Frequency-domain inspiral-merger-ringdown phenomenological waveforms for eccentric binary black holes

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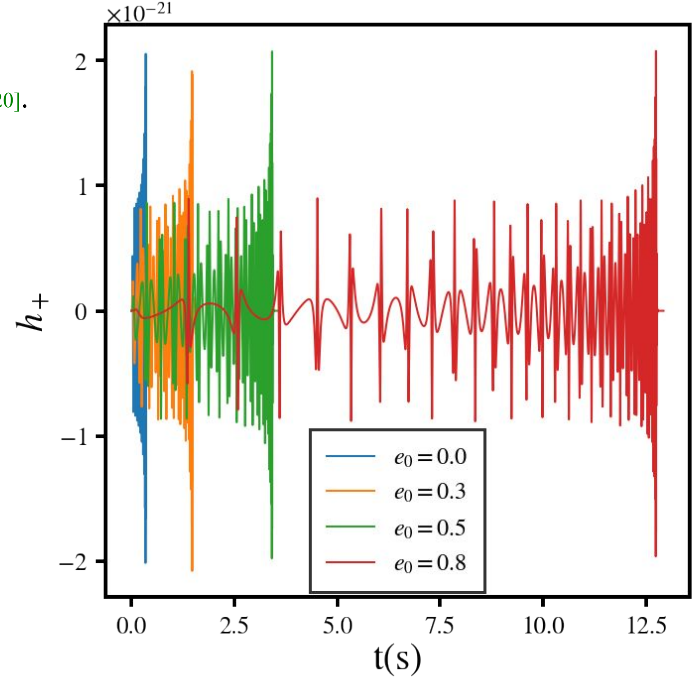
Inspiral-merger-ringdown eccentric waveform models

- Last years a lot of progress on constructing **inspiral-merger-ringdown** (IMR) **eccentric** waveform models:
  - **EOB framework**: TEOBResumS-Dali [Nagar+2021,Gamba+24], SEOBNREHM [Liu+2021], SEOBNRv4EHM [Ramos-Buades+2021], SEOBNRPHM [Liu+2023], SEOBNRv5EHM [Gamboa+2024], …
  - **NR surrogates and others**: ESIGMAHM [Huerta+2017,Kaushik+2024], Eccentric IMR [Hinder+2017], NRSurEcc [Islam+2021], [Setyawati+2021], …

- **Quasi-circular phenomenological waveforms** standard tools in LVK analysis:
  - **IMRPhenomX**: frequency domain, fastest models [Garcia-Quiros+2020,Pratten+2020].
  - **IMRPhenomT**: time domain, more accurate [Estelles+2020,Estelles+2021].
  - Latest: **IMRPhenomXO4a/SpinTaylor** [Thompson+2023, Colleoni+2023].

- Recent development of eccentric models in phenomenological approach:
  - **IMRPhenomTEHM** [Planas+2025,Planas+2025] : fast time-domain multipolar eccentric model for aligned-spins (**See Maria de Lluc Planas's talk**) with direct data analysis applications [Planas+2025] .

- Here we present the **IMRPhenomXE :**
  - **Extension of** the (2,2)-mode quasi-circular non-precessing **IMRPhenomXAS model** [Pratten+2020] **to eccentricity in frequency-domain**.



IMRPhenomXE : overview

- **IMRPhenomXE:** Frequency-domain inspiral-Merger-Ringdown (2,|2|)-mode model with aligned spins.

- Model split into 3 regions (inspiral, merger, ringdown)

  - **Inspiral** :
    - Evolve numerically orbit-average dynamics in quasi-Keplerian parametrization.
    - Evaluate time-domain waveform modes.
    - Perform stationary phase approximation (SPA).

  - **Merger+Ringdown**: Assume circularization and use quasicircular IMRPhenomXAS model [Pratten+20].

- Input parameters:
  - Initial eccentricity e_0.
  - Initial orbit-average frequency $\bar{f}_{\rm min}$.
  - Reference orbit-average frequency l_0.
  - Initial mean anomaly $\bar{f}_{\rm ref}$.

