A Generic Algorithm for Building Hybrids of Quasi-Circular Precessing Systems

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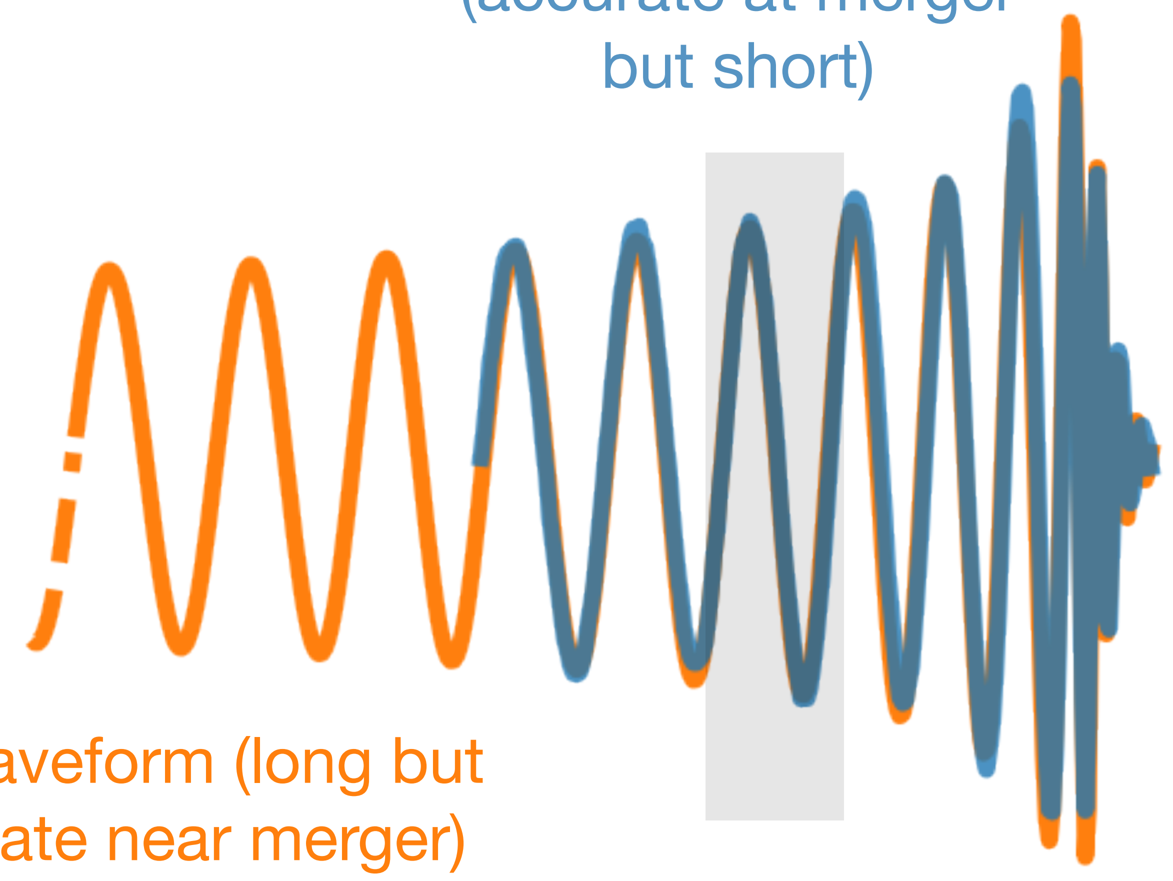
Introduction and motivation

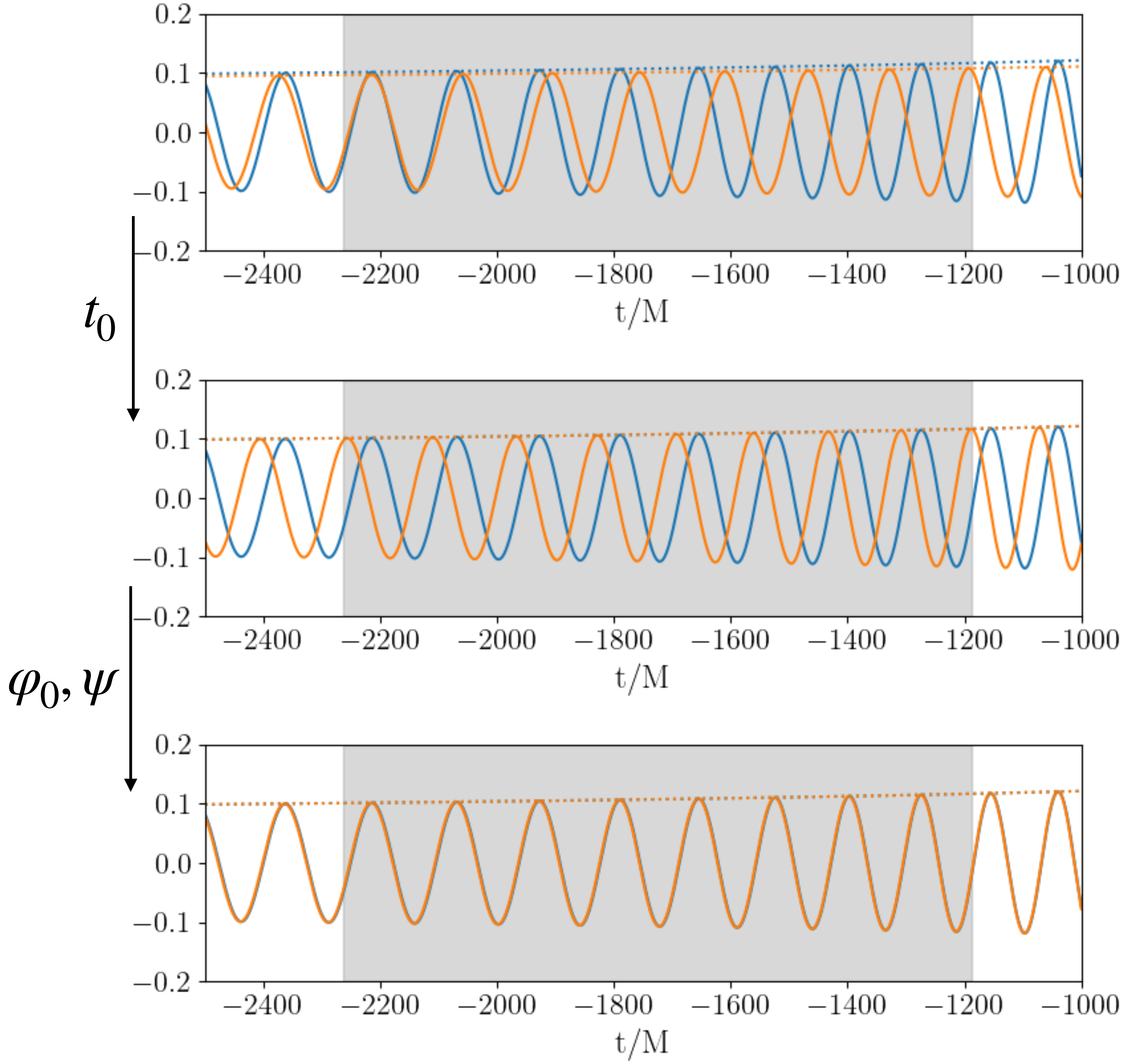
Why and what for?

