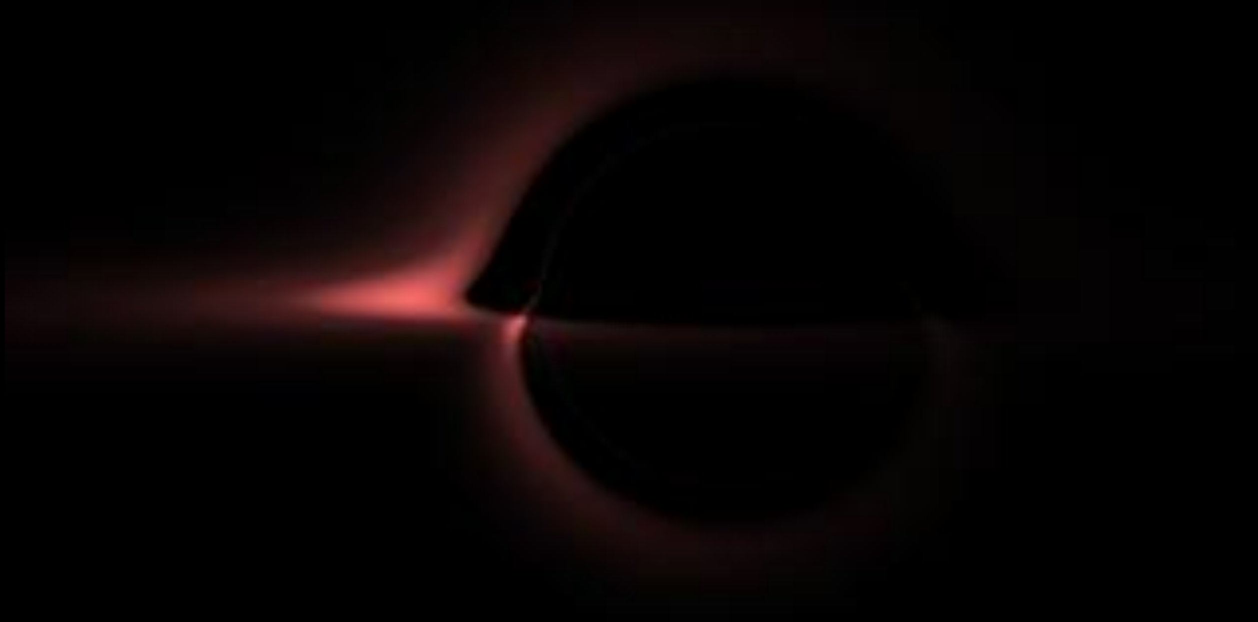
The QNM-Shadow correspondence in strong-gravity multimessenger astrophysics



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14th Iberian Gravitational Waves Meeting

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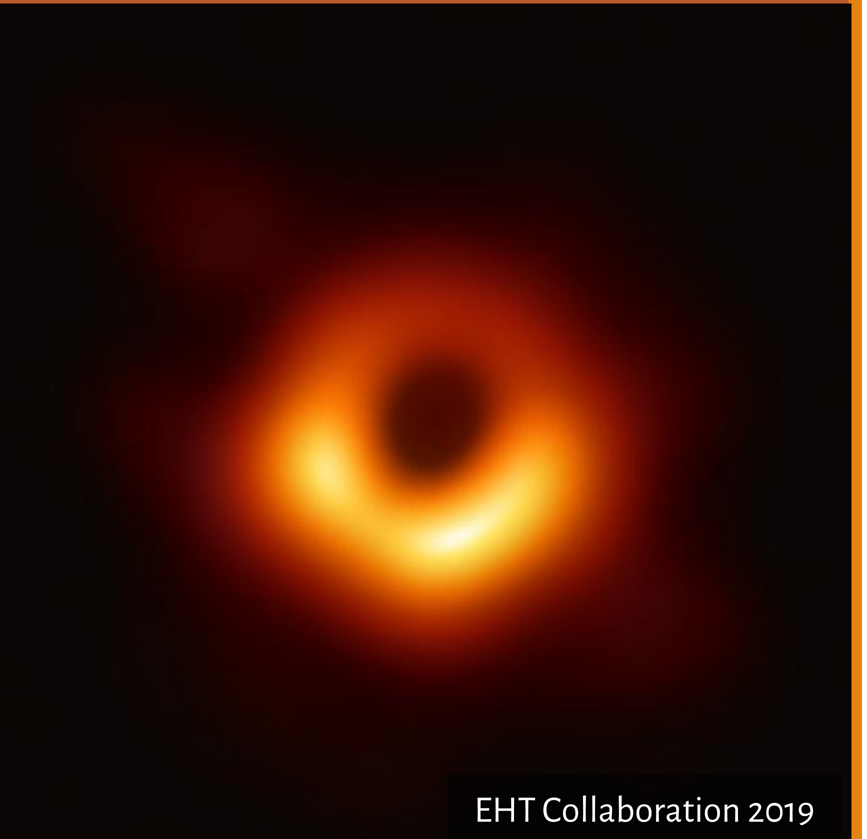
Imaging and gravitational waves – Testing strong gravity



