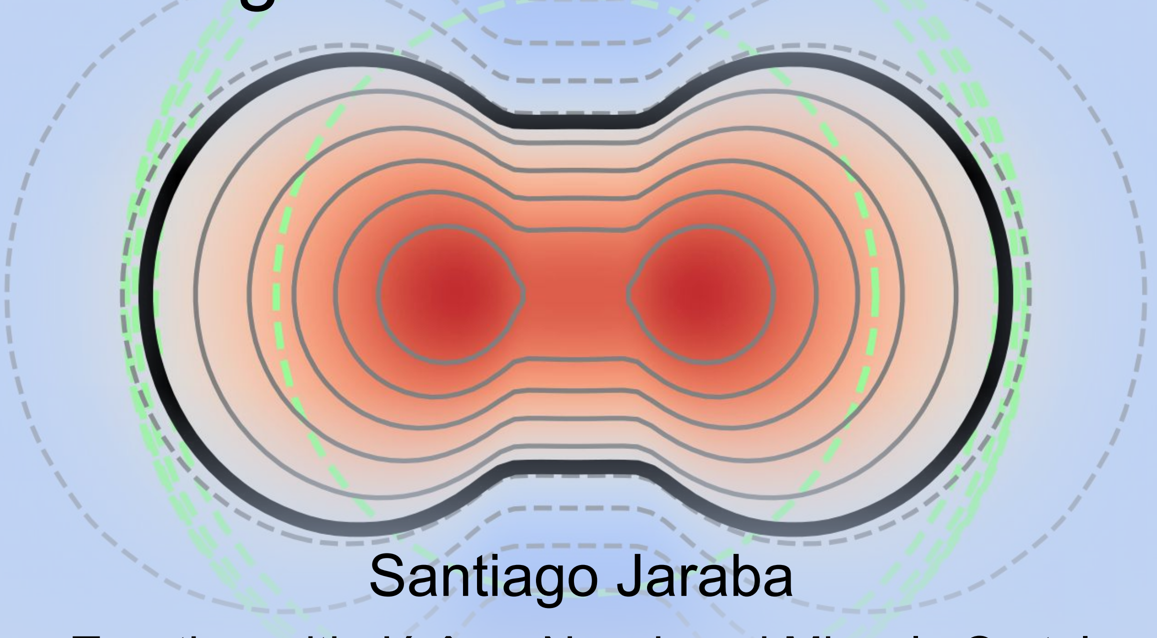
Upgrading ROXAS: a spectral code for rotating neutron star oscillations



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Together with Jérôme Novak and Micaela Oertel

14th Iberian Gravitational Waves Meeting

CNRS

Observatoire astronomique de Strasbourg | ObAS

Motivation and history

- Simulating perturbed, isolated neutron stars → BNS post-merger dynamics.
- Studied since the 1960s.
  - Chandrasekhar 1964 → radial oscillations.
  - Thorne, Campolattaro 1967 → non-radial modes.
- Progress on the numerical side has been drastic in the last few decades.
  - Valencia formulation (Banyuls et al., 1997): conservative formulation of GR hydrodynamical eqs.
  - First 3D binary neutron star merger simulation (Shibata, Uryu 2000).
  - Proliferation of different codes for Numerical Relativity:
    - CoCoNut
    - Einstein Toolkit
    - BAM
    - SpEC
    - SACRA
    - …

ROXAS

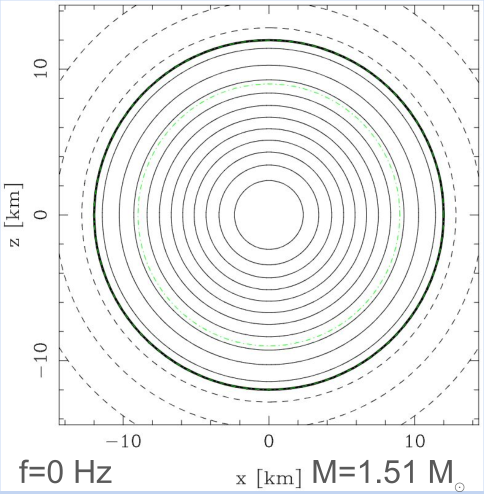
- **R**elativistic **O**scillations of non-a**X**isymmetric neutron st**A**r**S**.
- Developed mainly by Gaël Servignat, Servignat et al. 2025, together with Jérôme Novak.
- Based on formalism developed in Servignat et al. 2023 using primitive variables.
  - Energy density e, pressure p, baryon density n_B, log-enthalpy H, $H = \ln\left(\frac{e+p}{m_B n_B}\right)$
  - Coordinate velocity v_i, Eulerian velocity U_i.
  - Opposed to conserved variables, $D = m_B n_B \Gamma^2, \quad S_j = (e+p)\Gamma^2 U_j, \quad \tau = (e+p)\Gamma^2 - p$