- **Different approaches** to CBC waveform modelling: EOB formalism, phenom., surrogate…
- Waveform descriptions that are accurate at merger are also expensive.
- **Hybrid waveforms** are built joining two waveforms that represent the same system.
- Long waveforms are useful for calibration and validation of FD models, 3G detectors…
- Hybridization requires **alignment** => limited by waveform systematics.
- While non-precessing hybrids are routine, precession poses significant **challenges** to alignment:
  - Frame selection
  - Breaking of mode hierarchy
  - Gauge choices and parameters' definitions…
- In this work, we present a **general approach** to building hybrids.
- We don't discuss BMS frame fixing or gauge issues. [Sun+2024]



Non-precessing alignment and hybridization

Well-established and routine



- Natural frame of reference ($\overrightarrow{L} \,||\, \hat{z}$).
- Non-precessing alignment relies on **3 d.o.f.**:
  1. Time shift - t_0
  2. Phase shift - $m \cdot \varphi_0$ (for each (ℓ, m) mode)
  3. Polarization - $\psi \in \{0, \pi\}$.
- **Multi-modal**: optimize w/ all available (ℓ, m) modes.
- **Hybridization**: smooth transition in interval (t_1, t_2)

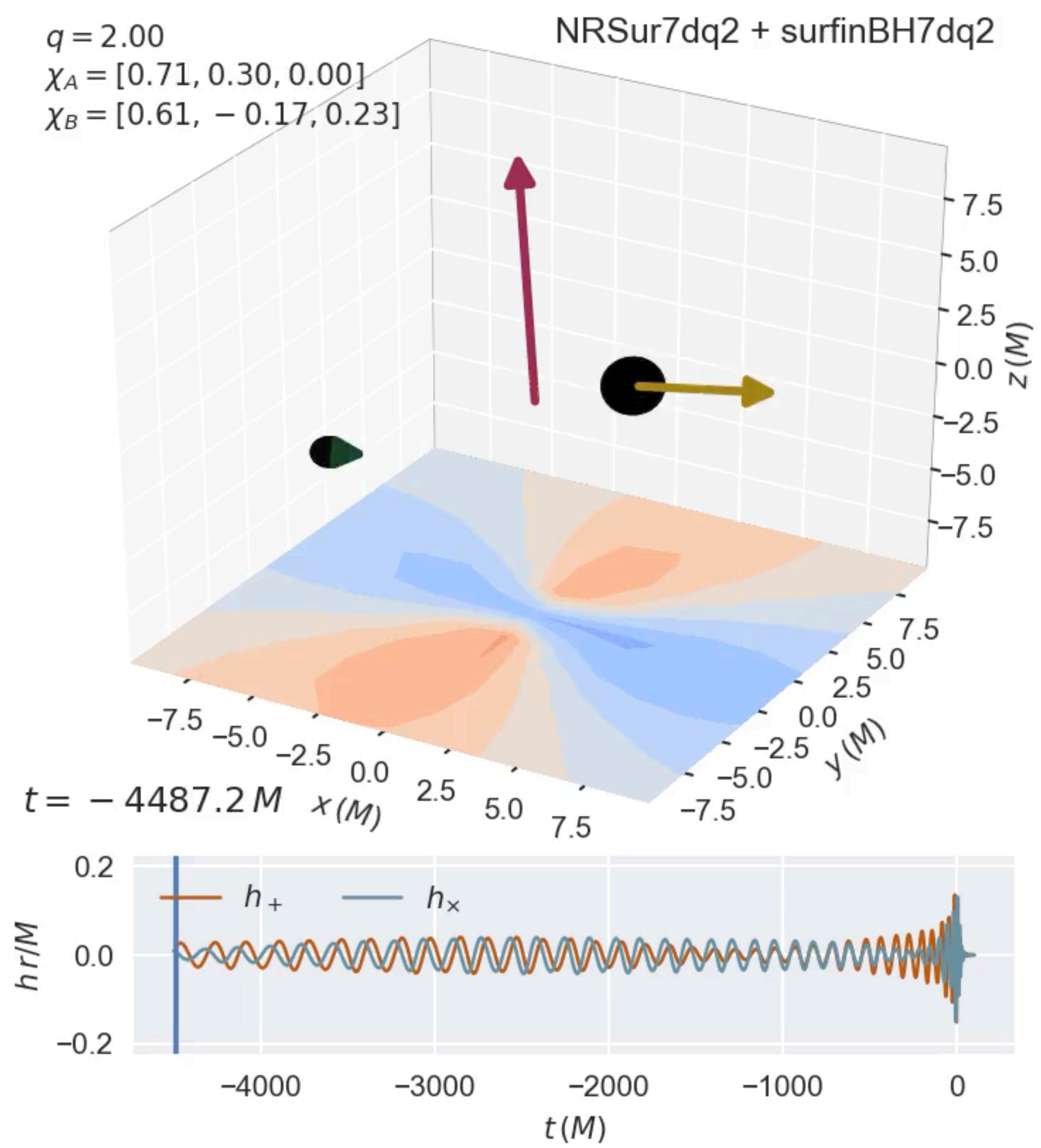
$$\text{Hybrid} = w(t) \cdot \text{WF}_1 + \left[1 - w(t)\right] \cdot \text{WF}_2 \text{ with}$$

$$w(t) = \begin{cases} 1 & \text{if } t < t_1 \\ 1/2 \left[1 + \cos\left(\pi(t - t_1)/(t_2 - t_1)\right)\right] & \\ 0 & \text{if } t > t_2 \end{cases}$$