IMRPhenomXE inspiral

- **Dynamics:** Evolve post-Newtonian (PN) [Henry+2023] /Effective One Body (EOB) [Gamboa+2025] orbit-averaged equations using quasi-Keplerian parametrization :

$$\frac{dx}{dt} = \dot{x}^{\rm PhenomT}(t) - \delta\dot{x}^{\rm 3PN}_{\rm ecc}, \quad \frac{de}{dt} = \mathcal{G}^{\rm 3PN}(x,e), \quad \frac{dl}{dt} = \mathcal{H}^{\rm 3PN}(x,e), \quad \frac{d\lambda}{dt} = x^{3/2}.$$

x: orbit-averaged frequency (x=$\boldsymbol{\omega}^{2/3}$)	ℓ: mean anomaly
e: orbital eccentricity	$\boldsymbol{\lambda}$: orbit-averaged orbital phase

- **Waveform modes:** Derived (2,2)-mode up to 3PN (w/o oscillatory memory) expanded up to O(e^{12}) :

$$h \equiv h_+ - ih_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{+l} Y^{-2}_{lm}(\theta,\phi) h_{lm}. \qquad h_{22} = e^{-2i\lambda} \sum_{j=-N}^{+N} A_j e^{-ijl}. \qquad A_j \equiv A^{\rm 3PN}_j(x,e,\eta,\chi_1,\chi_2) + \mathcal{O}(e^{13}).$$

- **Frequency-domain modes:** Numerical Stationary Phase Approximation (SPA) on individual harmonics:

$$\tilde{h}_{22}(f) = \sum_{s=-N}^{-N} \mathcal{A}^{\rm SPA}_s e^{i\Psi_s} \qquad \Psi_s = 2\pi f t(f) - [2\lambda(f) + s l(f)] \qquad \mathcal{A}^{\rm SPA}_s = A_s \sqrt{\frac{2\pi}{|\ddot{\Psi}|}}, \qquad |\ddot{\Psi}| = |2\dot{\omega} + s\ddot{l}|.$$

IMRPhenomXE : Inspiral-Merger-Ringdown waveform

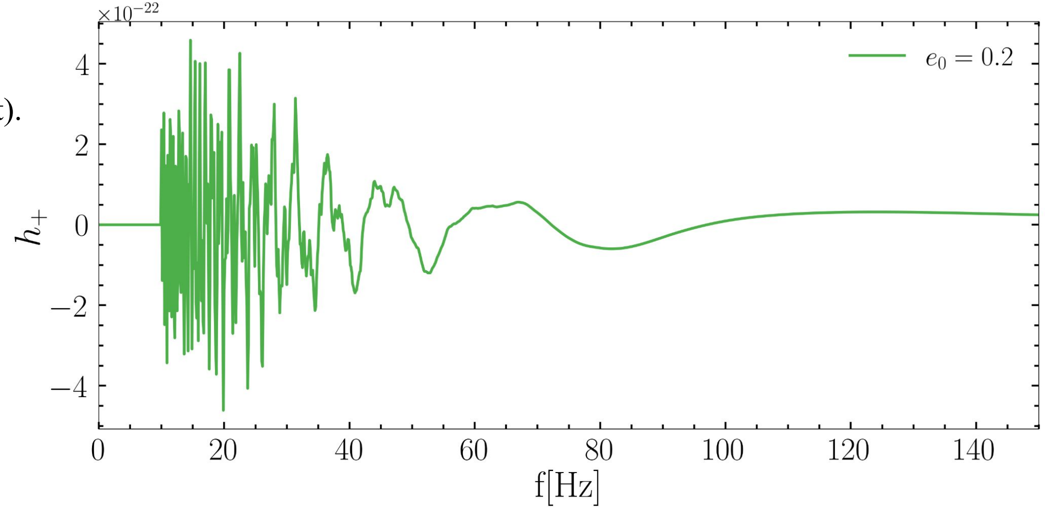
- We implement the eccentric corrections to the IMRPhenomXAS waveform model [Pratten+2020].

$$h_{22}^{\mathrm{XE}} = h_{s=0}^{\mathrm{IMR}} + \sum_{s=-N, s\neq 0}^{+N} \mathcal{A}_s^{\mathrm{SPA}} e^{-i\Psi_s}, \quad h_{s=0}^{\mathrm{IMR}} = A_{s=0}^{\mathrm{IMR}} e^{-i\Psi_{s=0}^{\mathrm{IMR}}}$$

$$\mathcal{A}_{s=0}^{\mathrm{IMR}} = \mathcal{A}^{\mathrm{XAS}} + \delta\mathcal{A}_{s=0}^{\mathrm{ecc}}\theta(f - f_{\mathrm{insp}}^{\mathrm{max}}). \qquad \Psi_{s=0}^{\mathrm{IMR}} = \phi_{22}^{\mathrm{XAS}} + \delta\Psi_{s=0}^{\mathrm{ecc}}\theta(f - f_{\mathrm{insp}}^{\mathrm{max}}).$$

- In e→0 limit, s=0 only harmonic (quasicircular limit).