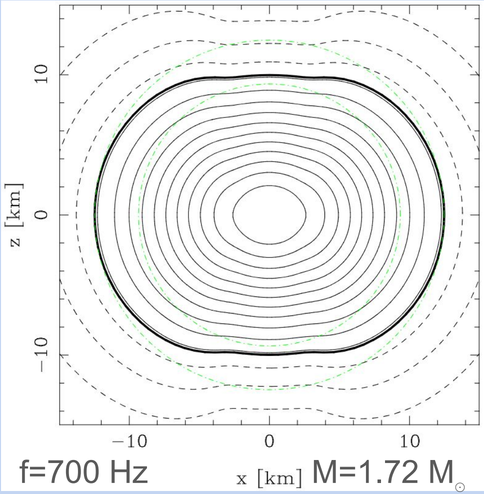
$$\Gamma = (1 - U_j U^j)^{-1/2}$$

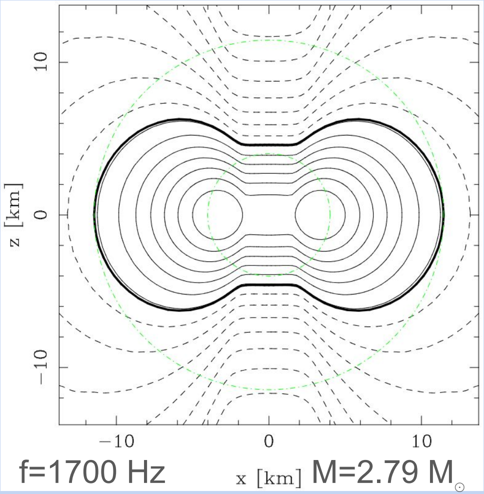
- Uses spectral methods (Chebyshev).
  - ! Unable to handle shocks: Gibbs phenomenon.
- Aims to be as lightweight as possible. Runs in laptops, O(hours, few days).
- Publicly available at https://zenodo.org/records/14849547

LORENE (Langage Objet pour la RElativité NumériquE)

- ROXAS is implemented on top of LORENE.
- LORENE: set of C++ classes for numerical relativity.
- Continuous development from 1997, mainly in Paris Observatory.
- LORENE can be used to find equilibrium configurations of neutron stars.







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- LORENE: set of C++ classes for numerical relativity.
- Continuous development from 1997, mainly in Paris Observatory.
- LORENE can be used to find equilibrium configurations of neutron stars.
- A perturbation mixing l=m=0 and l=m=2 modes is added and dynamically evolved with ROXAS.

$$\delta H = \varepsilon \left(x^2 - y^2 - \frac{2}{3} r^2 \right) \left(1 - \frac{r^2}{R_S(\theta)^2} \right)$$

$$H = \ln \left(\frac{e + p}{m_B n_B} \right)$$

First-step assumptions

- Cold NS (T=0) in β-equilibrium → one-parameter equation of state (EoS), p=p(ρ).
  - ROXAS currently supports 2-parameter (ρ, Y_e) EoS, accounting for out of β-equilibrium NS.
  - 3-parameter EoS (ρ, Y_e, T) are planned to be implemented.
  - Assumed polytropic ($p=\kappa\rho^{\gamma}$) for the results in this presentation.
- Equilibrium configurations for stars are rigidly rotating, $\vec{v}(r,\theta) = \Omega_0\, r \sin\theta\, \hat{\varphi}$
  - Differential rotation recently implemented and being tested.
- No electromagnetic fields.
- Hydrodynamical equations: $\partial_t H = -v^i D_i H - c_s^2 \frac{\Gamma^2 N}{\Gamma^2 - c_s^2(\Gamma^2-1)}\left[K_{ij}U^iU^j - K + D_iU^i - \frac{U^i}{\Gamma^2}D_iH\right]$

