When SPH meets SII

Numerical Hydrodynamics for Close Binary Star Systems

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Beyond the Point-Source Approximation

No more privileges.

No more idealized uniform stellar disk.

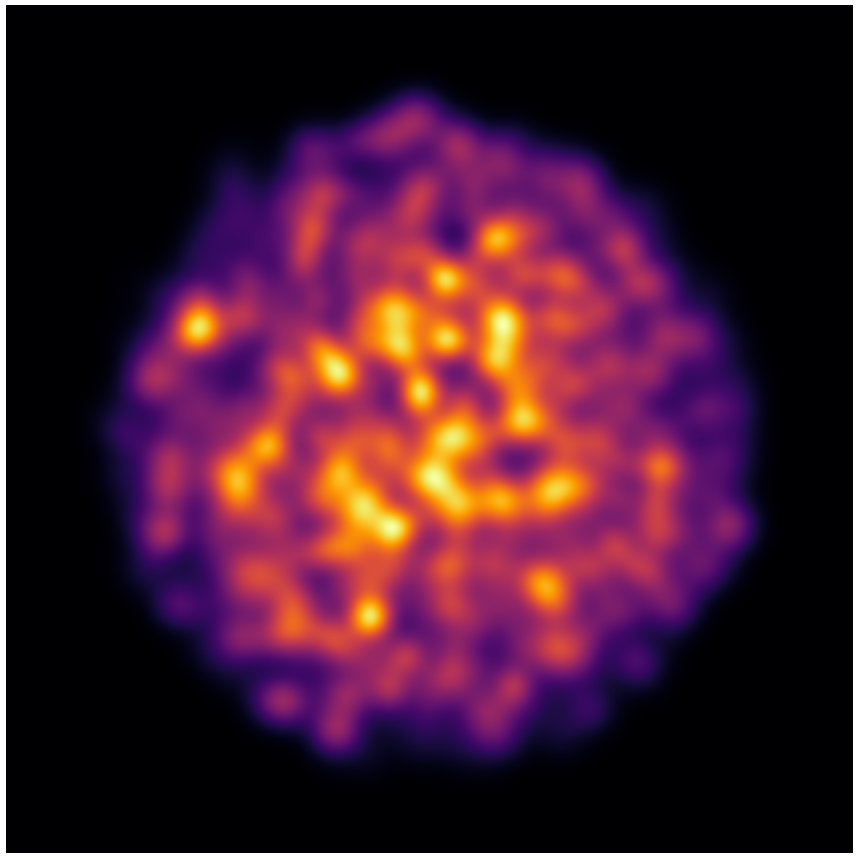
Time to deal with real sources.

Philip Mocz, 2011, MNRAS, 10(1), 1–9
AMUSE (Portegies Zwart et al., 2013)
ChatGPT (OpenAI, GPT-5.6)

From Point to Extended Source

- A star is a collection of particles - **FLUID**.
- It forms a **finite, spatially extended source**.
- Particle's mass in a volume - **DENSITY**.
- Density determines the **pressure** through

$$P = K\rho^{\gamma}.$$



Extended stellar surface

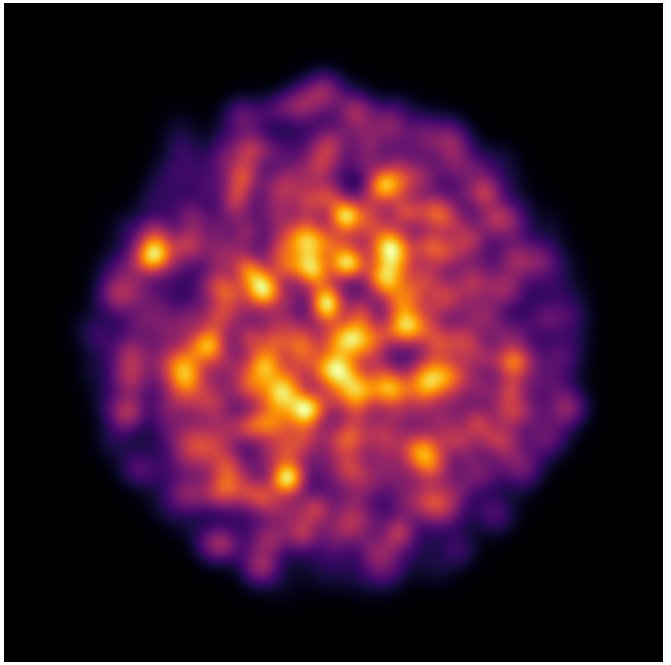
How Do We Study an Extended Source and Signal?

