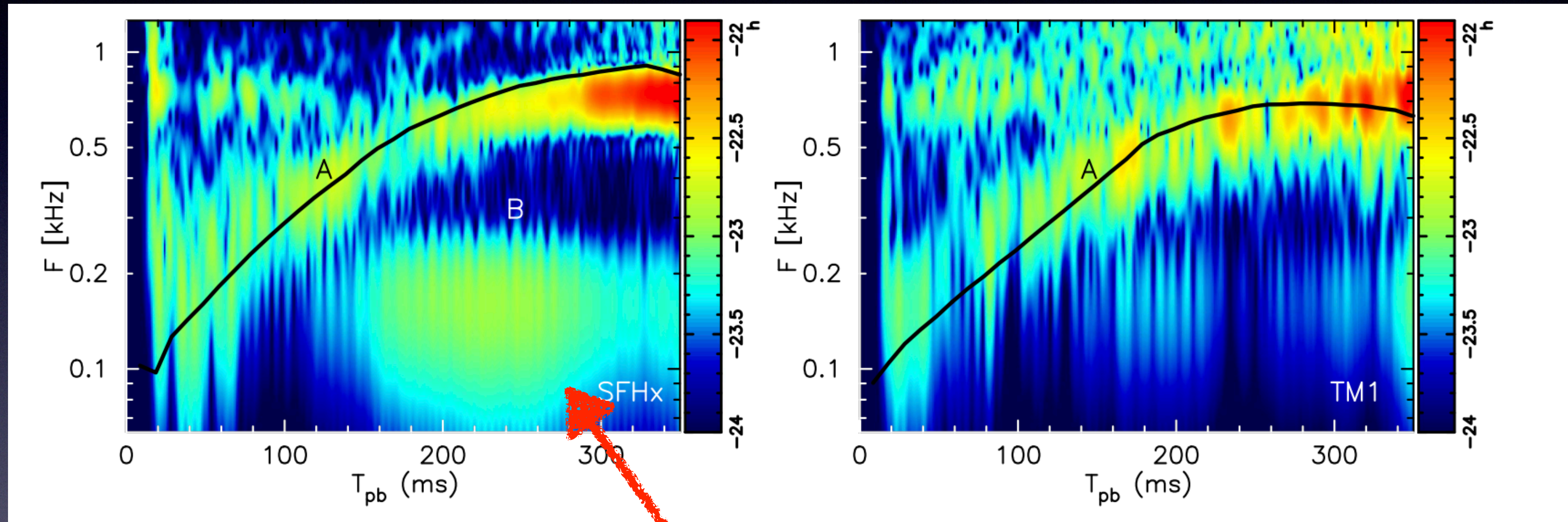
Advective-acoustic cycle between
the PNS surface & standing accretion shock



Foglizzo et al. (2012)

SASI and EoS

Kuroda et al. (2016)



Soft EoS

Stiff EoS

SASI

System of equations

General Relativistic Euler equations:

$$\frac{1}{\sqrt{\gamma}} \partial_t [\sqrt{\gamma} D] + \frac{1}{\sqrt{\gamma}} \partial_i [\sqrt{\gamma} D v^{*i}] = 0$$

$$\frac{1}{\sqrt{\gamma}} \partial_t [\sqrt{\gamma} S_j] + \frac{1}{\sqrt{\gamma}} \partial_i [\sqrt{\gamma} S_j v^{*i}] + \alpha \partial_j p = \frac{\alpha \rho h}{2} u^\mu u^\nu \partial_j g_{\mu\nu}$$

Background configuration:

- ▶ Non-rotating spherically symmetric star
- ▶ In equilibrium
- ▶ But **NOT** static

Perturbations

$$q \rightarrow q + \delta q$$

Decomposition of the perturbations:

$$\delta q = \delta \hat{q}(r) Y_{lm}(\theta, \phi) e^{-i\sigma t}$$

$$\xi^r = \eta_1 Y_{lm} e^{-i\sigma t},$$

$$\xi^\theta = \eta_2 \frac{1}{r} \partial_\theta Y_{lm} e^{-i\sigma t},$$

$$\xi^\phi = \eta_2 \frac{1}{r \sin^2 \theta} \partial_\phi Y_{lm} e^{-i\sigma t},$$