EHT Collaboration 2019

Black hole shadows and gravitational waves explore the same aspects — the near-horizon regime — but from different angles.

- **Ringdown (from GWs):** captures the dynamical response of spacetime after a merger.
- **Shadow (from imaging):** reveals the stationary structure of spacetime through light bending.

- Use both tools to test General Relativity and its extensions:
  - Modified gravity: e.g., Gauss–Bonnet, f(R)f(R), Lovelock.
  - Scalar–tensor theories: Brans–Dicke, dilaton, Horndeski.
  - Metric-affine formulations and non-Riemannian geometries.
- Search for exotic compact objects: boson stars, gravastars, fuzzballs…
- Look for signatures of quantum gravity near the event horizon.

Some questions…

If both fields are measuring the spacetime at its extreme…

- ❑ Is there any quantity from imaging that relates to one from gravitational waves?
- ❑ Can we use it to check quantities between fields?
- ❑ Can it show extra physics?

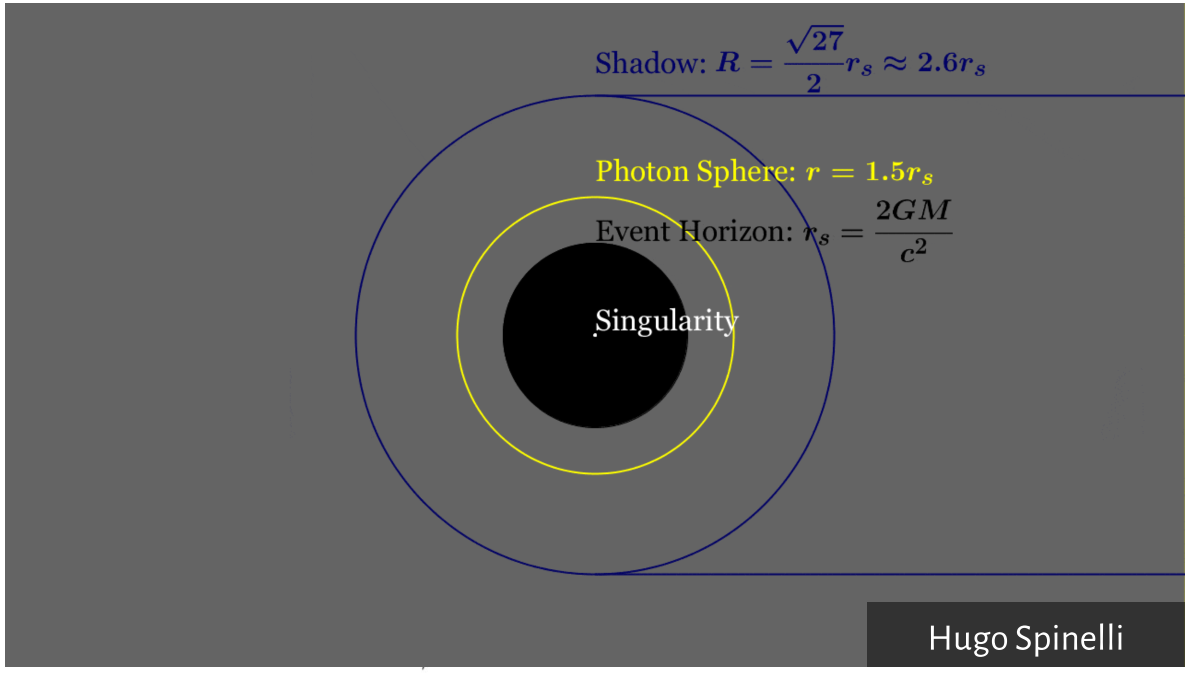
Null geodesics on curved spacetimes

Extreme lensing effects:

- Strong bending near a particular region: the **photon sphere.**
- The photon sphere limits the region where:
  - photons go back to infinity, and
  - photons that are captured (by a horizon or a surface).