- Implementation in python package *phenomxpy* [Garcia-Quiros+25].



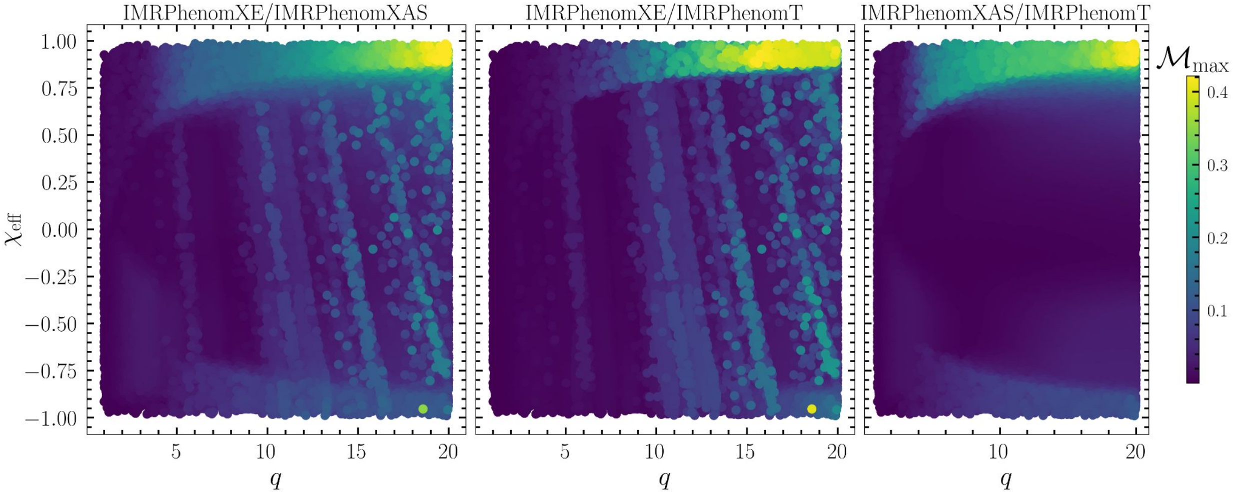
Quasicircular limit

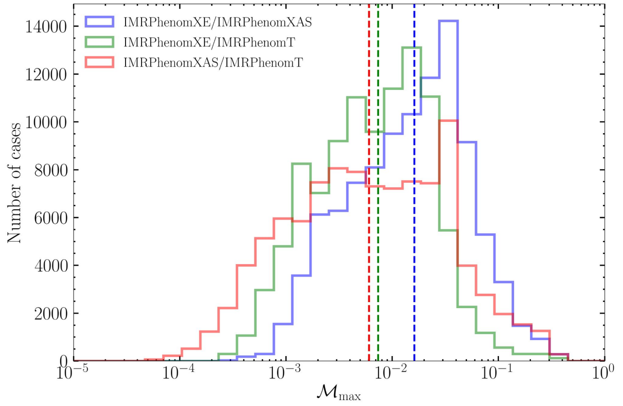
- Compare two waveforms using mismatch $\mathcal{M}$:
    - $\mathcal{M}$=0 same waveforms
    - $\mathcal{M}$=1 completely different waveforms

- Compute the mismatch for e=0 between IMRPhenomXE, IMRPhenomT,IMRPhenomXAS.

- Parameter space 10^5 configurations with $q\epsilon[0,20]$, $\chi_{1,2}\epsilon[-0.99,0.99]$, $M_T\epsilon[20,200]$, f_{min}=20Hz.

- Use Advanced LIGO design sensitivity power Spectral Density (PSD).