Assumptions:

► Cowling approximation

$$\delta \hat{\alpha} = 0 = \delta \hat{\psi}$$

► Adiabatic perturbations

$$\left. \frac{\partial p}{\partial \rho} \right|_{\text{adiabatic}} = hc_s^2 = \frac{p}{\rho} \Gamma_1$$

Rankine-Hugoniot conditions

General Relativistic

$$[[\rho u^\mu]] n_\mu = 0,$$

$$[[T^{\mu\nu}]] n_\nu = 0,$$



► Normal vector to the shock

$$[[F]] = F_{\text{int}} - F_{\text{ext}}$$

Standing Accretion
shock

Interior:
subsonic

Exterior:
supersonic

ρ_{int}

ρ_{ext}

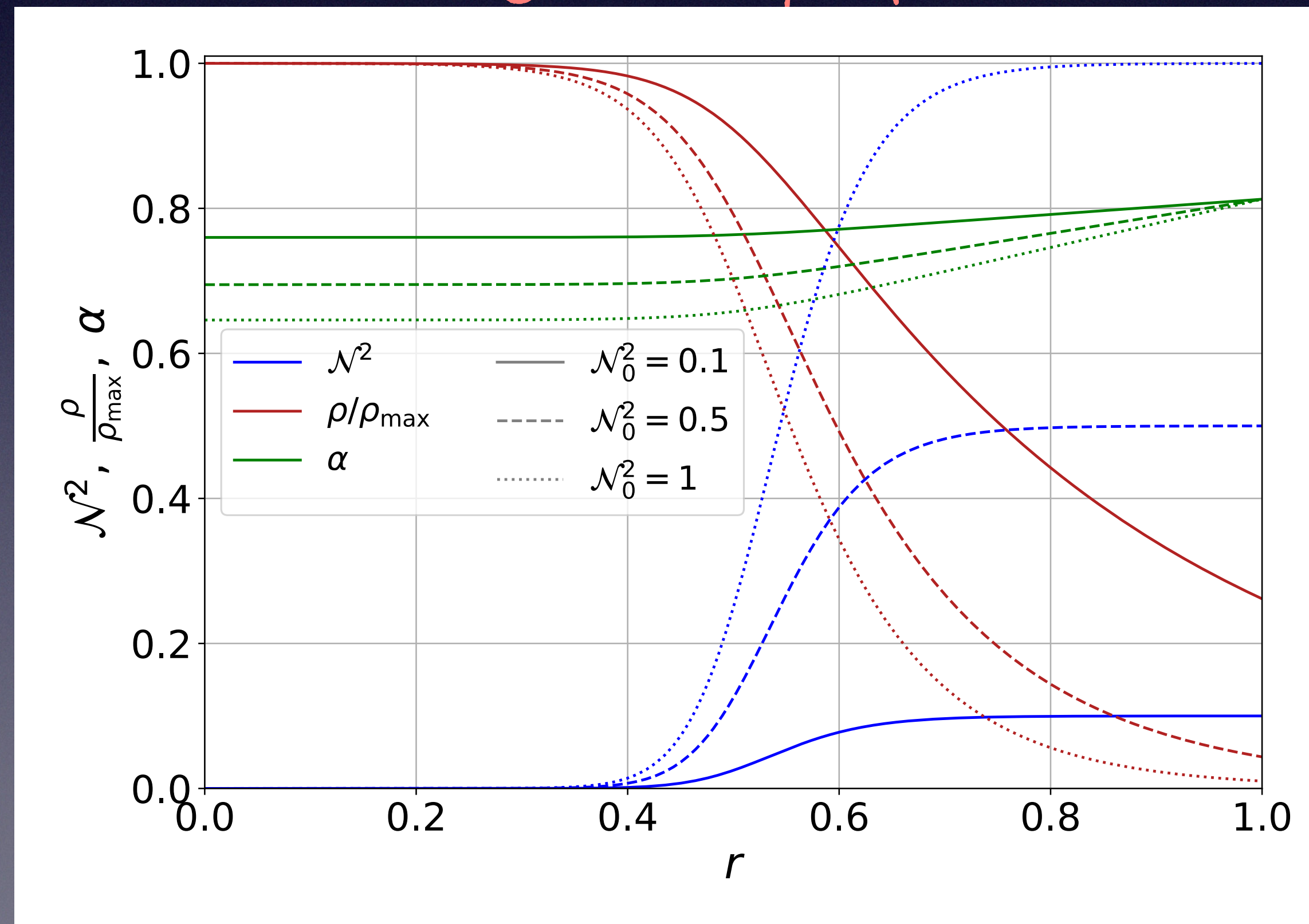
DISCONTINUITY

Ensure continuity of mass and conservation of energy-momentum fluxes

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- > Buoyant region
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Background profiles