THE PARTICLES

Smoothed Particle Hydrodynamics

Benefit of SPH

- **Mesh-free method**
  - No fixed grid is required.
- **Lagrangian description**
  - Particles move with the stellar fluid.
- **Natural stellar surface**
  - The stellar surface can deform naturally.
- **Complex geometry**
  - Suitable for non-spherical, distorted stars.
- **Particle-based information**
  - Position, density and velocity are available for evolving surface.



Projected surface of a rotating star

Advantage

SPH $\rightarrow$ Natural treatment of deformation in Stellar Objects.

How does the SPH Works for each particles?

SET OF PARTICLES

Local Neighbourhood

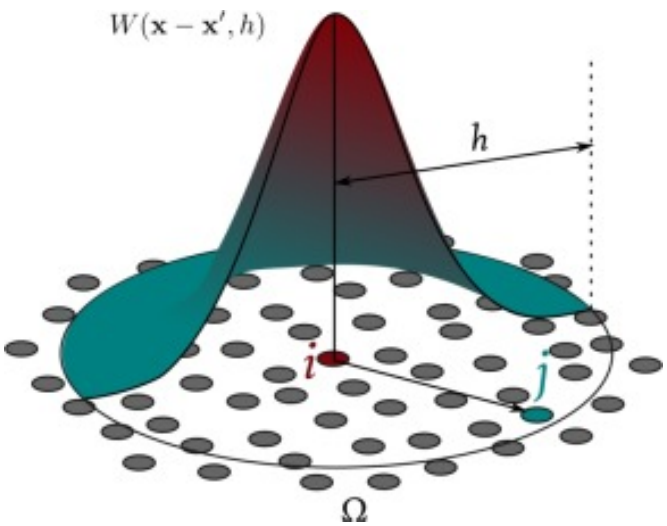
Local Neighbourhood

- Properties of star using **Local Neighbourhood** of particles.
- **Smoothing Length** h sets the size of it.
- Kernel distributes the contribution of **each particle smoothly**:

$$W(r,h) = \frac{1}{\pi^{3/2}h^3} \exp\left(-\frac{r^2}{h^2}\right).$$

$$\int W(r,h)\, d^3r = 1.$$

- Particles closer to the point of interest contribute **more strongly**.



Gaussian smoothing kernel

Density and Acceleration

- **Density** using the smoothing kernel:

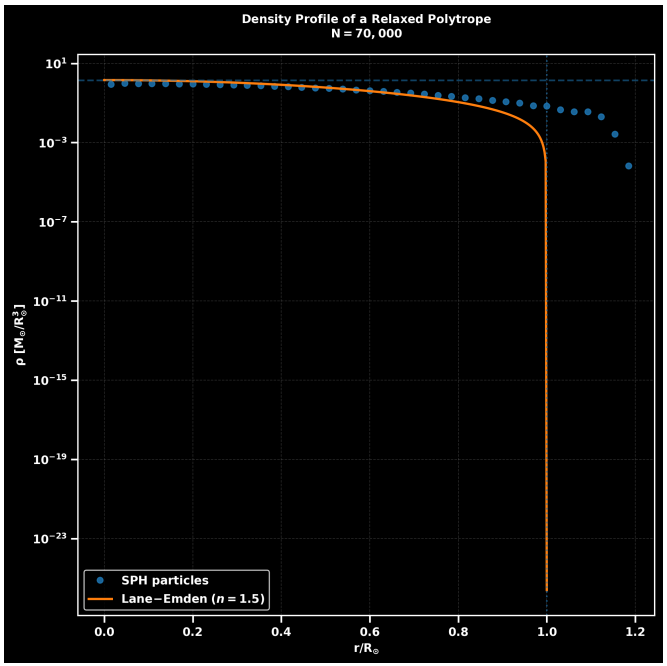
$$\rho_i = \sum_j m_j W(\mathbf{r}_i - \mathbf{r}_j, h).$$

- Pressure gradient produce the **hydrodynamic acceleration**:

$$\mathbf{a}_i^{\mathrm{P}} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij}.$$