The projected region (for the observer) from where no light comes from is the **shadow.**

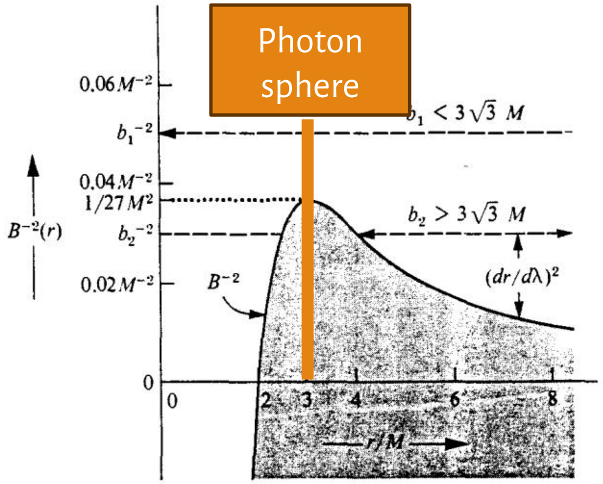
Near the photon sphere, photons can turn around the object multiple times: leading to multiple images: the **photon rings.**



The instability of photon paths

These orbits around the photon sphere are **not stable** — they sit at a **maximum of the effective null geodesic radial potential**.

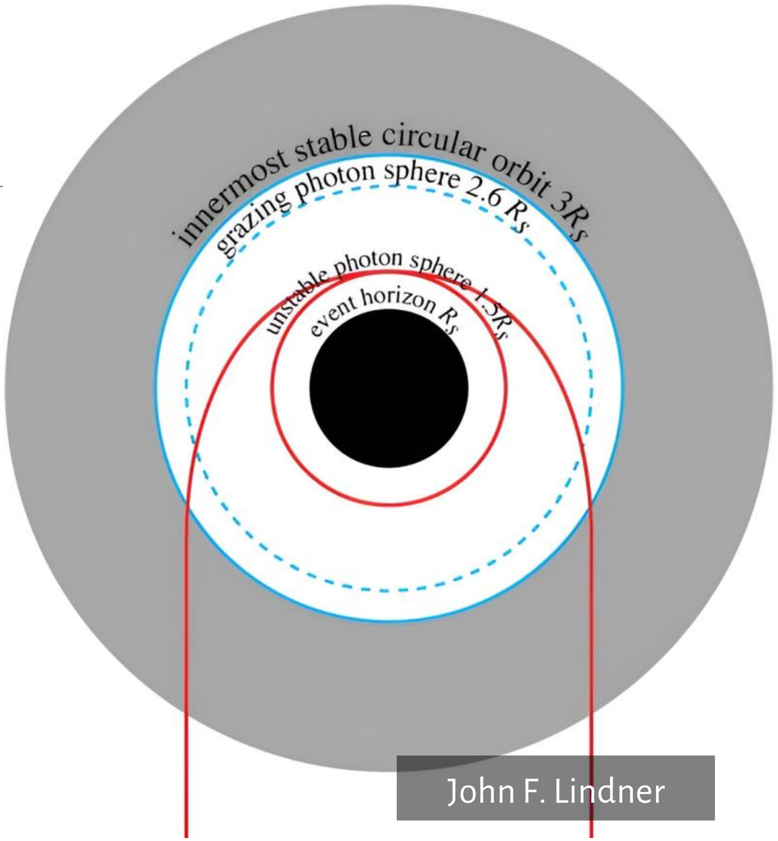
- A small inward or outward deviation causes the photon to be captured or escaping.



MTW

The **Lyapunov exponent** quantifies the **exponential rate at which nearby trajectories diverge**:

$$\delta r_n \propto e^{\gamma_{ps} n}$$



John F. Lindner

The QNM-Shadow correspondence

The **instability** of the photon paths near the photon sphere of the object is related to the instability of the gravitational radiation through the quasinormal modes of oscillation in the **eikonal limit**.