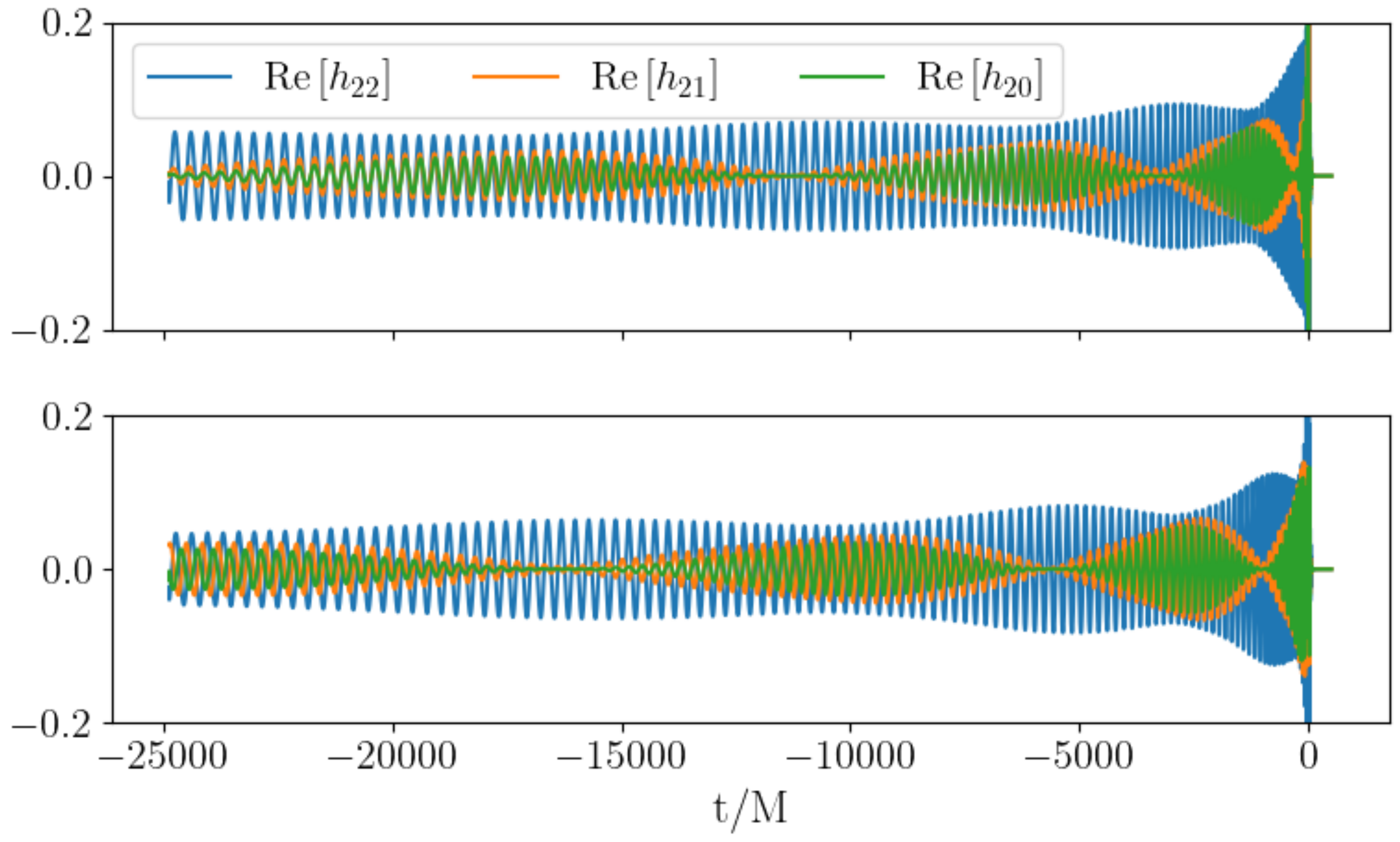
Precessing systems' challenges

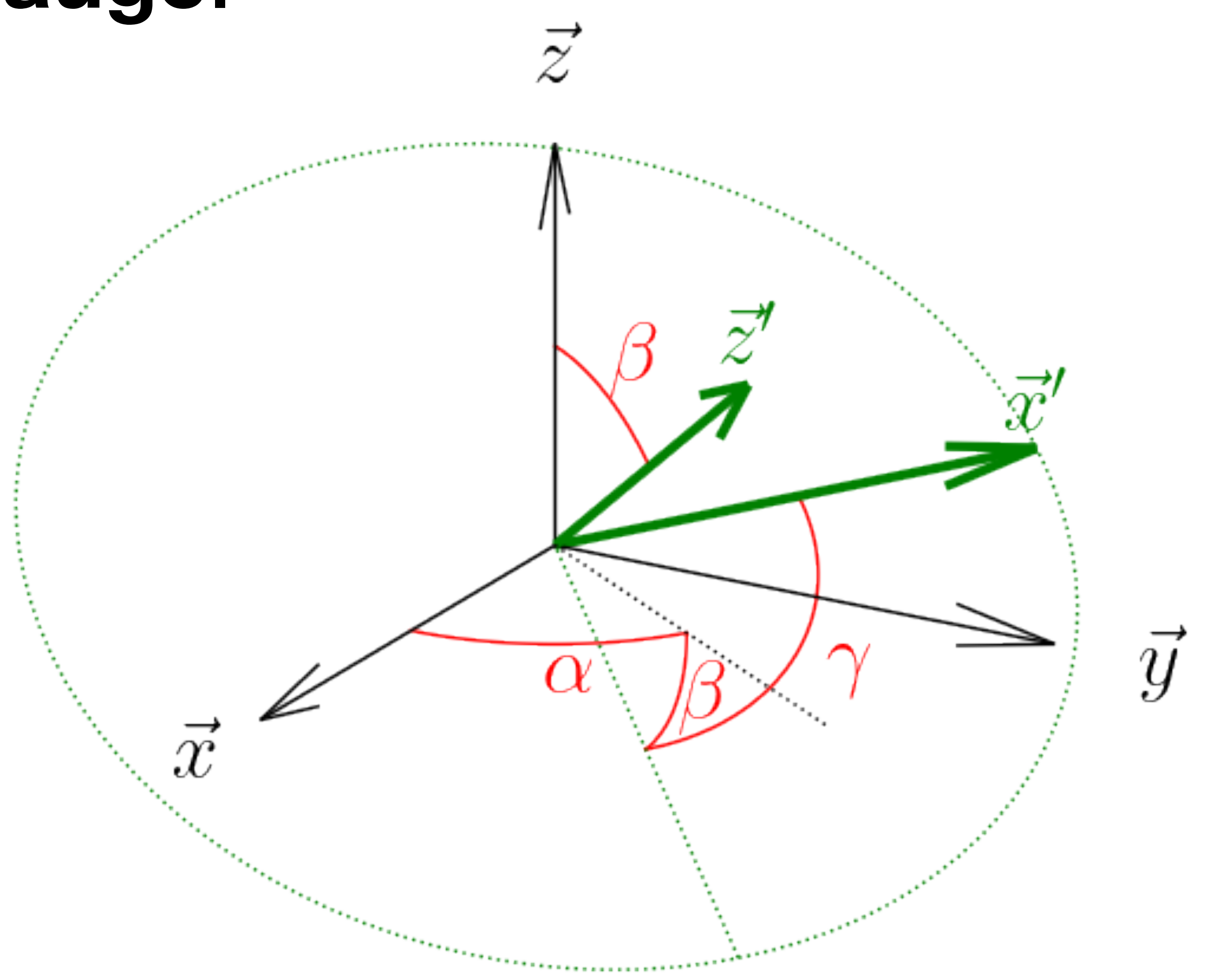
When $\vec{S} \neq \vec{L}$...