$$\partial_t U_i = -\, v^j D_j U_i - U_j D_i v^j + N U_j D_i U^j - \frac{N}{\Gamma^2} D_i(H + \ln N) + U_i U^j D_j N$$
$$+ \frac{c_s^2 N U_i}{\Gamma^2 - c_s^2(\Gamma^2-1)}(D_jU^j - K) + U_i \frac{\Gamma^2(c_s^2-1)}{\Gamma^2 - c_s^2(\Gamma^2-1)} N U^l U^j K_{lj} + \frac{N(1-c_s^2)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j \frac{D_j p}{e+p}$$

Well-balanced formulation

- We divide quantities in equilibrium term + “perturbation”

$$U_i = U_{i,\mathrm{eq}} + \bar{U}_i, \quad H = H_{\mathrm{eq}} + \bar{H}, \quad \beta^i = \beta^i_{\mathrm{eq}} + \bar{\beta}^i, \quad N = N_{\mathrm{eq}} + \bar{N}$$

- We can use identities that equilibrium quantities satisfy to get rid of some terms (order 0).

$$H_{\mathrm{eq}} + \ln N_{\mathrm{eq}} - \ln \Gamma_{\mathrm{eq}} = \mathrm{const.}$$

$$v^j_{\mathrm{eq}} D_j U_{i,\mathrm{eq}} + U_{j,\mathrm{eq}} D_i v^j_{\mathrm{eq}} = 0$$

$$v^i_{\mathrm{eq}} D_i H_{\mathrm{eq}} = 0$$

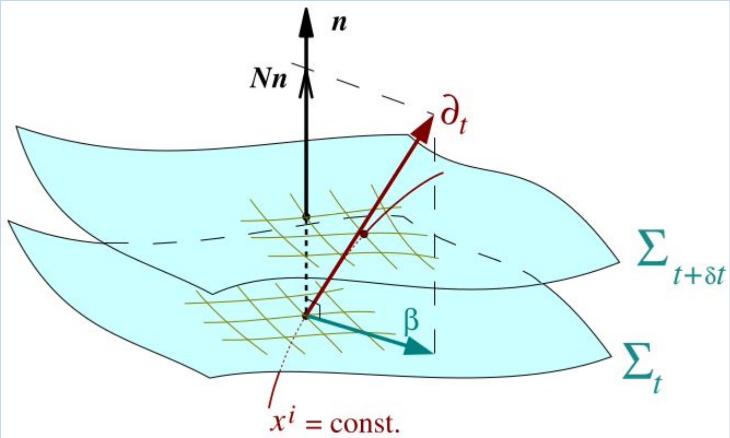
- **No** perturbative expansion: we keep all the terms.

3+1 formalism and CFC

- 3+1 decomposition:

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

Lapse N, shift β^i, spatial metric γ_{ij}.