Lyapunov exponent

$$\gamma_{ps} \propto \omega_I$$

Imaginary Freq. of QNM

V. Cardoso et.al. [0812.1806]

✓ Is there any quantity from imaging that relates to one from gravitational waves?

Yes, but...

Simulating exotic objects

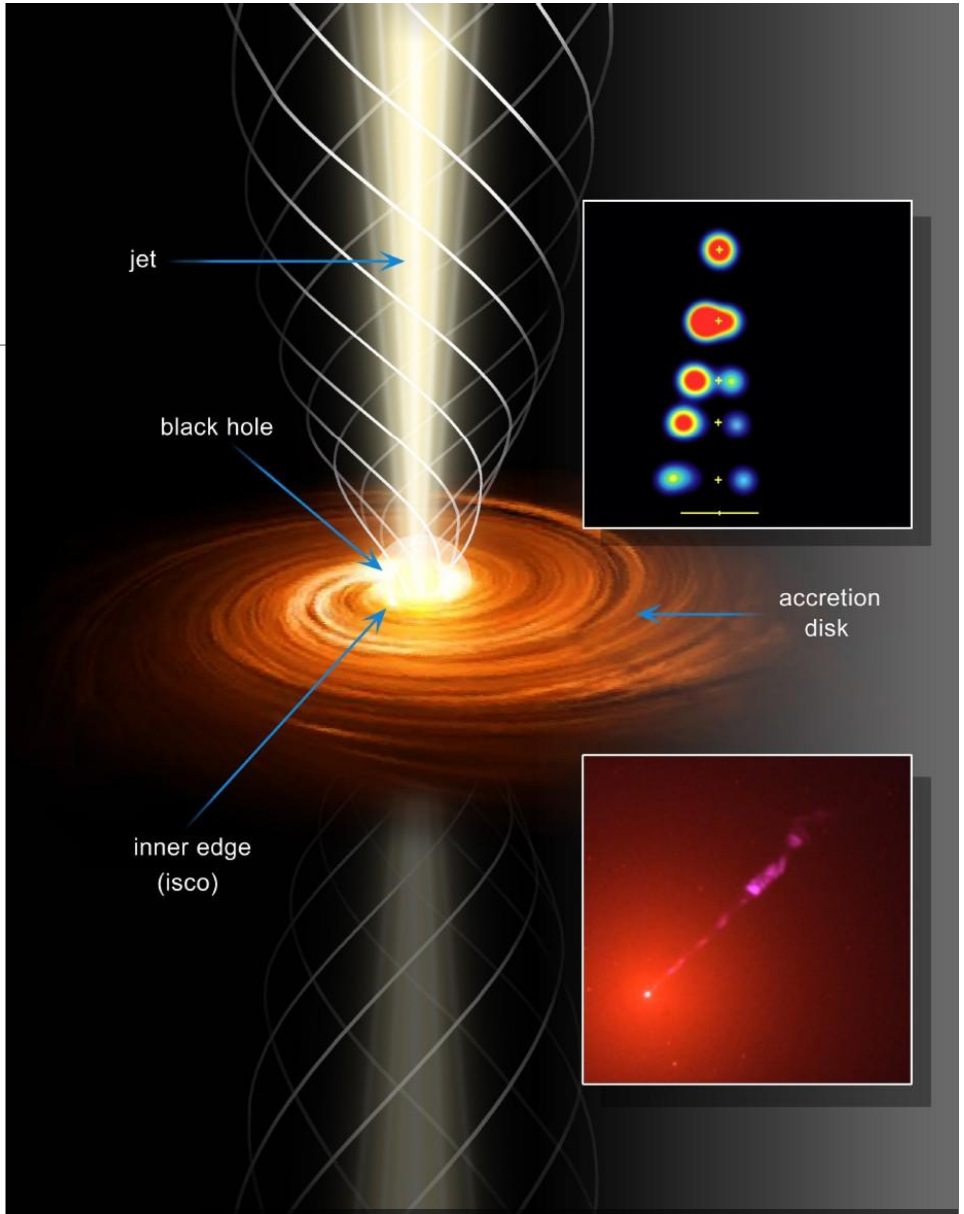
NULL GEODESICS + EMISSION = BLACK HOLE IMAGE

The emission

The matter that accretes into the black hole emits EM radiation.