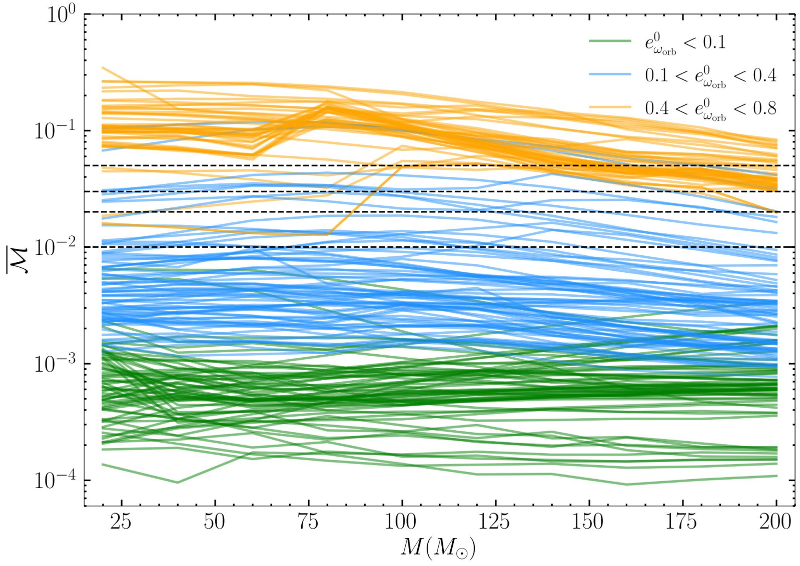


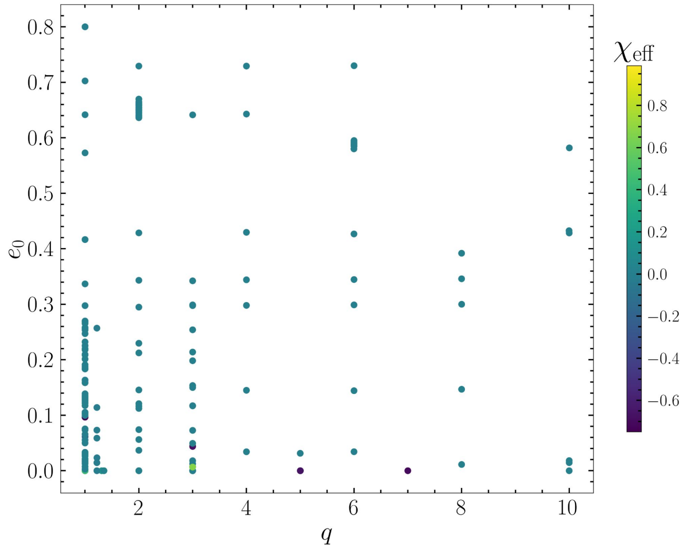


Comparison against numerical relativity

- Unfaithfulness of IMRPhenomXE against **186 public SXS NR waveforms** [Boyle+19, Scheel+25] with $e_0 \leq 0.8$.

- Maximization over eccentricity and mean anomaly of the model at the initial orbit-averaged freq. from NR.





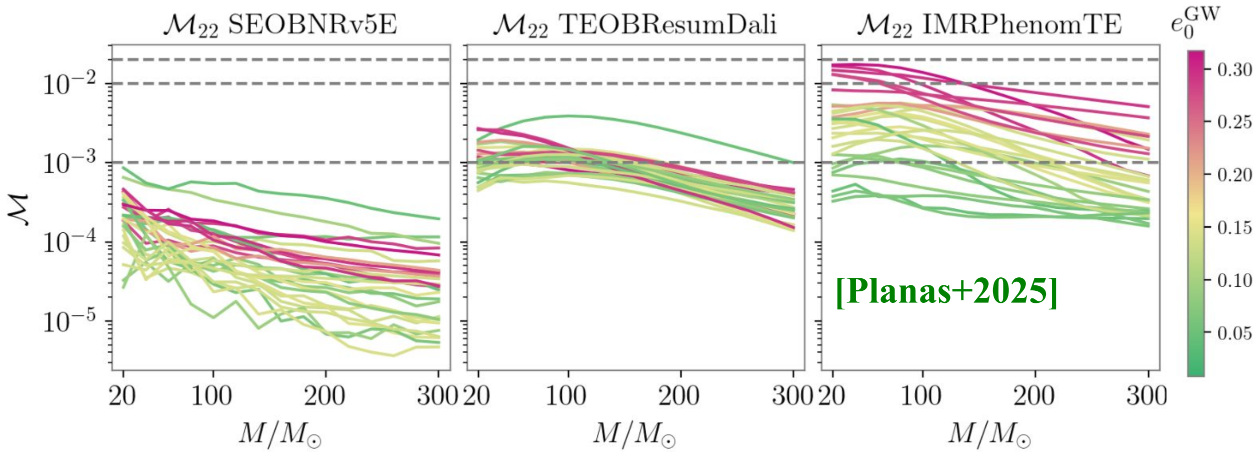
- For **low eccentricities** mismatch below 1%.