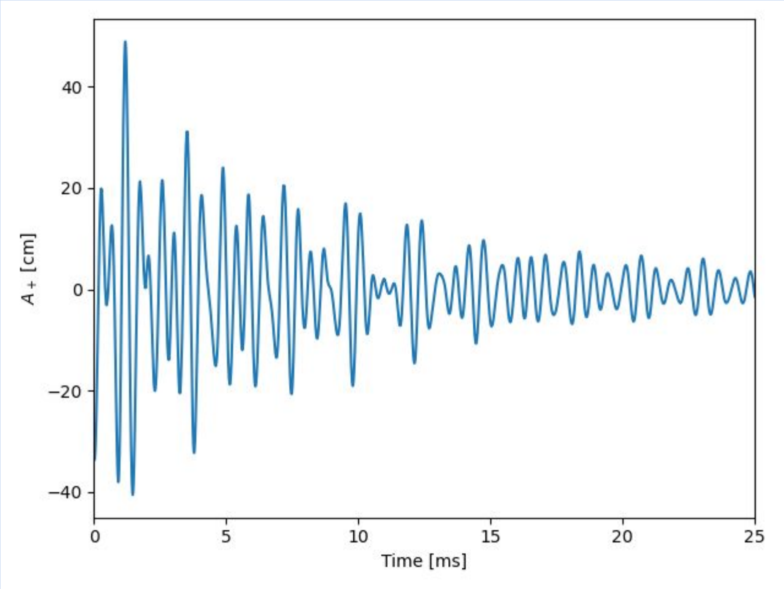


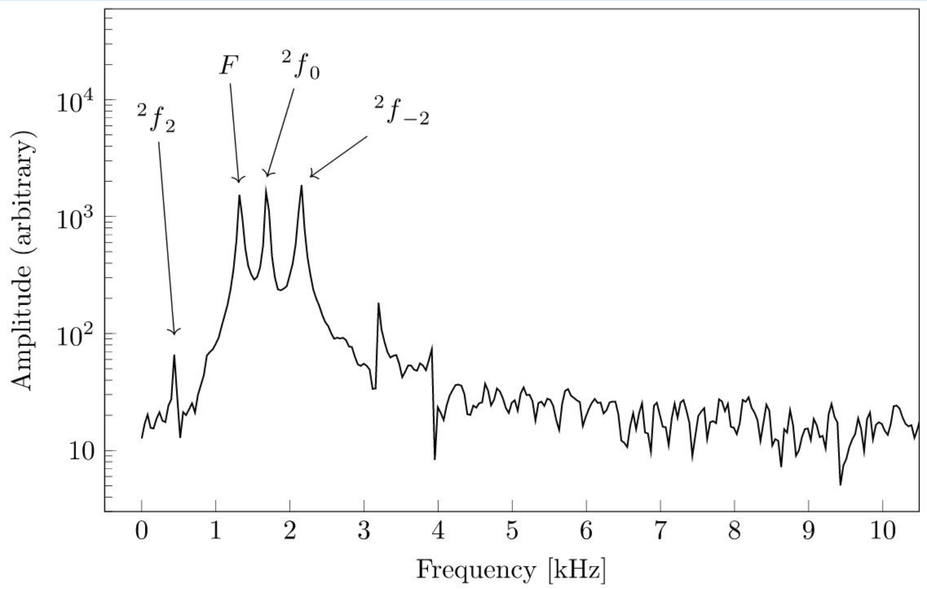
Credit: E. Gourgoulhon

- We assume conformal flatness (CFC), $\gamma_{ij} = \Psi^4 \tilde{\gamma}_{ij}$, with $\tilde{\gamma}_{ij}$ flat.
- Can be done as long as:
  - The deviation from axisymmetry is small enough.
  - The impact of GWs on the dynamics is negligible (no backreaction).
  - Both assumptions apply for an axisymmetric star + small perturbation.

Waveforms

- CFC removes the radiative degrees of freedom → no GW from the metric.
- GW extraction is still possible through the quadrupole formula.





Waveform spectra

- Comparison with axisymmetric modes from CFC simulations (Dimmelmeier et al. 2006) and non-axisymmetric modes from GR (Krüger, Kokkotas 2020) simulations.
- Most relative errors of order 1% or below.

Model	F	H_1	${}^2f_0$	${}^2f_2$	${}^2f_{-2}$	δF (%)	δH_1 (%)	$\delta\ {}^2f_0$ (%)	$\delta\ {}^2f_2$ (%)	$\delta\ {}^2f_{-2}$ (%)
BU0	1.446	3.956	1.596	1.565	1.565	0.83	0.38	0.63	0.83	0.83
BU1	1.423	3.883	1.609	1.08	1.953	0.7	0.82	0.12	0.09	0.56
BU2	1.396	3.884	1.642	0.84	2.052	1.15	0.59	0.43	0.48	0.1
BU3	1.36	3.915	1.68	0.64	2.123	1.25	0.15	0.65	0.63	0.24
BU4	1.322	3.934	1.715	0.451	2.153	1.36	0.4	0.99	1.11	0.09
BU5	1.282	3.958	1.723	0.28	2.158	0.08	0.15	0.52	4.64	1.44
BU6	1.258	3.998	1.744	0.13	2.143	3.1	0.3	0.86	12.3	1.82
BU7	1.202	4.001	1.751	$\times$	2.105	0.42	0.57	1.77	$\times$	3.94