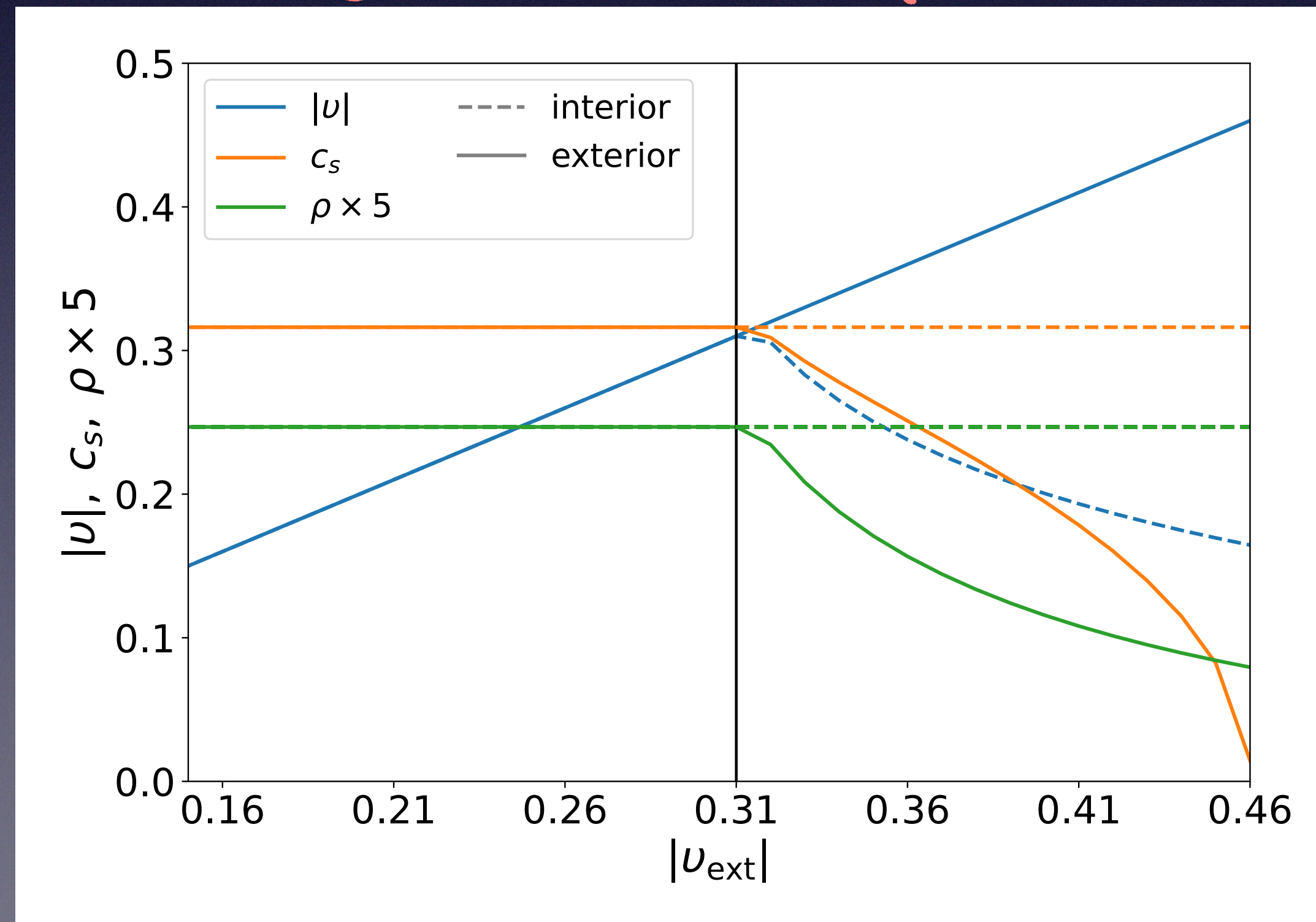


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