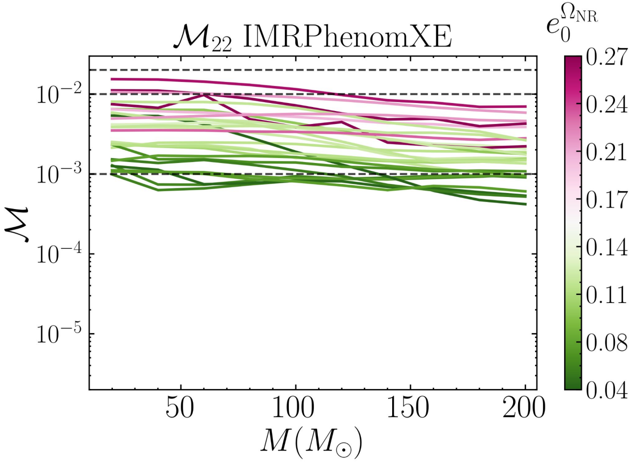
- For **intermediate eccentricities** most cases with mismatch < 3%.

- For **high eccentricities** poor performance:
  - Degrading of eccentricity expansions
  - More eccentric harmonics needed (used N=4)
  - Issues with the optimization procedure (under investigation)

Comparison against numerical relativity

- Comparison against other models IMRPhenomTEHM [Planas+25, Scheel+25] against 28 public SXS simulations with $e_0 \leq 0.3$.

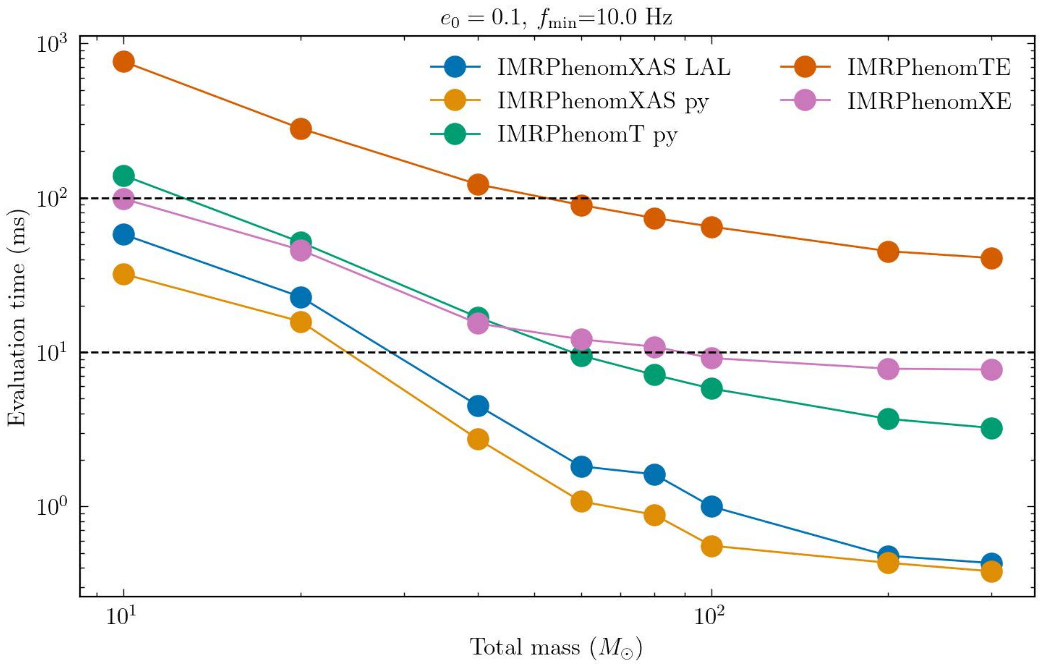




- SEOBNRv5EHM most accurate model, followed by TEOBResumS-Dali.
- IMRPhenomXE comparable accuracy to IMRPhenomTE, both with mismatches < 2%.