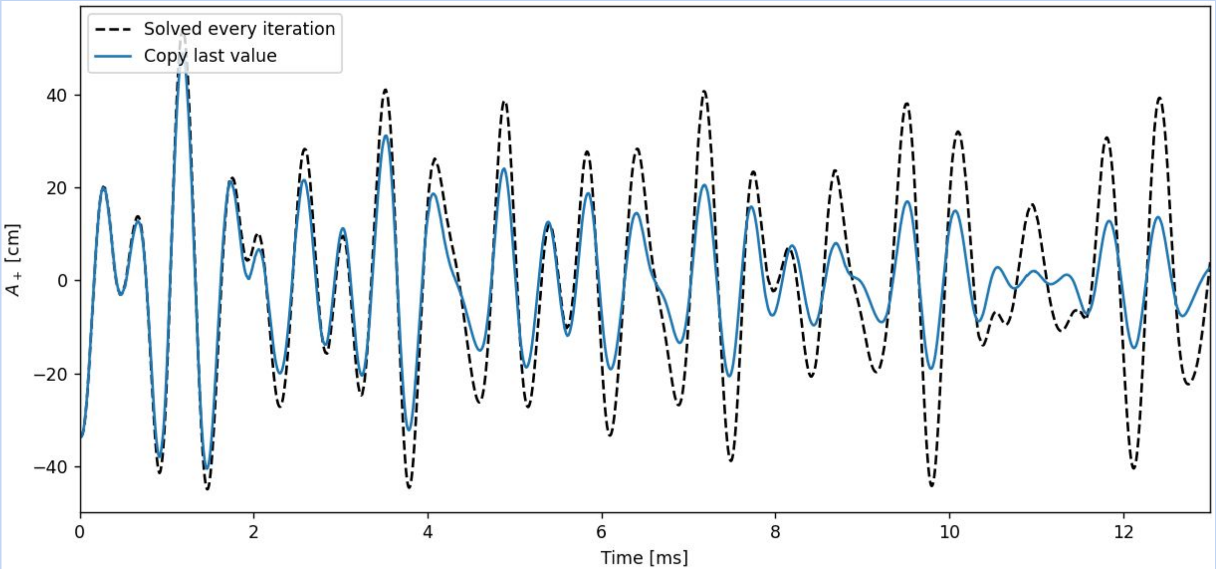
Recent progress: metric extrapolation

$$\Delta\Psi = -2\pi\Psi^5\left[E + \frac{1}{16\pi}K_{ij}K^{ij}\right],$$

$$\Delta(N\Psi) = 2\pi N\Psi^5\left[E + 2S_t + \frac{7}{16\pi}K_{ij}K^{ij}\right]$$

$$\Delta\beta^i + \frac{1}{3}\bar{D}^i\bar{D}_j\beta^j = 16\pi N\Psi^4 p^i + 2\Psi^{10}K^{ij}\bar{D}_j\left(\frac{N}{\Psi^6}\right)$$

- N and β^i are updated once every certain number of iterations.
- Old method: metric quantities are frozen → oscillation decays faster than it should.
- Recent addition: linear extrapolation → greater accuracy.