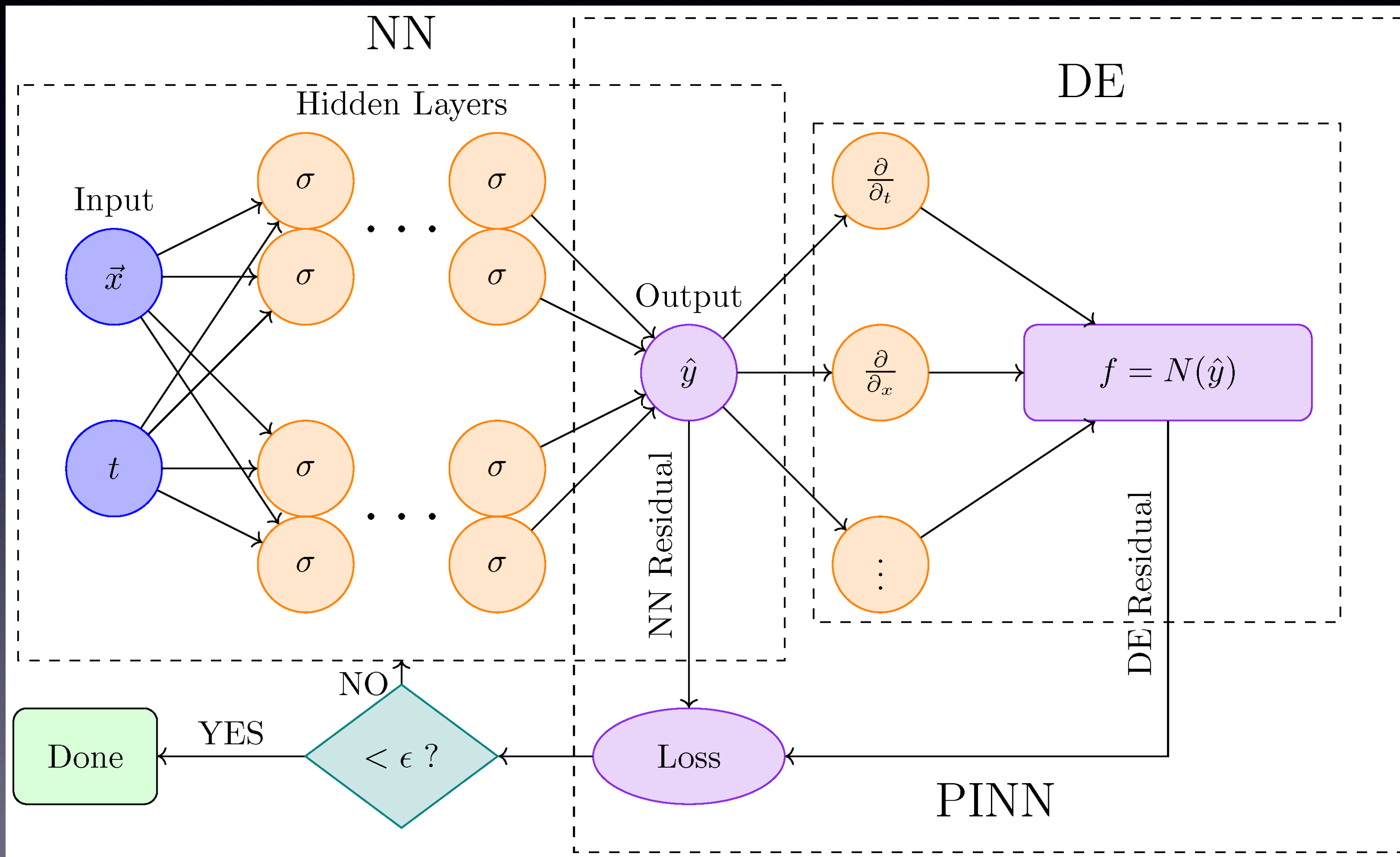
Case III: Advection