Benchmarks

- We compare the speed of the implementation of IMRPhenomXAS and IMRPhenomXE in phenomxpy against the IMRPhenomXAS in LALsuite.
- We include the fastest time-domain eccentric IMR model, IMRPhenomTE [Planas+2025].



- Time models for a binary configuration with moderately low eccentricity.
- IMRPhenomXE speed comparable to other models from IMRPhenomX family.
- **IMRPhenomXE fastest IMR eccentric model.**

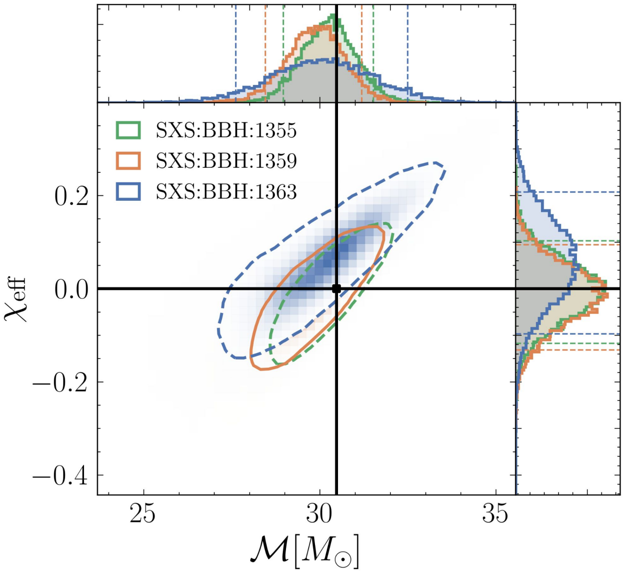
Parameter estimation: NR injections

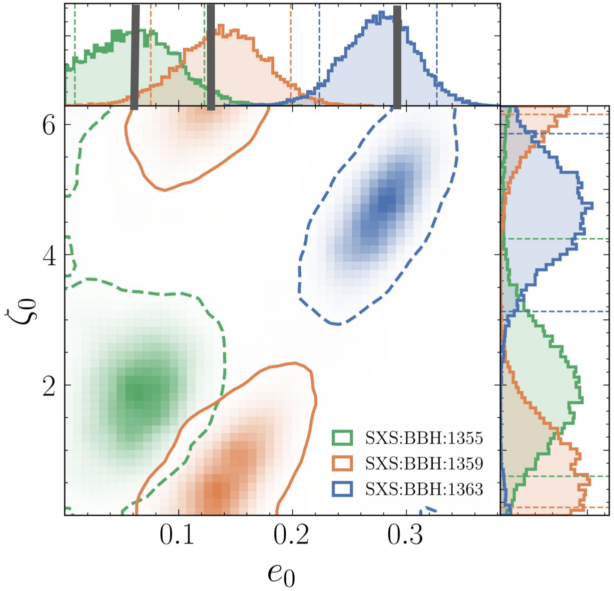
- Inject 3 NR waveforms (SXS:BBH:1355/1359/1363) into zero noise:

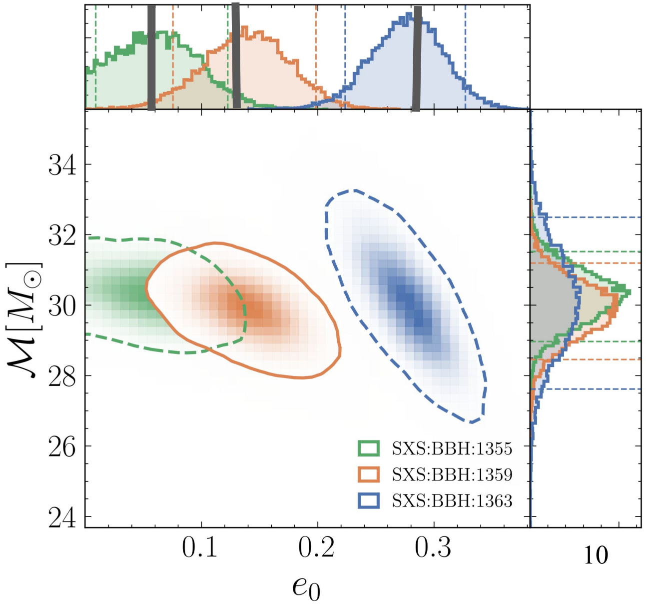
$$q = 1, \quad \chi_{1,2} = 0, \quad M = 70M_{\odot}, \quad \iota = 0, \quad \mathrm{SNR} = 20 \quad \text{at } 20\mathrm{Hz}.$$

- NR waveforms with modes **l ≤ 8 modes**, with increasing e_{gw} [20Hz](~0.06, 0.13, 0.28) and recover with **IMRPhenomXE**.