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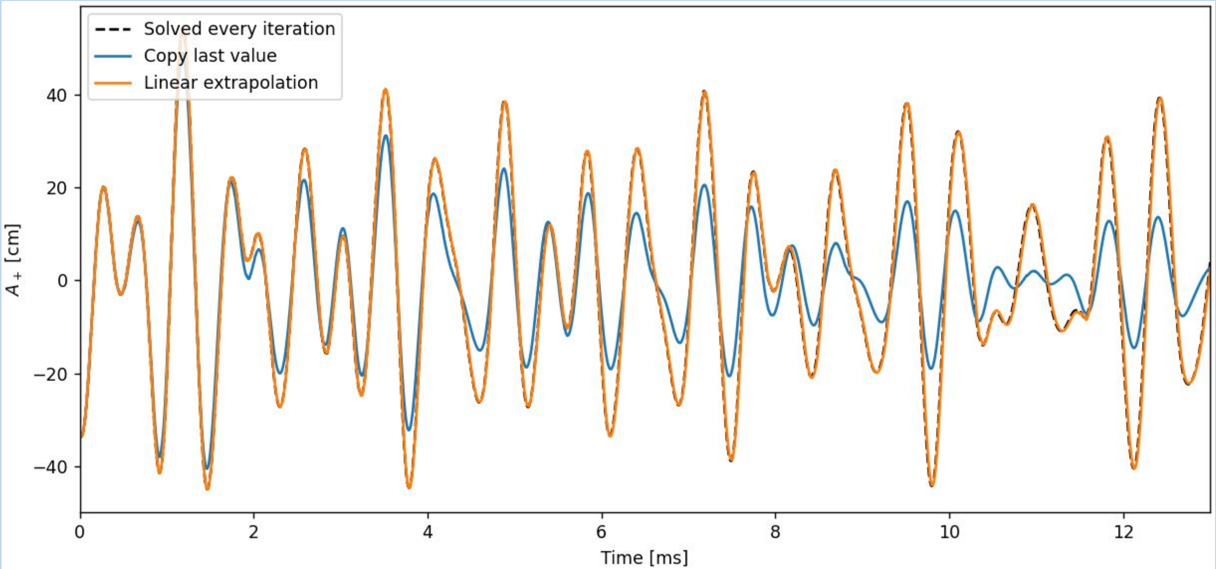
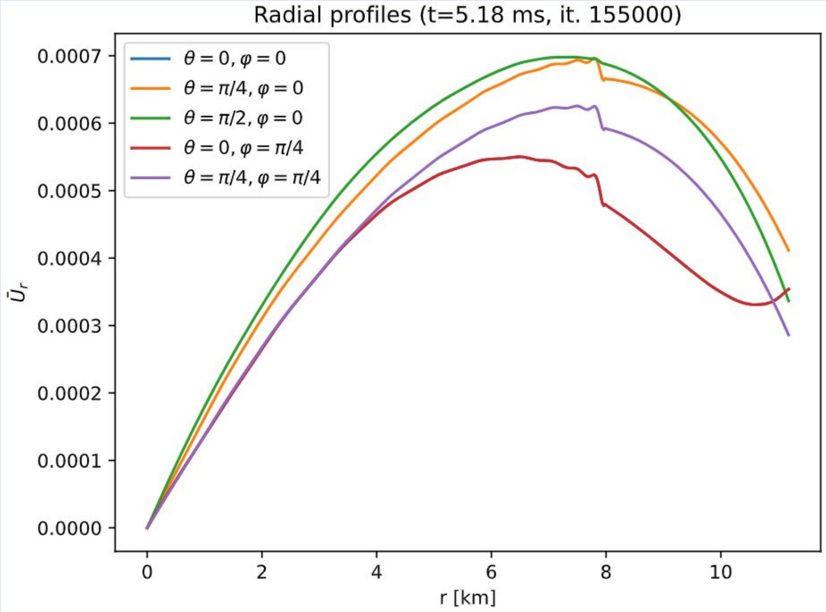
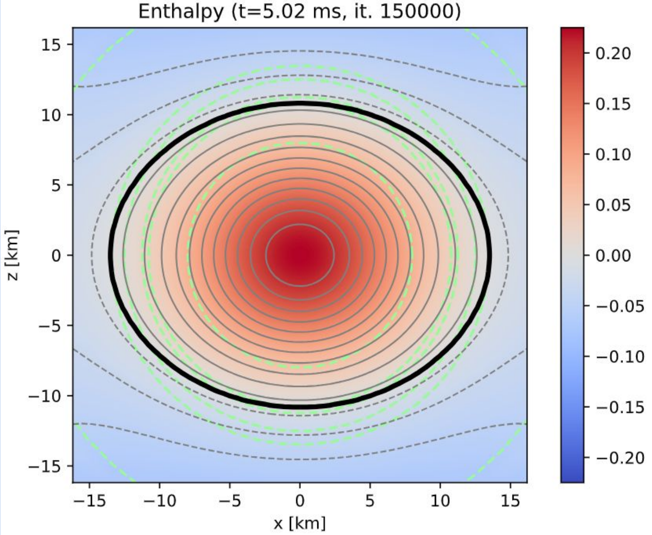
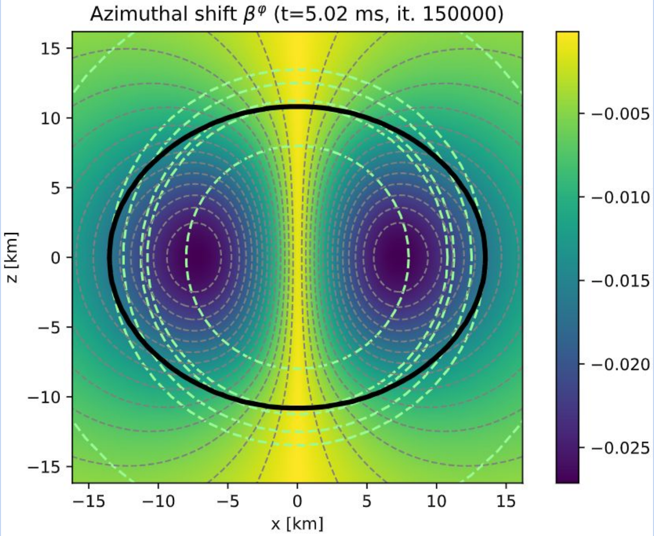