- Orbital plane precession strongly modulates the signal $h_{\ell m}$.
- In an inertial frame, there is frame-dependent **mode-mixing.**
- In the **non-inertial coprecessing frame**, the signal is similar to non-precessing.
- Widely used modeling strategy: **twist-up coprecessing modes**.
- Spin directions strongly depend on the **gauge**.



[Animation credit: V. Varma]





α and β define the instantaneous $\hat{z}$-axis ($\hat{L}$ or the maximum emission direction $\hat{Q}$).

γ lies in the orbital plane.

MRC: $\dot{\gamma} = -\dot{\alpha}\cos\beta$

Aligning precessing waveforms

Alignment in the coprecessing frame

- Twisting-up of coprecessing-frame modes:

$$h^{\mathrm{in}}_{\ell m}(t) = \sum_{m'=-\ell}^{\ell} h^{\mathrm{QA}}_{\ell m'}(t)\, \mathscr{D}^{\ell}_{mm'}\big(\alpha(t), \beta(t), \gamma(t)\big)$$

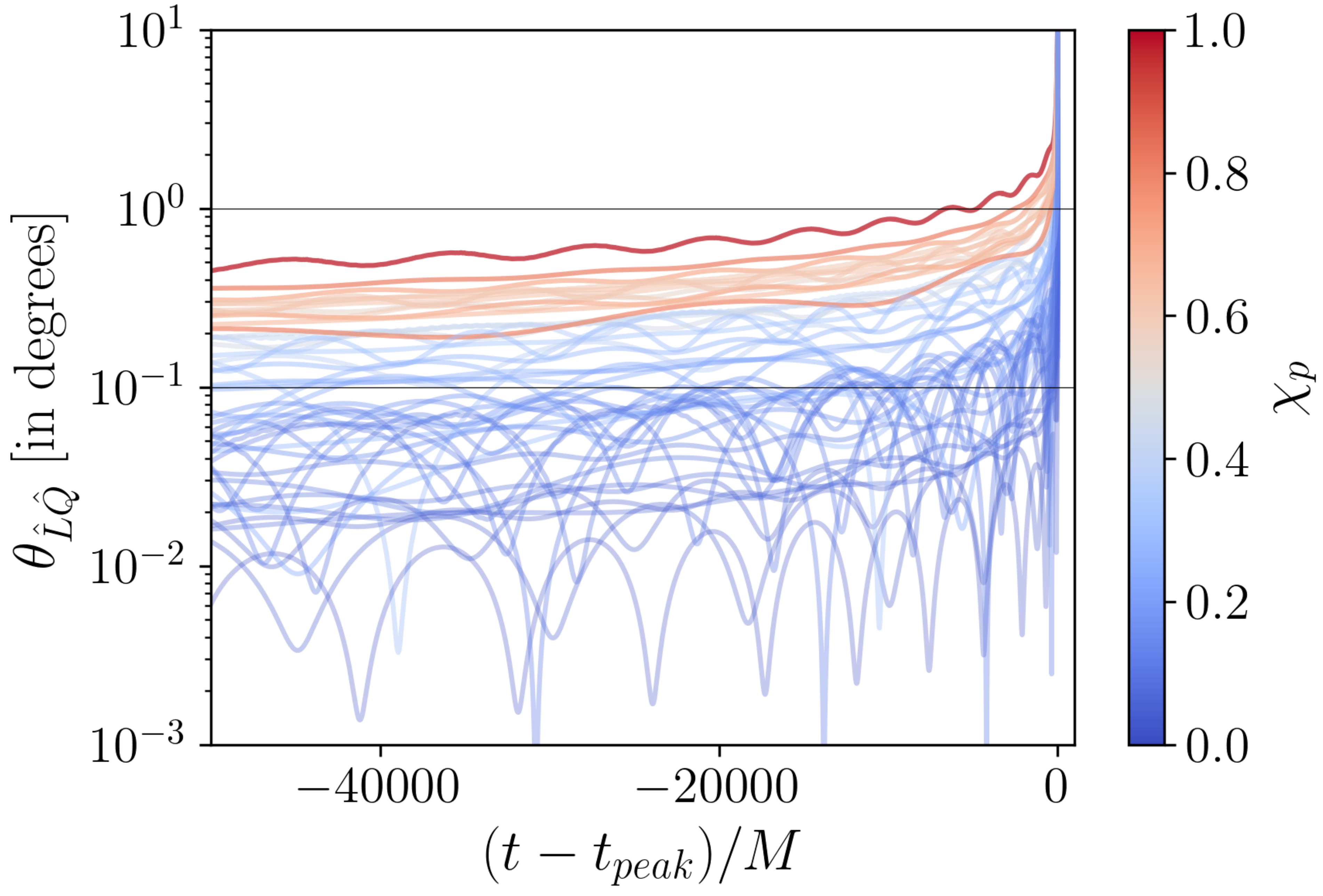
- Alignment of a precessing waveform requires aligning:
  1. The **coprecessing-frame waveform**.
  2. The **Euler angles** α, β, γ that define the inertial frame.
- We use the natural decomposition of these two problems and tackle them separately.
- An “all-at-once” optimization might be too expensive.