- Use serial **Bilby** [Ashton+19] to recover parameters. Uniform priors in eccentricity and mean anomaly.







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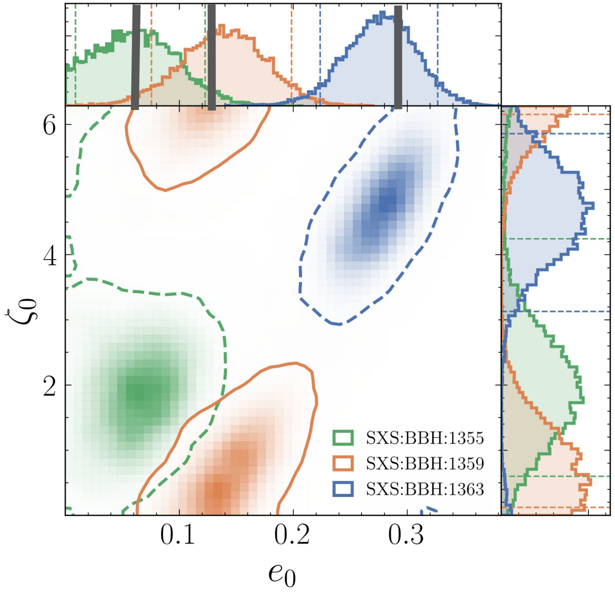
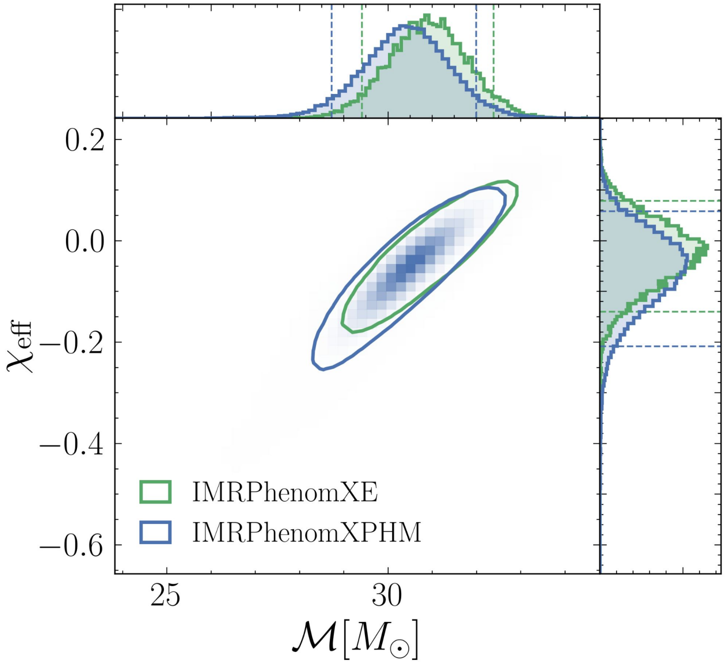