Recent progress: graphical output

- Snapshots and animations can now be plotted.
- Useful for problem diagnosis.







Recent progress: differential rotation

- Additional terms need to be taken into account in equilibrium wrt rigid rotation.

$$H_{\rm eq} + \ln N_{\rm eq} - \ln \Gamma_{\rm eq} + \int_{\Omega_p}^{\Omega} F(\Omega) d\Omega = \text{const.}$$

$$v^j_{\rm eq} D_j U_{i,\rm eq} + U_{j,\rm eq} D_i v^j_{\rm eq} = \frac{U_{\varphi,\rm eq} v^{\varphi}_{\rm eq}}{\Omega} D_i \Omega$$

$$\vec{v}(r,\theta) = \Omega(r,\theta)\, r \sin\theta\, \hat{\varphi}$$

$$F = u^t u_{\varphi}$$

- Must be accounted for both
  - LORENE equilibrium solvers → implemented and tested with literature (Iosif, Stergioulas 2021).
  - Hydrodynamical equations in ROXAS: extra term $\frac{\bar{N}}{N^2_{\rm eq}} \Psi^4_{\rm eq} r \sin\theta (\Omega r \sin\theta + \beta^{\varphi}_{\rm eq}) D_i \Omega$ in equation for U_i → Code is ready, still needs testing.

Recent progress: differential rotation

$$F = u^t u_\varphi$$

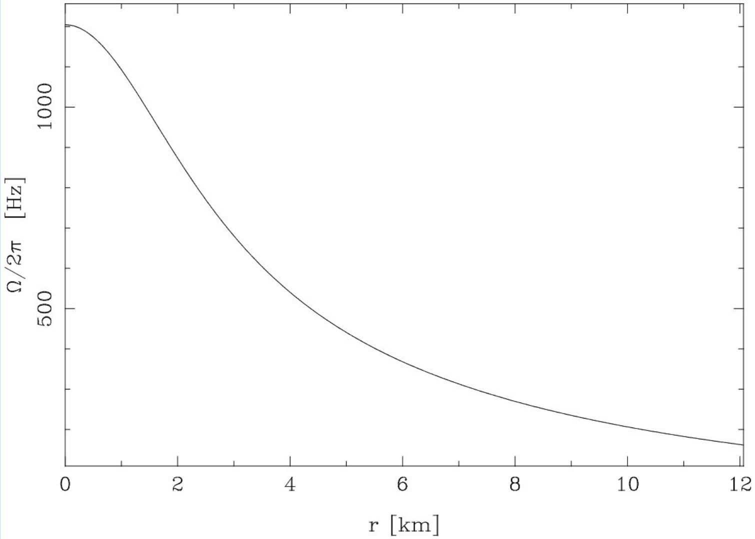
- Involves an additional choice: rotation profile F(Ω) (or Ω(F)).