Aligning precessing waveforms

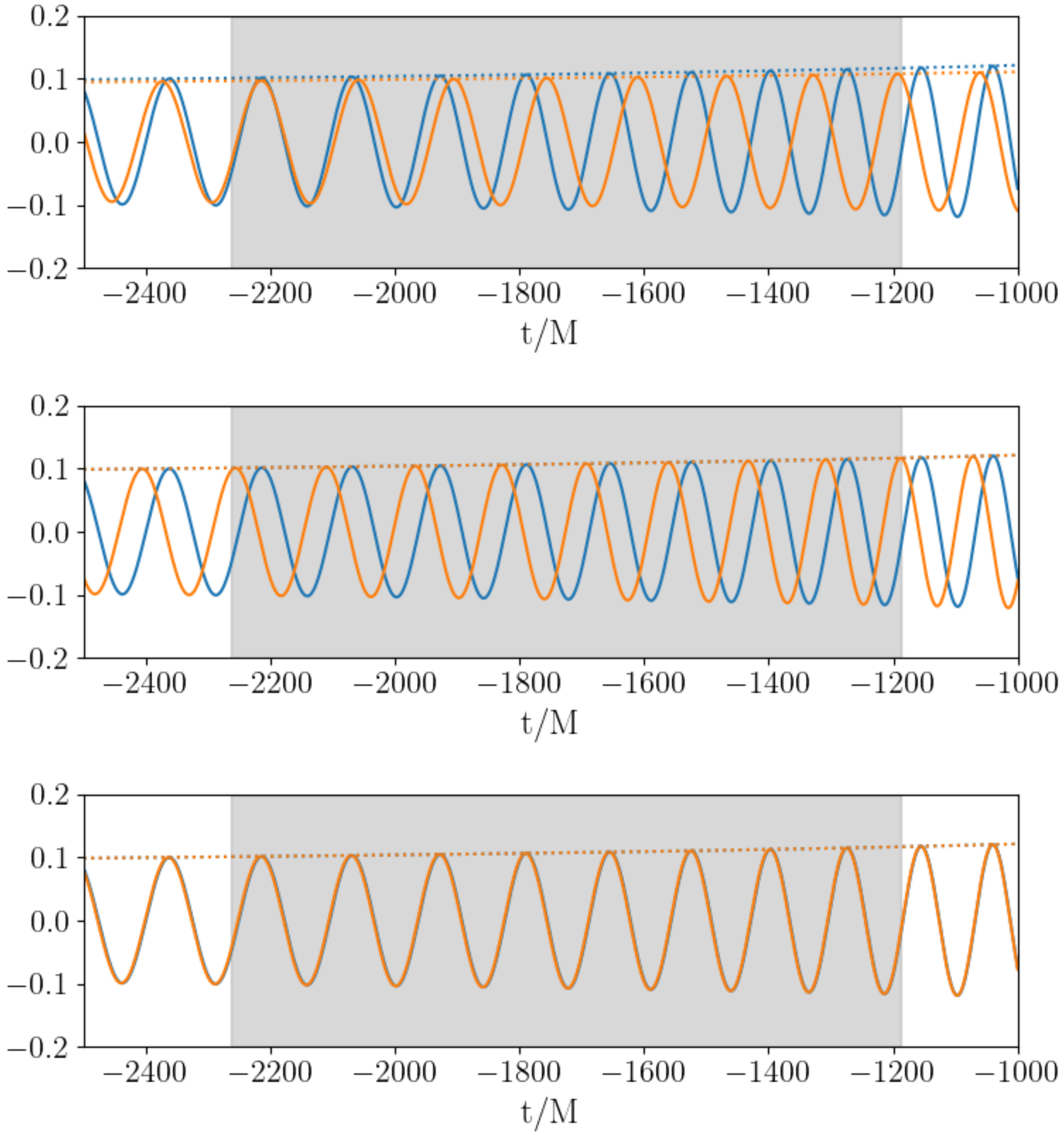
1. Aligning the coprecessing-frame waveform

✅ Retains the simplicity of the non-precessing case.

🤔 The **coprecessing frame** is defined with the maximum emission direction $\hat{Q}$ instead of $\hat{L}$.



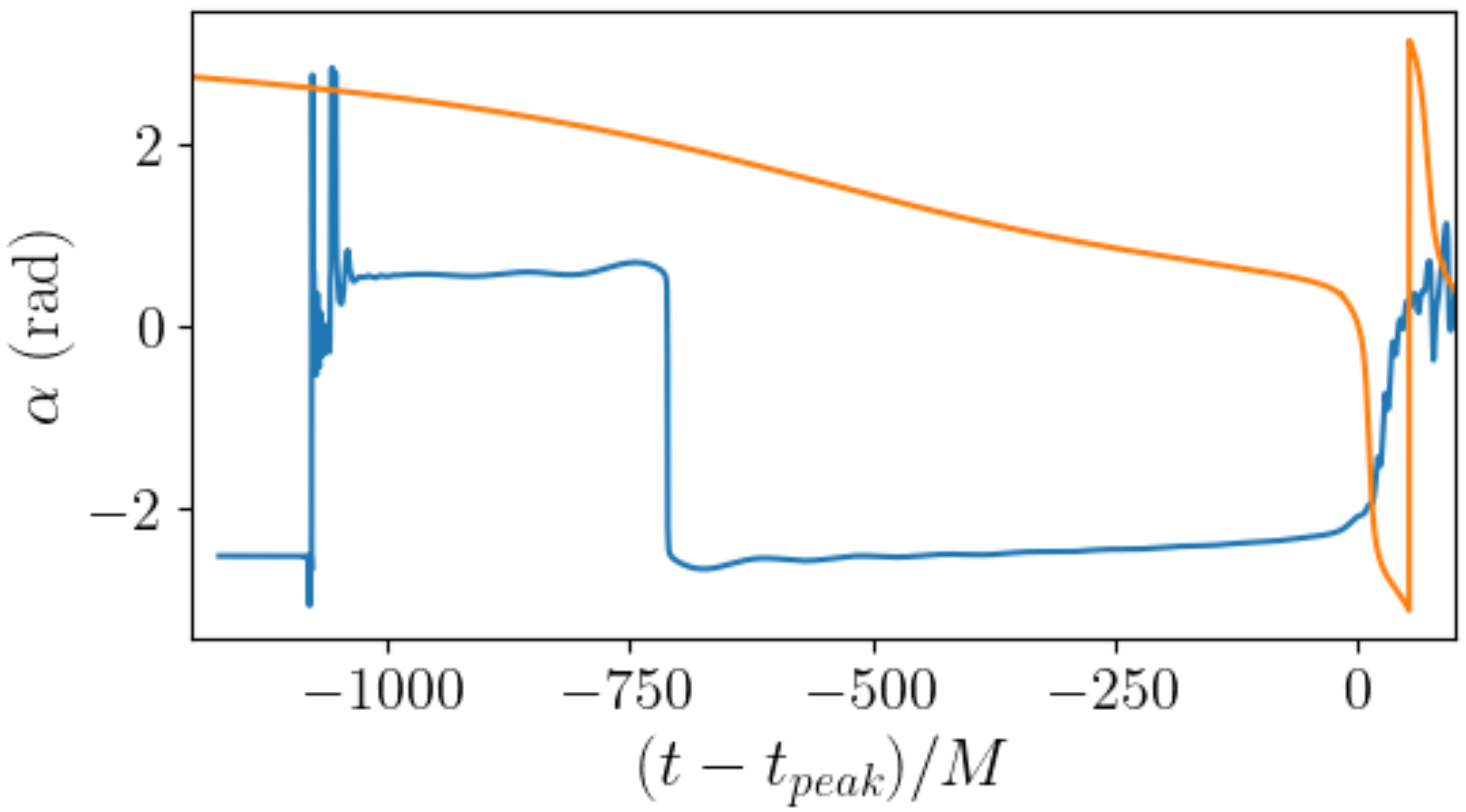
Angle between $\hat{L}$ and $\hat{Q}$ for SEOBNRv5PHM waveforms randomly distributed in $q \in [1,10], |\chi_i| \in [0,1), \hat{\chi}_i$ isotropic.