- The **particle acceleration** is then determined from the combined forces:

$$\boxed{\mathbf{a}_i = \mathbf{a}_i^{\mathrm{P}} + \mathbf{a}_i^{\mathrm{G}} - \nu \mathbf{v}_i}$$



From local neighbours to Density

Equilibrium: The Balancing Force

- Equilibrium - **pressure balances gravity**.
- The pressure gradient provides an outward force:

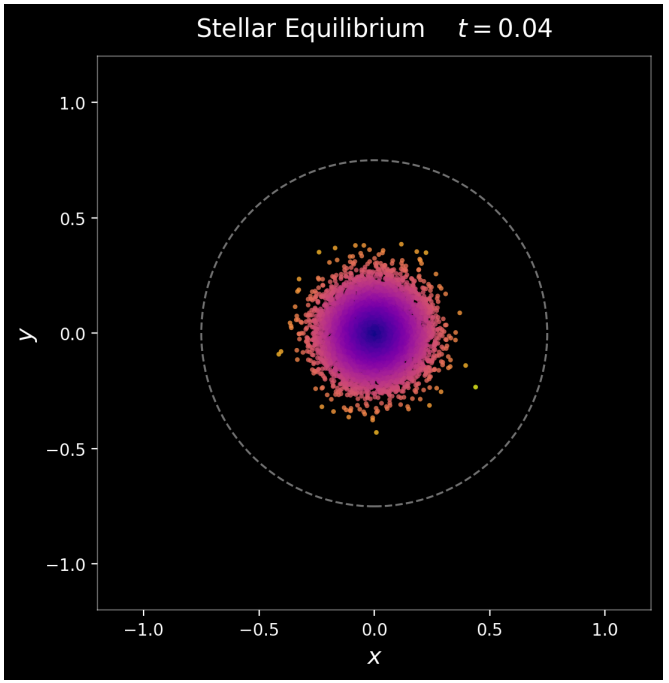
$$\mathbf{a}_P = -\frac{1}{\rho}\nabla P$$

- Gravity provides an inward acceleration:

$$\mathbf{a}_G = -\nabla\Phi$$

- At equilibrium:

$$\boxed{\mathbf{a}_P + \mathbf{a}_G = 0}$$



Reaching in Equilibrium

Evolution in Acceleration

- Particles are evolved using the **leap-frog method**.
- **Velocity** is updated by a half-step:

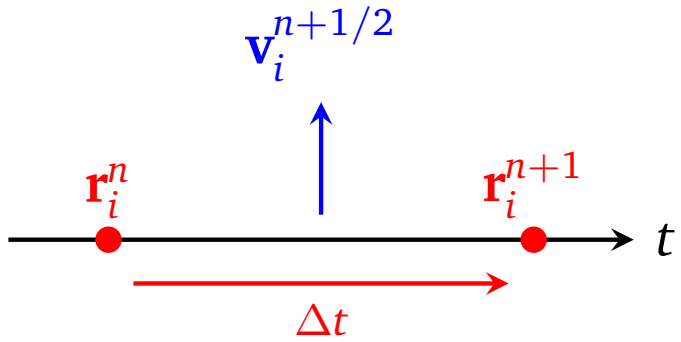
$$\mathbf{v}_i^{n+1/2} = \mathbf{v}_i^n + \frac{\Delta t}{2}\mathbf{a}_i^n$$

- **Position** is updated by a full step:

$$\mathbf{r}_i^{n+1} = \mathbf{r}_i^n + \Delta t\, \mathbf{v}_i^{n+1/2}$$

- **Acceleration** is then recomputed for the next step.

Leap-Frog



Key idea

r: full steps **v**: half steps

Extend the Idea to the Binary Star System

STARS ARE BOUND IN AN ORBIT

Analytical Equations of Signal

$$\mathbf{V(u,v)} = \frac{\mathbf{I_A V_A}(\rho) + \mathbf{I_B V_B}(\rho)\exp\left[\mathbf{2\pi i\,(uX + vY)/D}\right]}{\mathbf{I_A + I_B}}$$

Binary System with SPH Particles

- Each star is represented by a collection of SPH particles, which carries:

$$m_i, \quad \mathbf{r}_i, \quad \mathbf{v}_i, \quad \rho_i, \quad u_i, \quad h_i$$