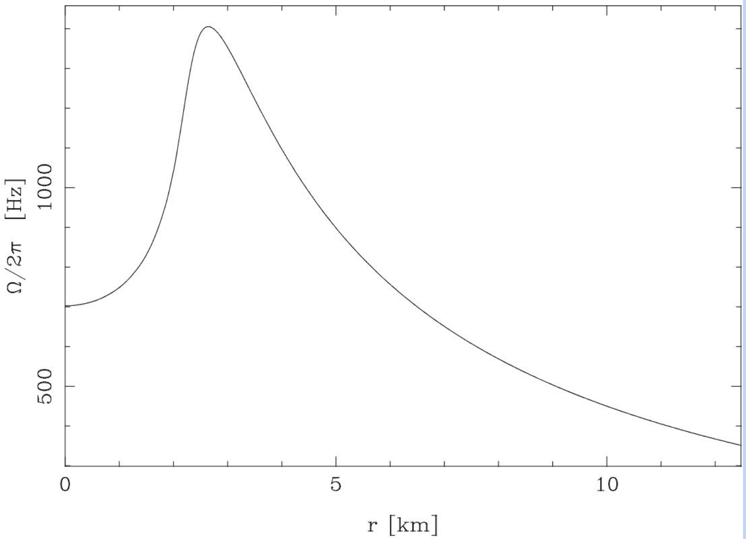
KEH profile (Komatsu, Eriguchi, Hachisu 1989)

$$F(\Omega) = A^2(\Omega_c - \Omega)$$

Uryu profiles, more realistic (Uryu et al. 2017)

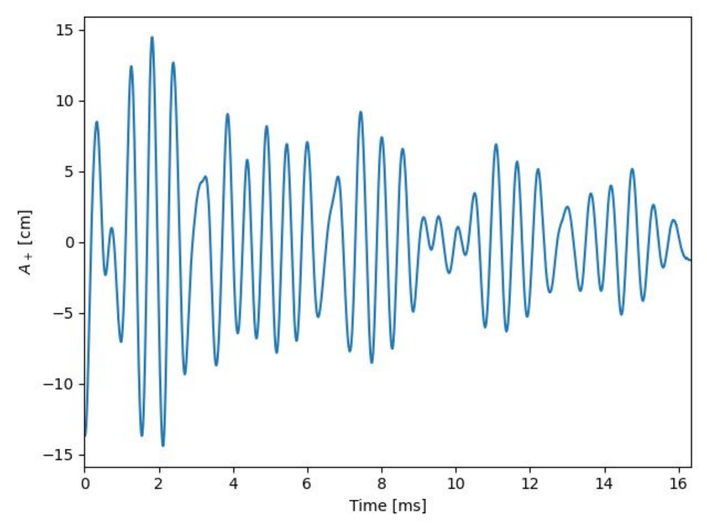
$$\Omega = \Omega_c \frac{1 + \left(\frac{F}{B^2\Omega_c}\right)^p}{1 + \left(\frac{F}{A^2\Omega_c}\right)^{q+p}}$$

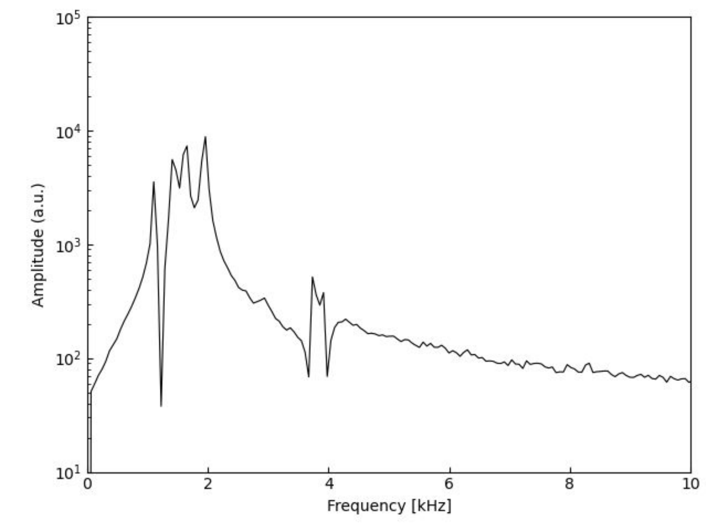




Recent progress: differential rotation

- Some simulations running for KEH profiles.





- Ongoing work: Uryu profiles, more tests.

Summary

- ROXAS: lightweight code to simulate isolated neutron star oscillations.
- Based on primitive variables, uses spectral methods, CFC, well-balanced formulation.
- Extracts waveforms through quadrupole formula.
- Reproduces results from the literature with rigid rotation.

✕ Unable to treat shocks.

✕ CFC limits applicability to small perturbations, no backreaction, no GW from the metric.

- Ongoing work:
  - Test differential rotation.
  - Improve accuracy and stability.
- Next steps:
  - Use realistic equations of state.
  - Generalize beyond CFC? Magnetic fields?