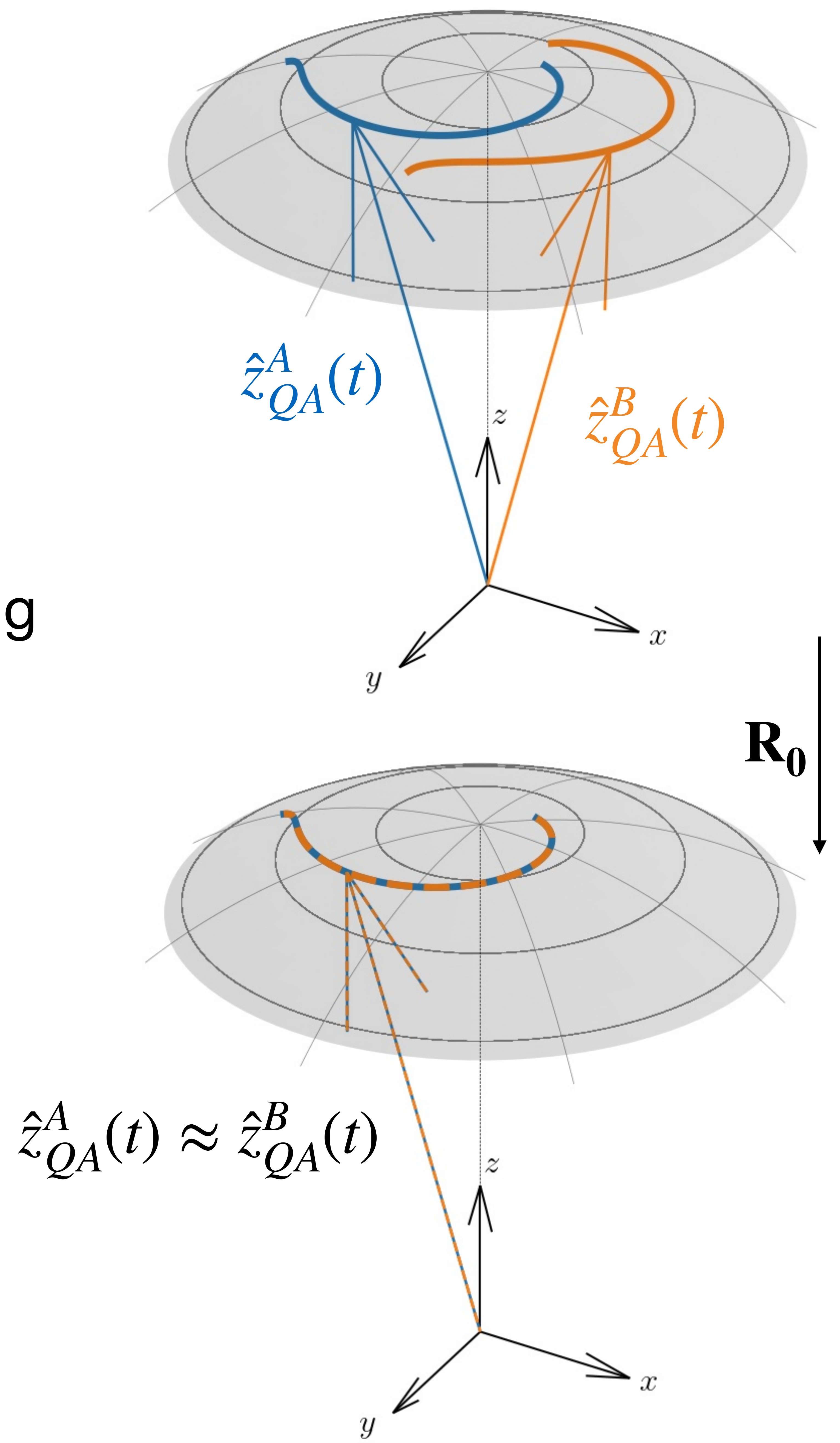


Aligning precessing waveforms

2. Aligning the Euler angles

- Euler angles α, β, γ track the evolution of the coprecessing frame as seen from a specific inertial frame.
- For two waveforms from the same binary system, the coprecessing frames' evolution is **approximately** similar.
- **However**, the angles themselves can differ significantly if the inertial frames used as references are not the same.