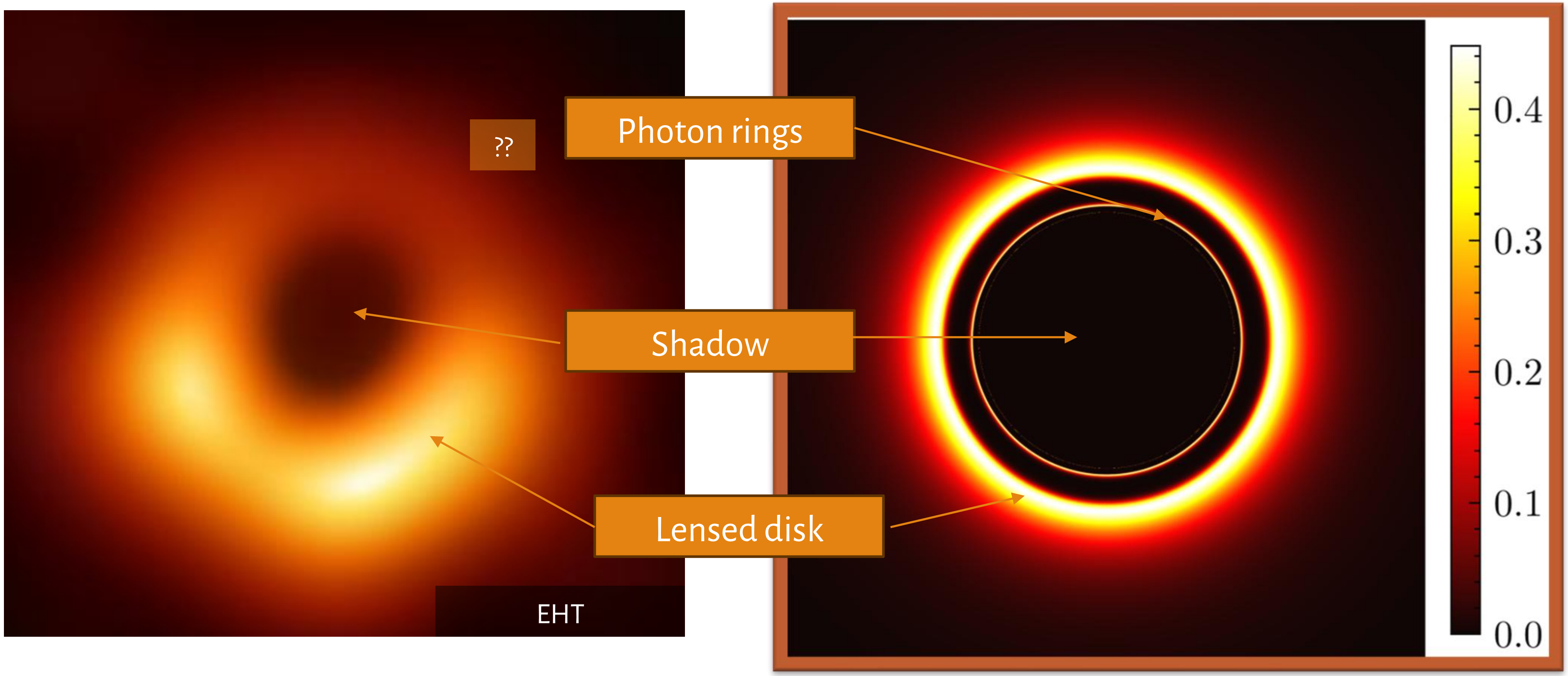
The accretion disk (to deep into fluid dynamics, plasma physics, astrophysical observations)
- Geometrically thin
- Optically thin
- Semi-analytic profile based on GRMHD simulations.