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- > Velocity $< c$ and subsonic

Background shock quantities



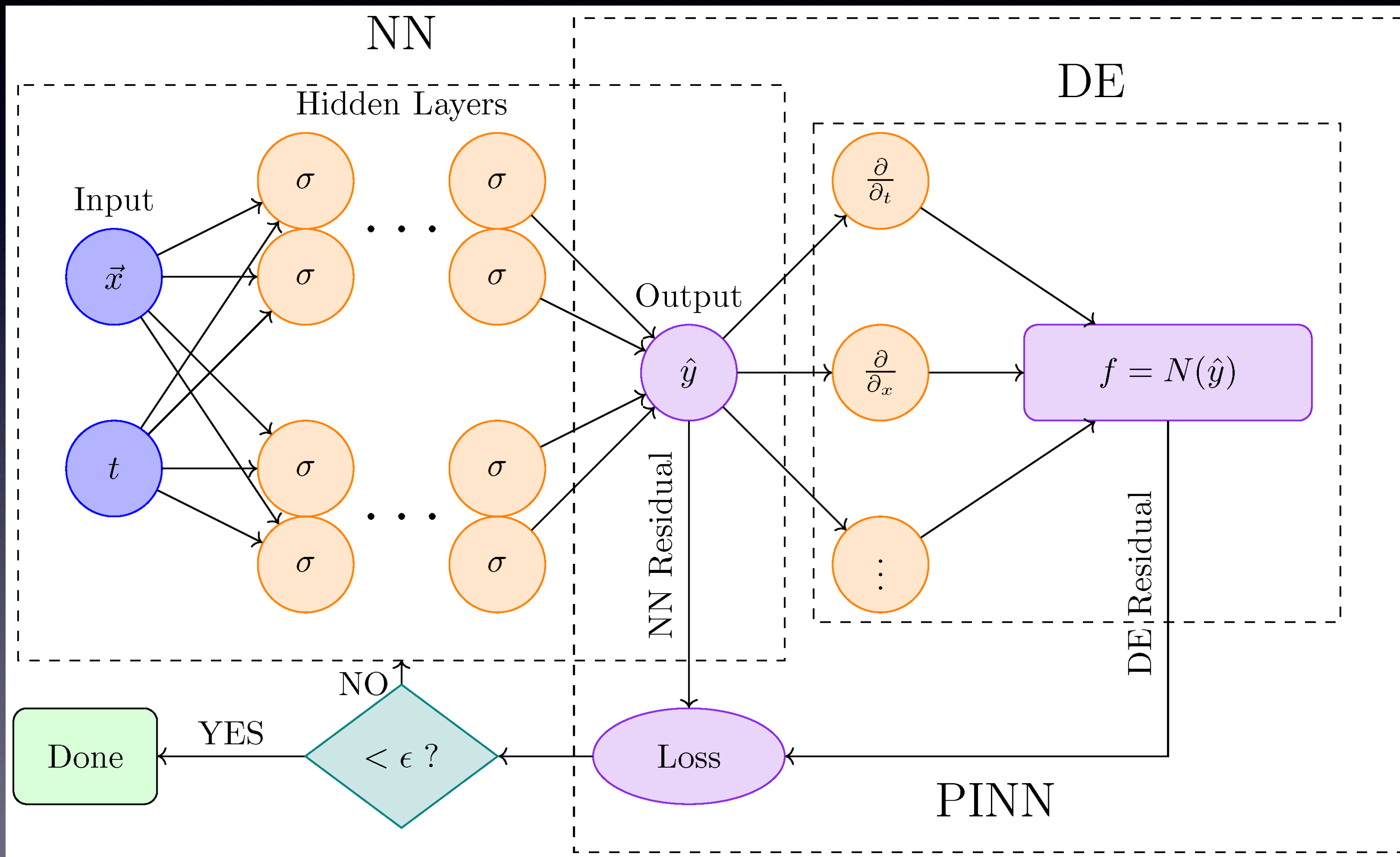
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ADVANTAGES

- > Grid free
- > Fast to code
- > Easy implementation of initial & boundary conditions
- > Flexible to add extra features
- > Error control

Physics Informed Neural Networks



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SMs

Preliminary!

Newtonian simulation, model s20