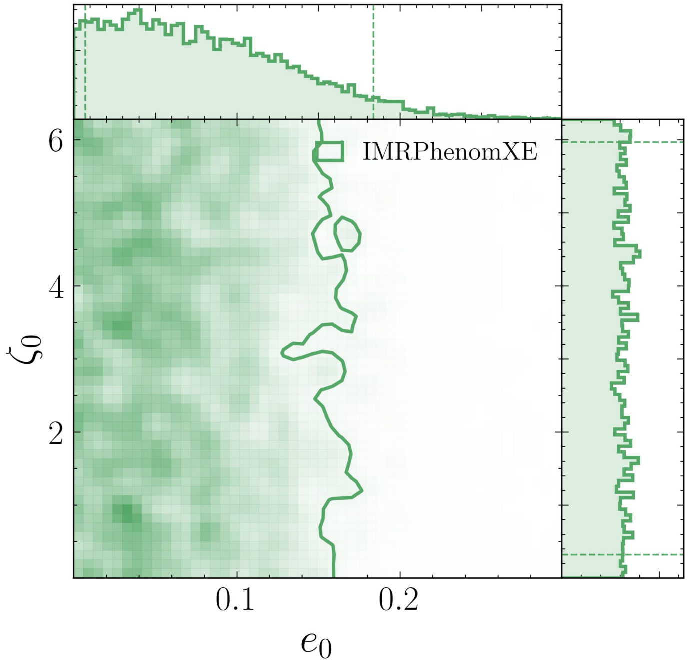
WALLTIME	IMRPhenomXE (128 cores)	IMRPhenomTE (50 cores Planas+25)
SXS:BBH:1355	**34 min**	**14h 38min**
SXS:BBH:1359	**45 min**	**15h 17min**
SXS:BBH:1363	**1h 35 min**	**15h 15min**

Parameter estimation: GW150914

Walltime (1 node):
- IMRPhenomTEHM (112 cores) 1d 3h
- IMRPhenomXE (128 cores) **1h 19min**

- Analyze GW150914 with **IMRPhenomXE** using starting frequency of 10Hz.
- Comparison against **IMRPhenomXPHM** from GWTC-2.1.
- Overall **good agreement with IMRPhenomXPHM**, and uninformative eccentricity posterior.





Conclusions

- We present a new phenomenological IMR frequency domain model for the dominant mode IMRPhenomXE.
- Derived 3PN spinning expressions for the amplitudes of h_{22} expanded up to $O(e^{12})$.
- Model valid for small eccentricities up to e~0.4 .
- The model is implemented in a new python infrastructure ***phenomxpy*** and is similar in speed to other frequency domain models.

Ongoing work:

- Parameter estimation analysis.
- Extension to higher order modes and spin-precession.
- Optimization and explore applications for 3G and LISA.
- Targeting a complete model for O5.

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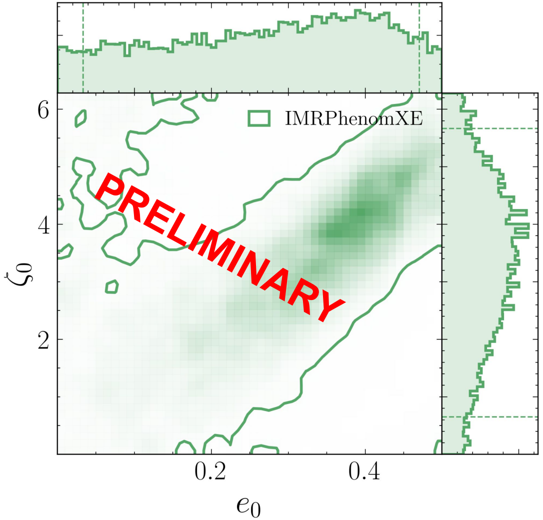
EXTRA SLIDES

GW190521 & GW200208

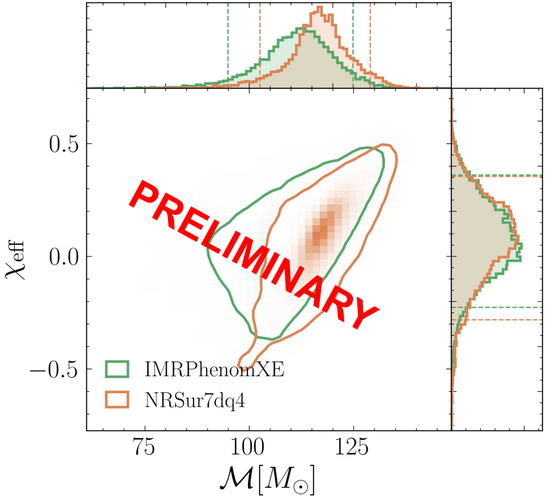
WALLTIME	**IMRPhenomXE** (128 cores)	**IMRPhenomTEHM** (112 cores Planas+25)
GW190521	**1h 15 min**	**11h 6min**
GW200208	**5h 24min**	**19h 24min**

Compare against IMRPhenomTEHM [Planas+25] results from public data release https://zenodo.org/records/15259941

GW190521



GW190521



GW200208