All together in a GPU-accelerated code (some metrics are expensive)



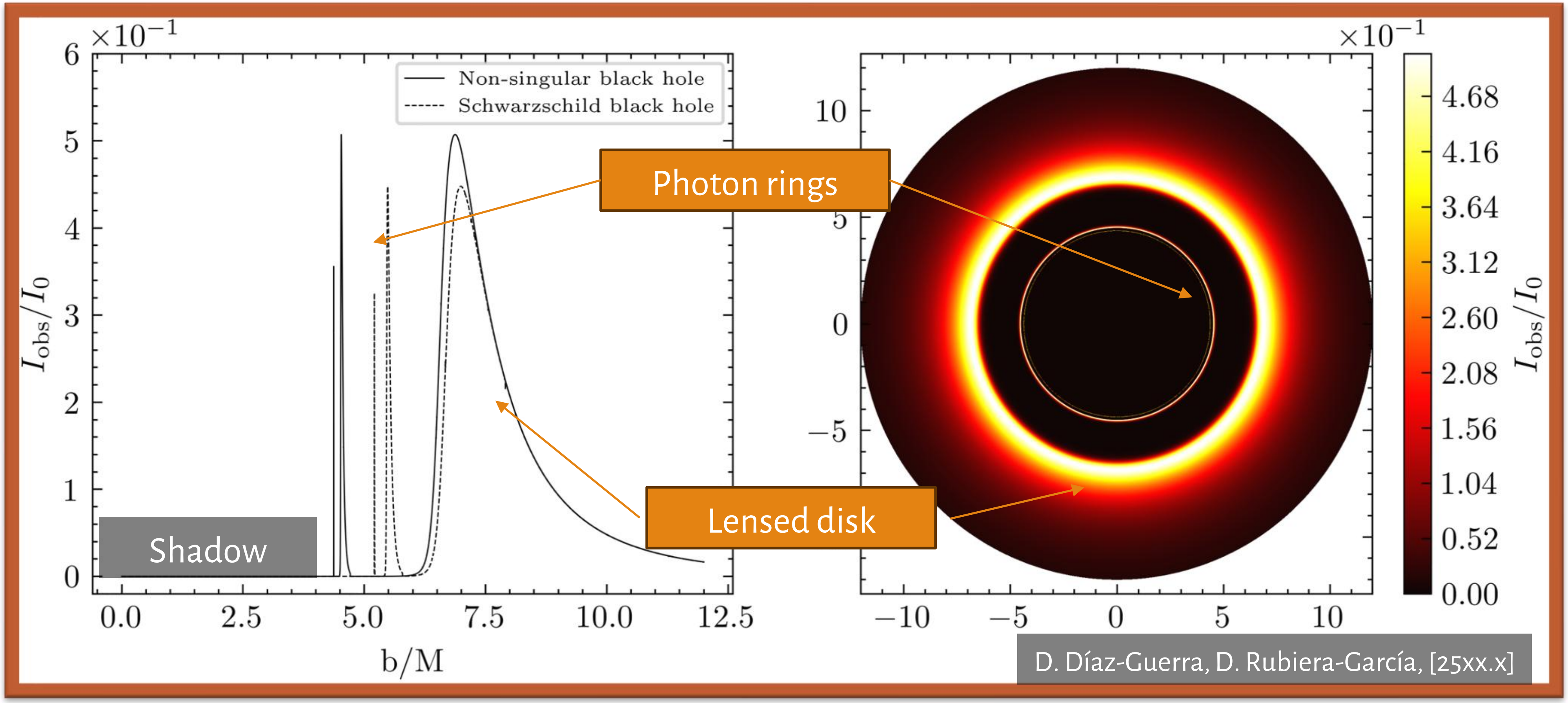
J.A. Biretta et al., Hubble Heritage Tea