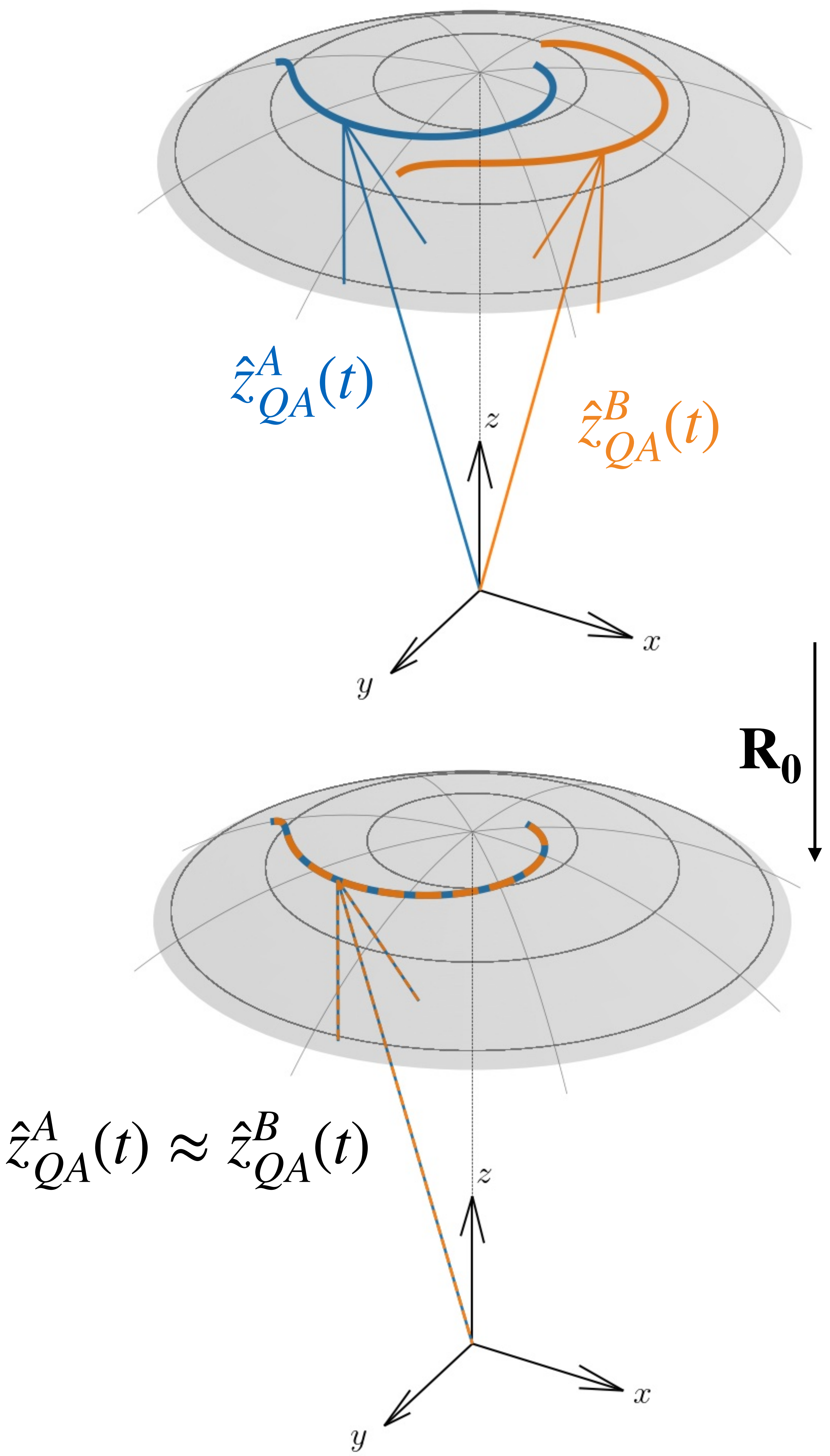
Aligning precessing waveforms

2. Aligning the Euler angles

- For two waveforms of the same system, $h^A_{\mathrm{QA}} \simeq h^B_{\mathrm{QA}}$.
- There are $\{\alpha_A(t), \beta_A(t), \gamma_A(t)\}$, $\{\alpha_B(t), \beta_B(t), \gamma_B(t)\}$ such that

$$\vec{v}_{in} = \mathbf{R}(\alpha, \beta, \gamma)\ \vec{v}_{\mathrm{QA}}, \qquad h_{in} = \sum_{(\ell,m)} h_{QA}\mathscr{D}(\alpha, \beta, \gamma)$$

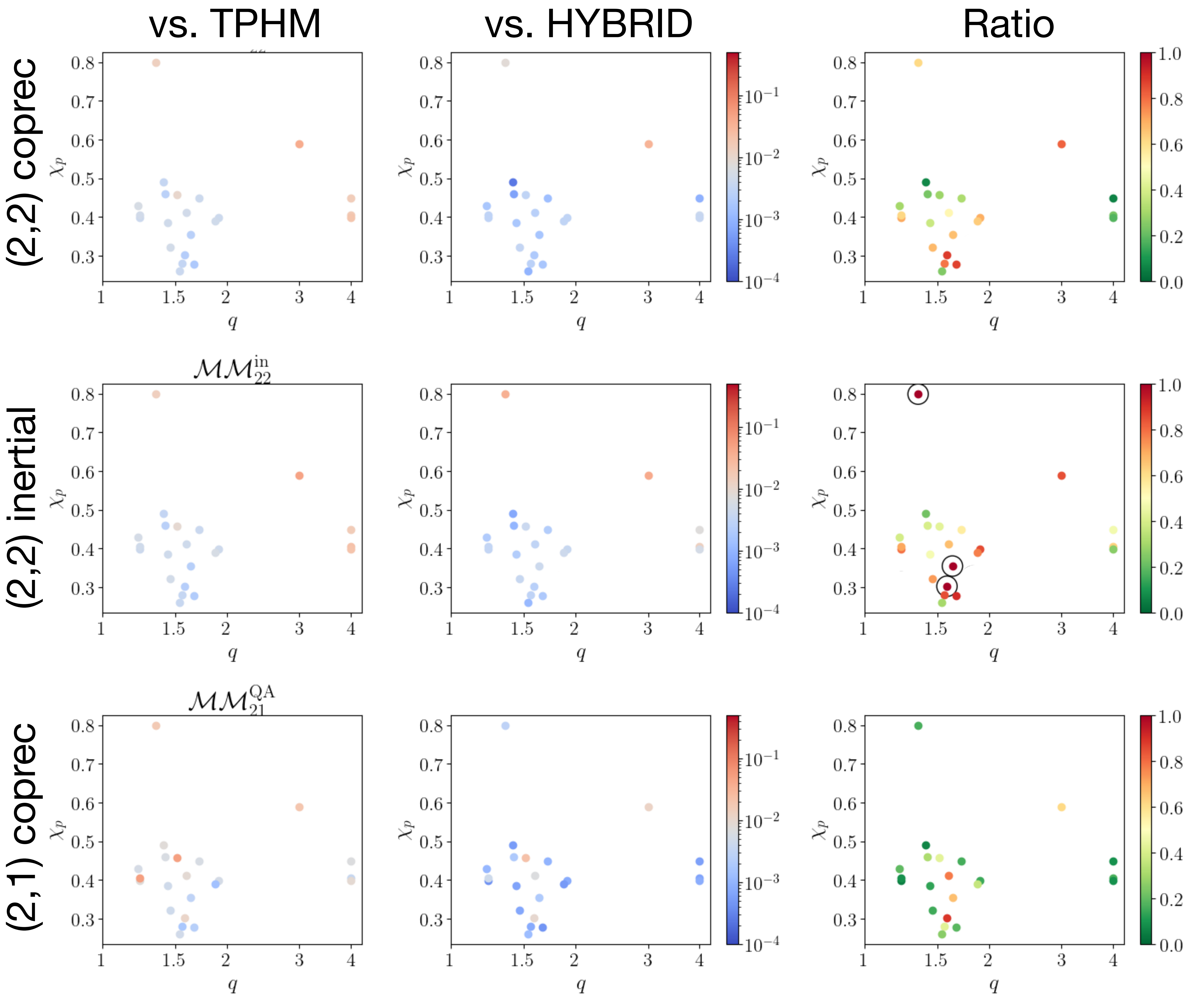
- We want the rotation matrix $\mathbf{R_0}$ that **approximately** transforms coordinate vectors $\vec{v}_A \approx \mathbf{R_0}\,\vec{v}_B$.
- Since $\vec{v}^A_{\mathrm{QA}} = \vec{v}^B_{\mathrm{QA}}$, $\quad \underline{\mathbf{R_0} \approx \mathbf{R}(\alpha_A, \beta_A, \gamma_A) \cdot \left[\mathbf{R}(\alpha_B, \beta_B, \gamma_B)\right]^{\mathrm{T}}}$
- Then, $\mathbf{R_0} \cdot \mathbf{R}(\alpha_B, \beta_B, \gamma_B) = \mathbf{R}(\tilde{\alpha}_B, \tilde{\beta}_B, \tilde{\gamma}_B) \approx \mathbf{R}(\alpha_A, \beta_A, \gamma_A)$
- **Challenges:**
  - Unstable, singular behaviour for certain values of $\{\alpha_i, \beta_i, \gamma_i\}$.
  - Fixing of γ degeneracy: find $\Delta\gamma_B$ such that $\mathbf{R}$ is "the most constant".