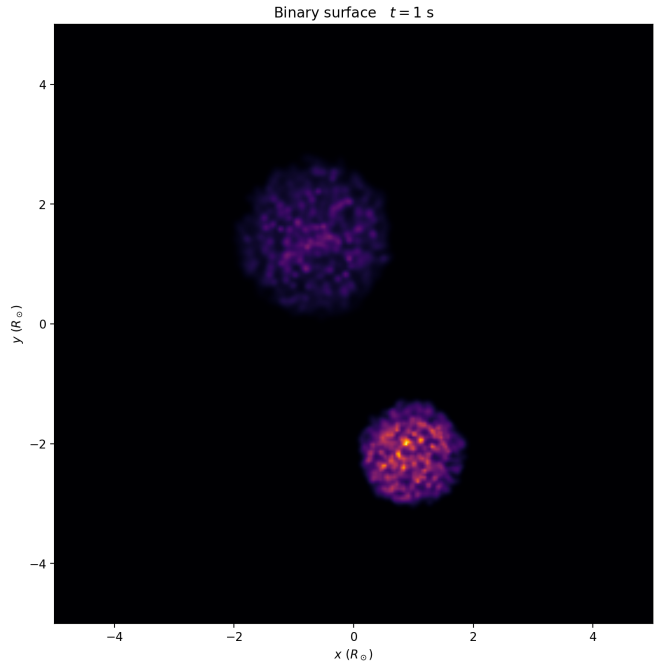
- The particles interact through:
  - gravitational force,
  - pressure force,
  - hydrodynamic interactions.

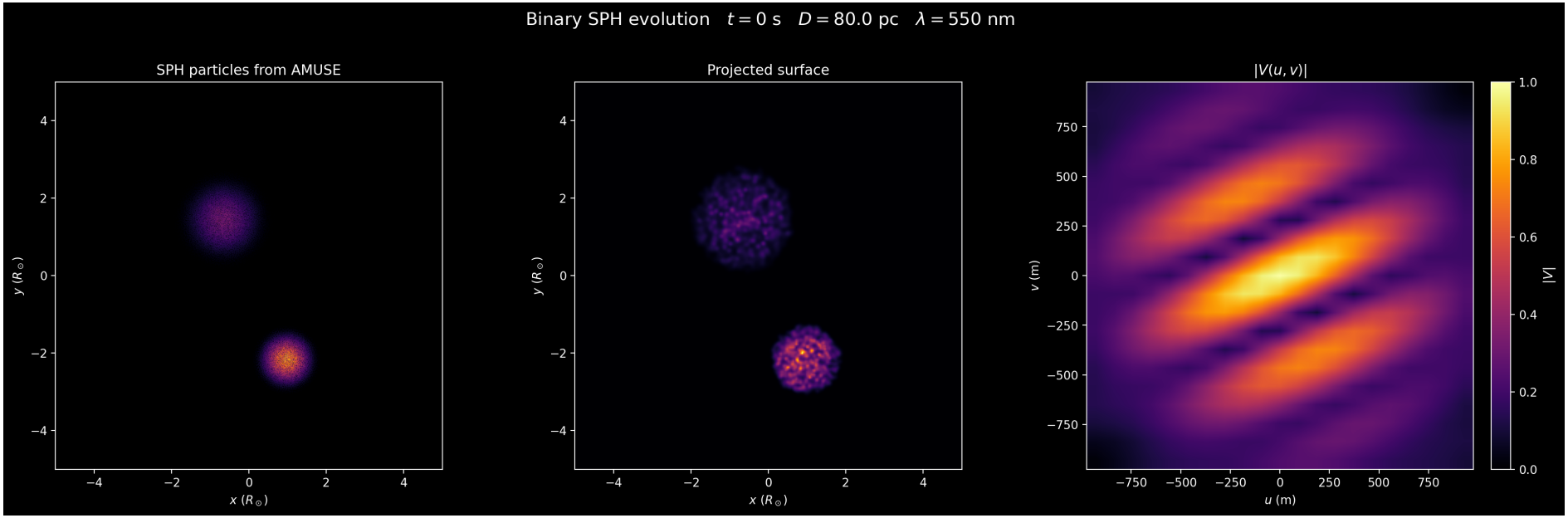
Advantage

The stellar object can be described analytically, while SPH provides a natural representation of the stellar surface and its complex geometry.