The black hole image



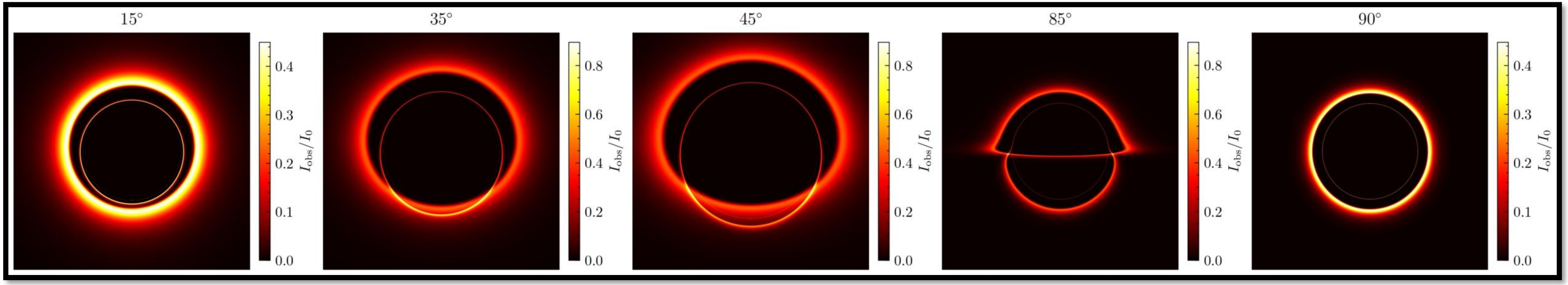
Testing GR with imaging

NON-SINGULAR BLACK HOLE
- Palatini: $g_{\mu\nu}$ and $\Gamma^{\mu}_{\nu\sigma}$ are independent
- Anisotropic fluid

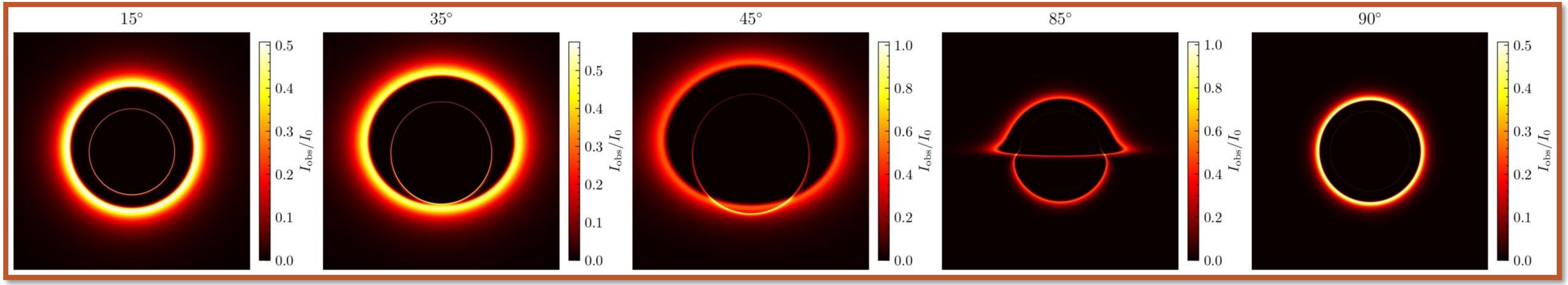


Quantitative Features:
- Lensed disk
- The shadow
- The photon rings

Schwarzschild black hole



Non-singular black hole



Measuring the Lyapunov

The exact value of the Lyapunov exponent needs the limit of $n \to \infty$, but...

From the rings obtained with the image, we can **approximate** the Lyapunov exponent,

The ratio of widths, ω, of subsequent rings,

The ratio of intensities of the peaks,

$$\frac{\omega_{n+1}}{\omega_n} \approx \frac{I_{\nu;n+1}}{I_{\nu;n}} \approx e^{-\gamma_{ps}}$$

As we go beyond $n > 1$, the rings do not depend on the physics of the accretion disk, yielding a pure **spacetime** measurement.

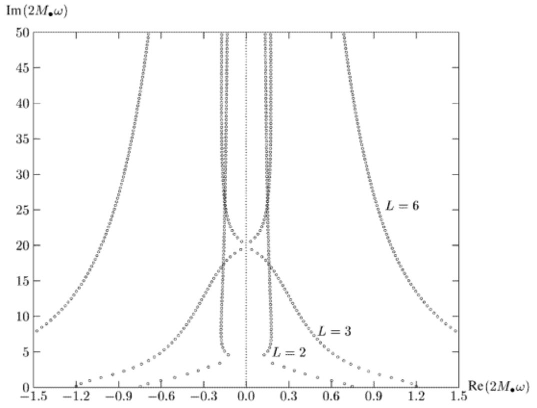
Using the QNM-Shadow correspondence

1. We measure the **shadow**, and **photon ring's** location on the image.

2. From the photon rings, we can approximate the Lyapunov exponent.

3. **Use** the correspondence

$$\gamma_{ps} \propto \omega_I$$



Nollert [CQG16.R159]

✓ Can we use it to check quantities between fields?
Yes, approximately.

BH Shadows aren't just still images — they encode the background spacetime where GWs propagate.
We might combine both to cross-check and constrain the black hole geometry.

Discussion and cautionary tale

AS OF TODAY, we can NOT resolve the rings.

- ngEHT and Black Hole explorer will resolve the first ring, hopefully also the second ring.