Results

Hybridizing SXS sims

- Hybridize SXS simulations with IMRPhenom TPHM.
- Comparing SXS vs. TPHM (1st column) and Hybrid (2nd column).
- Except for a handful of cases, Mismatch lowers when comparing the Hybrid instead on the model waveform.



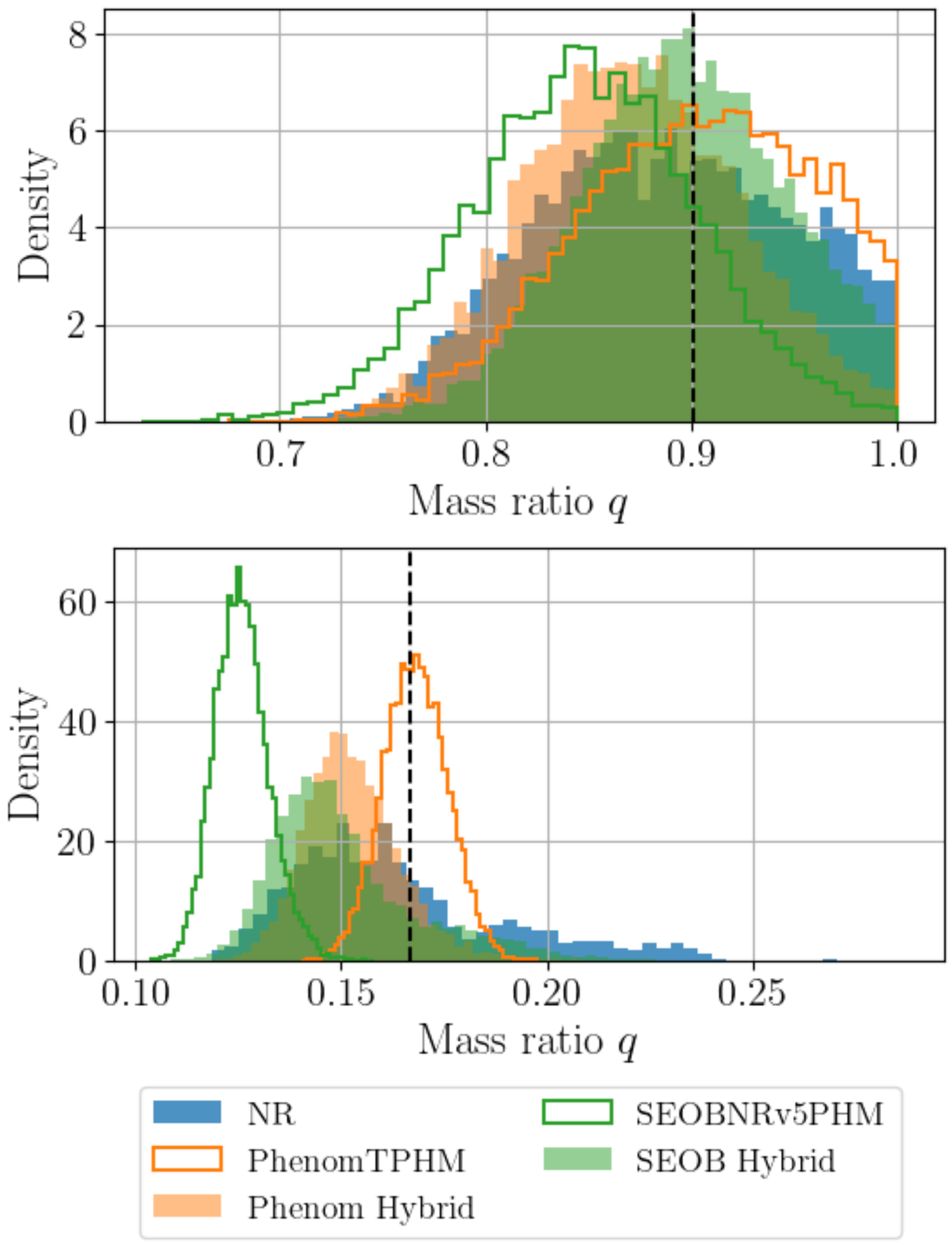
Results

PE injection studies

- We perform injections of NR, model and hybrid waveforms.
    - $M = 50\ \mathrm{M}_{\odot}$, network SNRs ~ 40-60, recovery with IMRPhenomTPHM, dynesty sampler.
- Top panel: when hybridizing long NR waveforms where models and NR posteriors agree, **hybridizing does not affect parameter recovery**.
- Bottom panel: when hybridizing short NR waveforms where posteriors disagree, **hybridizing signals unreliability**.

Self-consistency studies

- Different numerical optimizations
- Length and position of the alignment interval…



PE posterior of the mass ratio q for two systems corresponding to SXS:BBH:0623 (top) and SXS:BBH:0165 (bottom) and 5 different injected waveforms each.

Conclusions

- We can **align** and **build hybrid waveforms** for any quasi-circular system and any two waveform models with reasonable accuracy.
- We understand **challenges of defining precessing systems**. We don't discuss BMS frame fixing or gauge issues.
- The performance is limited by the waveform model differences => including the BMS frame fixing would be useful on top of this but probably impractical.
- Hybridization depends on numerical optimization of parameters, sometimes unreliable.
- Metaparameters (position and length of alignment interval) play a key role.
- Intended for quasi-circular systems, so the user needs to guarantee $e \sim 0$. Applicability to eccentric binaries is to be studied.

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