Initial Binary Configuration

- Two relaxed polytropic stars in a **orbit**.
- Binary parameters sets the **Separation**.
- Initial velocities provide **orbital motion**.
- At each time step:
  1. Particle positions are updated.
  2. Velocities are updated.
  3. Density and pressure are recalculated.
  4. Gravitational and hydrodynamic forces are evaluated.





The binary star, its projected surface and signal on earth.

Extend the Idea to the Close Binary Star System

THE MODELING PROBLEM OF COMPLEX SYSTEM

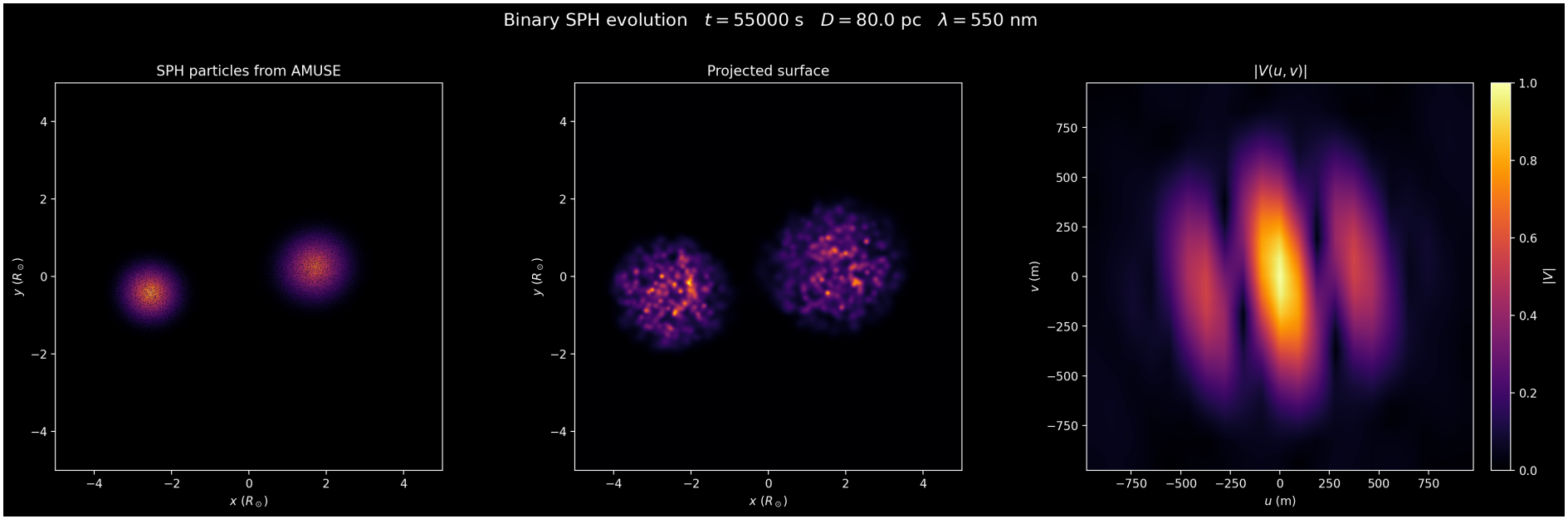
In Absence of Analytical Equations

Why a Close Binary Star System?

- The gravitational field is **no longer spherically symmetric**.
- The stars can undergo **strong tidal interaction**.
- Their **surfaces can become distorted** from spherical symmetry.
- The observed **brightness distribution changes** with orbital phase.

Key Idea

A binary system requires the **dynamics and surface structure** to be evolved together.



Differential Gravity: close binary system, its projected surface and signal.

$$\boxed{\mathbf{a}_i = \mathbf{a}_{P,i} + \mathbf{a}^{(1)}_{G,i} + \mathbf{a}^{(2)}_{G,i} \neq 0}$$

$$\mathbf{a}_{P,i} = -\frac{1}{\rho_i}\nabla P_i, \qquad \mathbf{a}^{(1)}_{G,i} = -\nabla\Phi_1, \qquad \mathbf{a}^{(2)}_{G,i} = -\nabla\Phi_2$$

$$\boxed{\text{Pressure force} + \text{self-gravity} + \text{companion gravity} \neq 0}$$

Thank You!

Thanks to the Audience
and **ChatGPT** too!!

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