Even measuring the two photon rings, the Lyapunov exponent calculated is an approximation.

- Useful, but still an approximation!

Gravitational waves do NOT follow ray-optics. The Eikonal limit just yields an idealized high frequency wave of any spin on curved spacetime.

We need to keep study the theoretical implications of this correspondence, and new ways to exploit it. Such that, in the future, we can do **strong-gravity multimessenger astrophysics**.

✓ Or do they show extra physics? We do not now, yet…

Conclusions

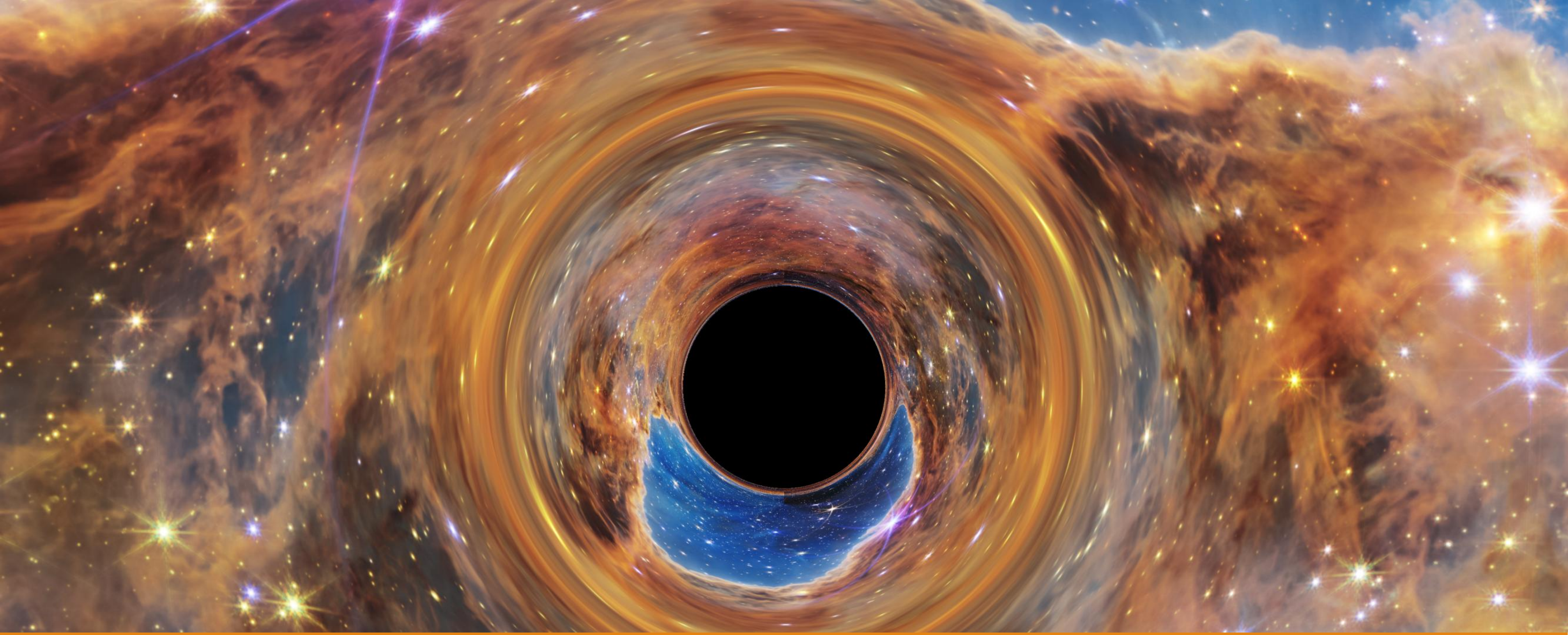
Black Hole Imaging offers a direct look at strong-field spacetime.

- Null geodesics + lensing → shadow, photon ring, Lyapunov
- In the inner photon ring, emission is secondary; geometry is the main aspect.

GWs and imaging are **deeply complementary tools** for strong gravity:

- The QNM-shadow correspondence is useful to compare and test results between the shadow and GW, or even test new physics.
- We can keep looking for new hints of deviations of the standard picture in strong-gravity.

We are still taking the **first steps for strong-gravity multimessenger astrophysics**, but we there is a lot of work to do…



Thank you for your attention!